Chapter 6 Test Review

AP Calculus AB
Chapter 6 Test Review

Name Heinl 2017
Date

MULTIPLE CHOICE
NO CALCULATOR

1. $\int (\sin(2x) + \cos(2x))\,dx =$ $-\cos(2x) \cdot \frac{1}{2} + \frac{1}{2}\sin 2x + C$

(A) $\frac{1}{2}\cos(2x) + \frac{1}{2}\sin(2x) + C$

(B) $-\frac{1}{2}\cos(2x) + \frac{1}{2}\sin(2x) + C$

(C) $2\cos(2x) + 2\sin(2x) + C$

(D) $2\cos(2x) - 2\sin(2x) + C$

(E) $-2\cos(2x) + 2\sin(2x) + C$

$s(t) = \frac{1}{3}\cdot 3t^3 + \frac{1}{2}\cdot 6t^2 + c$
$2 = c$
$s(t) = t^3 + 3t^2 + 2$
$s(1) = 1 + 3\cdot 1 + 2 = 6$

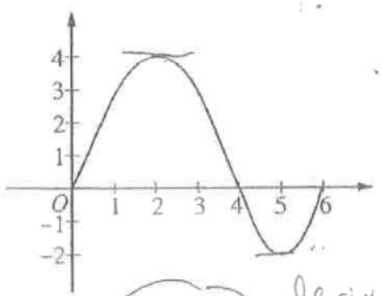
2. A particle moves along the x-axis with velocity given by $v(t) = 3t^2 + 6t$ for time $t \geq 0$. If the particle is at position $x = 2$ at time $t = 0$, what is the position of the particle at time $t = 1$?

(A) 4 $\boxed{\text{(B) } 6}$ (C) 9 (D) 11 (E) 12

3. If $f(x) = \cos(3x)$, then $f'\left(\frac{\pi}{9}\right) =$ $f'(x) = -3\sin\left(3\cdot\frac{\pi}{9}\right) = -3\sin\frac{\pi}{3} = -3\cdot\frac{\sqrt{3}}{2}$

(A) $\frac{3\sqrt{3}}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $-\frac{\sqrt{3}}{2}$ (D) $-\frac{3}{2}$ (E) $-\frac{3\sqrt{3}}{2}$

4.



Graph of f

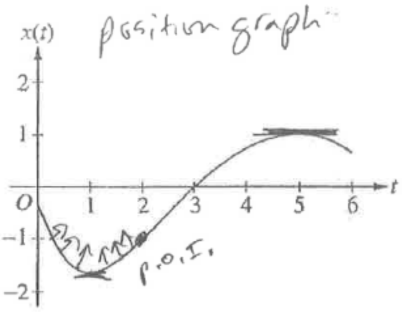
$f(x) = g'(x)$ derivative of g max/mins of g'

The graph of the function f shown above has horizontal tangents at $x = 2$ and $x = 5$. Let g be the function defined by $g(x) = \int_0^x f(t)\,dt$. For what values of x does the graph of g have a point of inflection?

(A) 2 only (B) 4 only (C) 2 and 5 only (D) 2, 4, and 5 (E) 0, 4, and 6

OVER →

5.



A particle moves along a straight line. The graph of the particle's position $x(t)$ at time t is shown above for $0 < t < 6$. The graph has horizontal tangents at $t = 1$ and $t = 5$ and a point of inflection at $t = 2$. For what values of t is the velocity of the particle increasing?

(A) $0 < t < 2$

(B) $1 < t < 5$

(C) $2 < t < 6$

(D) $3 < t < 5$ only

(E) $1 < t < 2$ and $5 < t < 6$

CC ↑

[concave up]

