AP Calculus AB Name\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Lesson 2-4: *Rates of Change and Tangent Lines* Date \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Learning Goals:**

* *I can calculate average rates of change (*$A\_{RoC})$ *and instantaneous rates of change (*$I\_{RoC})$*.*
* *I can find equations of tangent lines and normal lines to a curve at a given point.*

<http://webspace.ship.edu/msrenault/GeoGebraCalculus/derivative_avg_ROC.html>



A person stands on a cliff and watches a hot air balloon (the balloon is far away, it is not just really small).

* What is the *average* rate of change of the balloons height between any two points in time?
* What is the *instantaneous* rate of change of the balloons height at one particular moment in time?

Watch the animation and see how the movement of the balloon is related to the graph. Time moves at a steady rate, but the balloon rises and falls at different rates throughout its trip.

1. How would you describe those parts of the graph where the balloon is rising? Falling?

2. What quality does the graph have at times when the balloon is moving quickly? Slowly?

3. Is it clear when the average rate of change should be positive? Negative?

4. How do we calculate the average rate of change in height from time $t\_{1}$ to time $t\_{2}$?

5. Use the graph to estimate the average rate of change from $t=10$ to $t=20$. **Include units**.

6. Graphically represent the $A\_{RoC}$ from problem (5) on the graph.

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7. How would we find the $A\_{RoC}$ from any time $t$ to a time $h$ seconds after $t$, represented $t+h$. Write

formula using function notation. This formula is called the **difference quotient**.

While average rate of change is nice, we are not often interested in average rate of change. Instead, we want to know the rate of change *right now*, at one particular instant.

8. Look now at the slope of the tangent line to the height over time curve at time $t=t\_{1}$. Why is it reasonable to believe that the slope of the tangent line is the instantaneous rate of change of the balloon height at time $t\_{1}$?

9. Sadly ☹, we cannot compute the slope of the tangent line to a curve using the same formula that we developed in problem (7). Why not?

10. The really important question for Calculus then is: *how do we compute the slope of a tangent line?*

11. Find the slope of the tangent line to the parabola $f(x)=x^{2}+7x-3$ at $x=2$.

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12. Find the equation of the tangent line to the equation of the parabola from problem (11).

13. What information do you need to find the equation of a tangent line to a curve? **Knowing how to write the equation of a tangent to a curve is extremely important for the AP Exam!!**

**Practice**

14. Find the slope of the tangent line to the curve  at.

15. Find the equation of the tangent line to *g* at $x=-2$.

The **normal** **line** to a curve at a point is the line perpendicular to the tangent line at that point.

16. Find the equation for the normal line to the curve $g(x)$ at $x=-2$.

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17. Find the slope of the tangent line to the equation of at 

 *Hint #1: Substitute x = 2 at the beginning of the definition to simplify your work.*

*Hint #2: You will need to multiply by the conjugate!*