Chapter 7 Test Review

AP Calculus AB
Chapter 7 Test Practice (with review)

Name ~~Heinl~~ 2017
Date

Evaluate the following integrals

1. $\int \sin x \cdot e^{\cos x}\,dx$

$u = \cos x$
$\frac{du}{dx} = -\sin x$
$dx = \frac{du}{-\sin x}$
$\int \cancel{\sin x} \cdot e^{u} \cdot \frac{du}{-\cancel{\sin x}}$
$-\int e^{u}\,du =$
$-e^{u} + c = \boxed{-e^{\cos x} + C}$

2. $\int \frac{5x}{(x^2-3)^4}\,dx$

$u = x^2 - 3$
$\frac{du}{dx} = 2x$
$dx = \frac{du}{2x}$
$\int \frac{5x}{u^4} \cdot \frac{du}{2x} =$
$\frac{5}{2}\int \frac{1}{u^4} \cdot du$
$= \frac{5}{2} \cdot \frac{-1}{3} u^{-3} + C$
$= -\frac{5}{6} \cdot \frac{1}{(u^2-3)^3} + c = \boxed{\frac{-5}{6(u^2-3)^3} + C}$

3. $\int_0^1 \left[2(x^2+1)^2 + 5(x^2+1)\right] \cdot 2x\,dx$

$u = x^2 + 1$
$\frac{du}{dx} = 2x$
$dx = \frac{du}{2x}$
$\int_0^1 \left[2u^2 + 5u\right] \cdot \cancel{2x} \cdot \frac{du}{\cancel{2x}}$
$u(1) = 2$
$u(0) = 1$
$\int_1^2 2u^2 + 5u\,du$
$\frac{2}{3}u^3 + \frac{5}{2}u^2 \Big|_1^2 = \left(\frac{16}{3} + 10\right) - \left(\frac{2}{3} + \frac{5}{2}\right)$
$= \frac{46}{3} - \frac{19}{6} = \boxed{\frac{73}{6}}$

4. $\int \frac{1}{\cos^2 x}\,dx$

$= \int \sec^2 x$
$\boxed{= \tan x + C}$

5. $\int_0^2 (x+1)(x^2+2x-3)^3\,dx$

$u = x^2 + 2x - 3$
$\frac{du}{dx} = 2x + 2$
$dx = \frac{du}{2x+2} = \frac{du}{2(x+1)}$
$\int_0^2 \cancel{(x+1)} \cdot u^3 \cdot \frac{du}{2\cancel{(x+1)}}$
$u(2) = 5$
$u(0) = -3$
$\frac{1}{2}\int_{-3}^{5} u^3\,du =$
$\frac{1}{2} \cdot \frac{u^4}{4} \Big|_{-3}^{5} = \frac{1}{2}\left(\frac{625}{4} - \frac{81}{4}\right)$
$= \boxed{68}$

6. $\int_0^{\pi} \cos x \sqrt{\sin x}\,dx$

$u = \sin x$
$\frac{du}{dx} = \cos x$
$dx = \frac{du}{\cos x}$
$\int_0^{\pi} \cancel{\cos x}\sqrt{u} \cdot \frac{du}{\cancel{\cos x}}$
$u(\pi) = 0$
$u(0) = 0$
$\int_0^0 \sqrt{u}\,du = \boxed{0}$

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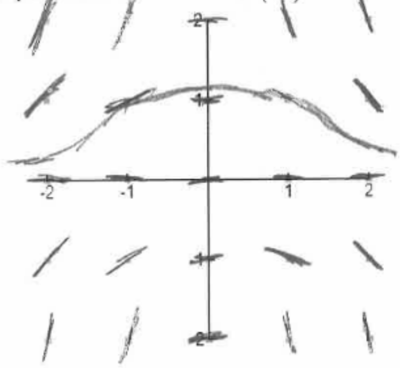
7. Consider the differential equation $\frac{dy}{dx} = \frac{-xy^2}{2}$. Let $y = f(x)$ be the particular solution to this differential equation with the initial condition $f(-1) = 2$.

a) On the axis below, sketch a slope field for the given differential equation at the points indicated. Sketch the particular solution for $f(-1) = 1$

$(-2,2)=4$ $\quad (1,2)=-2$
$(-2,1)=1$ $\quad (1,1)=\frac{-1}{2}$
$(-2,-1)=1$ $\quad (1,-1)=\frac{-1}{2}$
$(-2,-2)=4$ $\quad (1,-2)=-2$
$\quad (2,2)=4$
$(-1,2)=2$ $\quad (2,1)=-1$
$(-1,1)=\frac{1}{2}$ $\quad (2,-1)=-1$
$(-1,-1)=\frac{1}{2}$ $\quad (2,-2)=-4$
$(-1,-2)=2$



b) Find the solution $y = f(x)$ to the given differential equation with the initial condition $f(-1) = 1$.