6. If $\int_{-5}^{2} f(x)\,dx = -17$ and $\int_{5}^{2} f(x)\,dx = -4$, what is the value of $\int_{-5}^{5} f(x)\,dx$?

$-\int_{2}^{5} f(x)\,d(x) = 4$

use order of integration rule then Additivity rule

(A) −21 (B) −13 (C) 0 (D) 13 (E) 21

$-17 + 4$

7. If $G(x)$ is an antiderivative for $f(x)$ and $G(2) = -7$, then $G(4) =$

(A) $f'(4)$

(B) $-7 + f'(4)$

(C) $\int_{2}^{4} f(t)\,dt$

(D) $\int_{2}^{4} (-7 + f(t))\,dt$

(E) $-7 + \int_{2}^{4} f(t)\,dt$

FTC, pt 2

$G(x) = \int f(x)$

$\int_{2}^{4} f(x)\,dx = G(4) - G(2)$

$\int_{2}^{4} f(x)\,dx = G(4) - -7$

$\int_{2}^{4} f(x)\,dx = G(4) + 7$

$G(4) = \int_{2}^{4} f(x)\,dx - 7$

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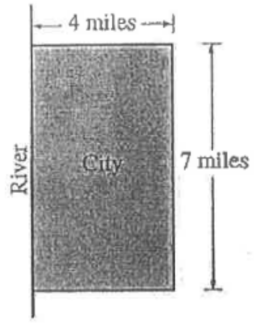
8.



A city located beside a river has a rectangular boundary as shown in the figure above. The population density of the city at any point along a strip x miles from the river's edge is $f(x)$ persons per square mile. Which of the following expressions gives the population of the city?

(A) $\int_0^4 f(x)\,dx$

(B) $7\int_0^4 f(x)\,dx$

(C) $28\int_0^4 f(x)\,dx$

(D) $\int_0^7 f(x)\,dx$

(E) $4\int_0^7 f(x)\,dx$

rate $f(x) = \frac{\text{persons}}{\text{sq. mile}}$

$\int f(x)\,dx$ = persons per mile

$\int_0^4 f(x)\,dx$ = persons per mile from 0 to 4

$7 \cdot \int_0^4 f(x)\,dx$

FREE RESPONSE
NO CALCULATOR

1. For $0 \le t \le 12$, a particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = \cos\left(\frac{\pi}{6}t\right)$. The particle is at position $x = -2$ at time $t = 0$.

(a) For $0 \le t \le 12$, when is the particle moving to the left? $3 < t < 9$

(b) Write, but do not evaluate, an integral expression that gives the total distance traveled by the particle from time $t = 0$ to time $t = 6$. $\int_0^6 |v(t)|\,dt$ absolute value

(c) Find the acceleration of the particle at time t. Is the speed of the particle increasing, decreasing, or neither at time $t = 4$? Explain your reasoning.

The speed is increasing at time $t=4$ b/c velocity & acceleration have the same sign.

(d) Find the position of the particle at time $t = 4$.

a. $v(t) = \cos(\frac{\pi}{6}t) = 0$

$\frac{\pi}{6}t = \frac{\pi}{2}$ or $\frac{\pi}{6}t = \frac{3\pi}{2}$

$t = 3$ or $t = 9$

$\cos(\frac{\pi}{6}t)$: + on (0, 3), − on (3, 9), + on (9, 12)

0 3 Left 9 12

c. $a(t) = v'(t) = -\frac{\pi}{6}\sin(\frac{\pi}{6}t)$

$a(4) = -\frac{\pi}{6}\sin(\frac{2\pi}{3})$

$= -\frac{\sqrt{3}}{12}\pi < 0$

$v(4) = \cos(\frac{2\pi}{3})$

$= -\frac{1}{2} < 0$

d. $x(4) = -2 + \int_0^4 \cos(\frac{\pi}{6}t)\,dt$

$= -2 + \left[\frac{6}{\pi}\sin(\frac{\pi}{6}t)\right]_0^4$

$= -2 + \frac{6}{\pi}\left[\sin(\frac{2\pi}{3}) - 0\right]$

$= -2 + \frac{6}{\pi}\cdot\frac{\sqrt{3}}{2} = \boxed{-2 + \frac{3\sqrt{3}}{\pi}}$

OVER →

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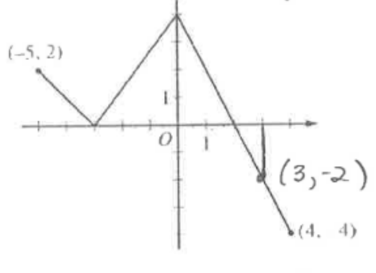
ANSWER KEY

1.

(a) $v(t) = \cos\left(\frac{\pi}{6}t\right) = 0 \Rightarrow t = 3,\ 9$ The particle is moving to the left when $v(t) < 0$. This occurs when $3 < t < 9$.	$2 : \begin{cases} 1 : \text{considers } v(t) = 0 \\ 1 : \text{interval} \end{cases}$
(b) $\int_0^6 \lvert v(t) \rvert \, dt$	1 : answer
(c) $a(t) = -\frac{\pi}{6}\sin\left(\frac{\pi}{6}t\right)$ $a(4) = -\frac{\pi}{6}\sin\left(\frac{2\pi}{3}\right) = -\frac{\sqrt{3}\pi}{12} < 0$ $v(4) = \cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2} < 0$ The speed is increasing at time $t = 4$, because velocity and acceleration have the same sign.	$3 : \begin{cases} 1 : a(t) \\ 2 : \text{conclusion with reason} \end{cases}$
(d) $x(4) = -2 + \int_0^4 \cos\left(\frac{\pi}{6}t\right) dt$ $= -2 + \left[\frac{6}{\pi}\sin\left(\frac{\pi}{6}t\right)\right]_0^4$ $= -2 + \frac{6}{\pi}\left[\sin\left(\frac{2\pi}{3}\right) - 0\right]$ $= -2 + \frac{6}{\pi} \cdot \frac{\sqrt{3}}{2} = -2 + \frac{3\sqrt{3}}{\pi}$	$3 : \begin{cases} 1 : \text{antiderivative} \\ 1 : \text{uses initial condition} \\ 1 : \text{answer} \end{cases}$

NO CALCULATOR

derivative graph



Graph of f = $g'(x)$

2. The function f is defined on the closed interval $[-5, 4]$. The graph of f consists of three line segments and is shown in the figure above.
 Let g be the function defined by $g(x) = \int_{-3}^{x} f(t)\,dt$.

 (a) Find $g(3)$. $\boxed{9}$

 (b) On what ~~open~~ intervals contained in $-5 < x < 4$ is the graph of g both increasing and concave down? Give a reason for your answer. See below.

 (c) The function h is defined by $h(x) = \dfrac{g(x)}{5x}$. Find $h'(3)$. $\boxed{-\frac{1}{3}}$

 (d) The function p is defined by $p(x) = f(x^2 - x)$. Find the slope of the line tangent to the graph of p at the point where $x = -1$. $\boxed{6}$

a. $g(3) = \int_{-3}^{3} f(t)\,dt$
$= \frac{1}{2}\cdot 5\cdot 4 - \frac{1}{2}\cdot 1\cdot 2$
$= 10 - 1$
$= 9$

b. $g'(x) = f(x)$
The graph of g is increasing and concave down on the intervals $-5 < x < -3$ and $0 < x < 2$ b/c $g' = f$ is positive and decreasing on these intervals.

c. $h'(x) = \dfrac{g'(x)\cdot 5x - 5\cdot g(x)}{25x^2}$

$h'(3) = \dfrac{g'(3)\cdot 5\cdot 3 - 5\cdot g(3)}{25(3^2)}$

$= \dfrac{-2\cdot 15 - 5\cdot 9}{225}$

$= -30 - 45$

$= \dfrac{-75}{225} = -\dfrac{1}{3}$

d.
$p'(x) = f'(x^2 - x)(2x-1)$
$p'(-1) = f'((-1)^2 - 1)(2\cdot -1 - 1)$
$= f'(2)(-3)$
$= -2\cdot -3$
$= 6$

CALCULATOR ACTIVE

3. For $0 \le t \le 31$, the rate of change of the number of mosquitoes on Tropical Island at time t days is modeled by $R(t) = 5\sqrt{t}\cos\left(\frac{t}{5}\right)$ mosquitoes per day. There are 1000 mosquitoes on Tropical Island at time $t = 0$.
 → derivative $M(0) = 1000$

 (a) Show that the number of mosquitoes is increasing at time $t = 6$. $R(t) > 0$

 (b) At time $t = 6$, is the number of mosquitoes increasing at an increasing rate, or is the number of mosquitoes increasing at a decreasing rate? Give a reason for your answer. → $R'(t)$

 (c) According to the model, how many mosquitoes will be on the island at time $t = 31$? Round your answer to the nearest whole number. $\int_0^{31} R(t)\,dt = M(31) - M(0)$

 (d) To the nearest whole number, what is the maximum number of mosquitoes for $0 \le t \le 31$? Show the analysis that leads to your conclusion.