$$\frac{dy}{dx} = \frac{-xy^2}{2}$$
$$dy = -\frac{xy^2}{2} \cdot dx$$
$$\frac{2}{y^2}\,dy = -x\,dx$$
$$\int 2y^{-2}dy = \int -x\,dx$$
$$-2y^{-1} = -\frac{1}{2}x^2 + C$$
$$-\frac{2}{1} = -\frac{1}{2}\cdot 1^2 + C$$
$$-2 = -\frac{1}{2} + C$$
$$C = -\frac{3}{2}$$

$$-\frac{2}{y} = -\frac{1}{2}x^2 - \frac{3}{2}$$
$$-2 = y\left(-\frac{1}{2}x^2 - \frac{3}{2}\right)$$
$$-2 = y\left(\frac{-x^2-3}{2}\right)$$
$$\frac{-4}{-x^2-3} = y$$
$$\boxed{y = \frac{4}{x^2+3}}$$

8. Find $\frac{dy}{dx}$ if $y = \int_{x^2}^{5}\left(3t^3 - \sin t\right)dt$.

$$y = -\int_5^{x^2}(3t^3 - \sin t)\,dt$$
$$u = x^2$$
$$\frac{du}{dx} = 2x$$
$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$$

$$\frac{dy}{dx} = -\frac{d}{dx}\int_5^{u}(3t^3 - \sin t)\,dt$$
$$= -(3u^3 - \sin u)\cdot 2x$$
$$= \left(-3(x^2)^3 + \sin x^2\right)2x$$
$$\boxed{= 2x\sin x^2 - 6x^7}$$

9. AP Free Response – **CALCULATOR ACTIVE**

At a certain height, a tree trunk has a circular cross section. The radius $R(t)$ of that cross section grows at a rate modeled by the function

$$\frac{dR}{dt} = \frac{1}{16}\left(3 + \sin\left(t^2\right)\right) \text{ centimeters per year}$$

for $0 \le t \le 3$, where time t is measured in years. At time $t = 0$, the radius is 6 centimeters. The area of the cross section at time t is denoted by $A(t)$. $r(0) = 6$

(a) Write an expression, involving an integral, for the radius $R(t)$ for $0 \le t \le 3$. Use your expression to find $R(3)$.

(b) Find the rate at which the cross-sectional area $A(t)$ is increasing at time $t = 3$ years. Indicate units of measure.

(c) Evaluate $\int_0^3 A'(t)\,dt$. Using appropriate units, interpret the meaning of that integral in terms of cross-sectional area.

a. $R(t) = 6 + \int_0^t \frac{1}{16}(3 + \sin(x^2))\,dx$

$R(3) = 6.610$ or 6.611

b. $A(t) = \pi(R(t))^2$

$A'(t) = 2\pi(R(t)) \cdot R'(t)$

$A'(3) = 2\pi(R(3)) \cdot R'(3)$

$= 8.858\ \text{cm}^2/\text{yr}$

c. $\int_0^3 A'(t)\,du = A(3) - A(0) = 24.200$ or 24.201

From time $t=0$ $t=3$ years, the cross-sectional area grows by 24.201 square cm.

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10. **2016 AP Exam – No Calculator**

4. Consider the differential equation $\frac{dy}{dx} = (y-2)(x^2+1)$.

(a) Find $y = g(x)$, the particular solution to the given differential equation with initial condition $g(0) = 5$.

(b) For the particular solution $y = g(x)$ found in part (a), find $\lim_{x \to -\infty} g(x)$.

(c) Let $y = f(x)$ be the particular solution to the given differential equation with initial condition $f(1) = 3$. Find the value of $\frac{d^2y}{dx^2}$ at the point $(1, 3)$. Is the graph of $y = f(x)$ concave up or concave down at the point $(1, 3)$? Give a reason for your answer.

5pts a. $\frac{dy}{dx} = (y-2)(x^2+1)$

$\frac{dy}{y-2} = (x^2+1)dx$

$\int \frac{1}{y-2}\,dy = \int (x^2+1)\,dx$

$\ln|y-2| = \frac{1}{3}x^3 + x + C$

$\ln 3 = C$

$\ln|y-2| = \frac{1}{3}x^3 + x + \ln 3$

$y-2 = \pm e^{(\frac{1}{3}x^3+x)} \cdot e^{\ln 3}$

$y = \pm 3e^{\frac{1}{3}x^3+x} + 2$

$\boxed{y = 3e^{\frac{1}{3}x^3+x} + 2}$

1pt b. $\lim_{x \to -\infty} 3e^{\frac{1}{3}x^3+x} + 2 = \boxed{2}$

approaches 0

3pts c. $\frac{d^2y}{dx^2} = (y-2)(2x) + 1 \cdot \frac{dy}{dx}(x^2+1)$

$\left.\frac{dy}{dx}\right|_{(1,3)} = (3-2)(1^2+1) = 2$

$\left.\frac{d^2y}{dx^2}\right|_{(1,3)} = (3-2)(2\cdot 1) + 1\cdot 2(1^2+1)$

$= 2 + 2\cdot 2$

$= 6$

Because $\left.\frac{d^2y}{dx^2}\right|_{(1,3)} > 0$ and $\frac{d^2y}{dx}$ is continuous, the graph of $y = f(x)$ is concave up at the point $(1, 3)$.