$\frac{t}{5} = \frac{\pi}{2} \Rightarrow t = \frac{5\pi}{2}$

$\frac{t}{5} = \frac{3\pi}{2}$
$t = \frac{15\pi}{2}$

a. $R(6) = 5\sqrt{6}\cdot\cos\left(\frac{6}{5}\right)$
$= 4.438 > 0.$
Since $R(6) > 0$, the # of mosquitoes is increasing at $t = 6$.

b. Find $R'(6)$.
$R'(6) = -1.913 < 0$
Since $R'(6) < 0$, the # of mosquitoes is increasing at a decreasing rate

c. $M(31) = M(0) + \int_0^{31} R(t)\,dt$
$M(31) = 1000 + -36 = 964$
There are 964 mosquitoes.

d. $R(t) = 0$ at $t = 0$, $t = 2.5\pi$ or $t = 7.5\pi$
$R(t) > 0$ on $0 < t < 2.5\pi$
$R(t) < 0$ on $2.5\pi < t < 7.5\pi$
$R(t) > 0$ on $7.5\pi < t < 31$
The absolute maximum # of mosquitoes occurs at $t = 2.5\pi$ or at $t = 31$.
$M(2.5\pi) = 1000 + \int_0^{2.5\pi} R(t)\,dt = 1039.351$
There are 964 mosquitoes at $t = 31$, so the max # of mosquitoes is 1039.

2.

(a) $g(3) = \int_{-3}^{3} f(t)\,dt = 6 + 4 - 1 = 9$	1 : answer
(b) $g'(x) = f(x)$ The graph of g is increasing and concave down on the intervals $-5 < x < -3$ and $0 < x < 2$ because $g' = f$ is positive and decreasing on these intervals.	$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$
(c) $h'(x) = \dfrac{5xg'(x) - g(x)5}{(5x)^2} = \dfrac{5xg'(x) - 5g(x)}{25x^2}$ $h'(3) = \dfrac{(5)(3)g'(3) - 5g(3)}{25 \cdot 3^2}$ $= \dfrac{15(-2) - 5(9)}{225} = \dfrac{-75}{225} = -\dfrac{1}{3}$	$3 : \begin{cases} 2 : h'(x) \\ 1 : \text{answer} \end{cases}$
(d) $p'(x) = f'(x^2 - x)(2x - 1)$ $p'(-1) = f'(2)(-3) = (-2)(-3) = 6$	$3 : \begin{cases} 2 : p'(x) \\ 1 : \text{answer} \end{cases}$

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3.

(a) Since $R(6) = 4.438 > 0$, the number of mosquitoes is increasing at $t = 6$.	1 : shows that $R(6) > 0$
(b) $R'(6) = -1.913$ Since $R'(6) < 0$, the number of mosquitoes is increasing at a decreasing rate at $t = 6$.	$2 : \begin{cases} 1 : \text{considers } R'(6) \\ 1 : \text{answer with reason} \end{cases}$
(c) $1000 + \int_0^{31} R(t)\,dt = 964.335$ To the nearest whole number, there are 964 mosquitoes.	$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$
(d) $R(t) = 0$ when $t = 0$, $t = 2.5\pi$, or $t = 7.5\pi$ $R(t) > 0$ on $0 < t < 2.5\pi$ $R(t) < 0$ on $2.5\pi < t < 7.5\pi$ $R(t) > 0$ on $7.5\pi < t < 31$ The absolute maximum number of mosquitoes occurs at $t = 2.5\pi$ or at $t = 31$. $1000 + \int_0^{2.5\pi} R(t)\,dt = 1039.357$, There are 964 mosquitoes at $t = 31$, so the maximum number of mosquitoes is 1039, to the nearest whole number.	$4 : \begin{cases} 2 : \text{absolute maximum value} \\ \quad 1 : \text{integral} \\ \quad 1 : \text{answer} \\ 2 : \text{analysis} \\ \quad 1 : \text{computes interior critical points} \\ \quad 1 : \text{completes analysis} \end{cases}$