AP Calculus AB Name\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Lesson 3-2: *Differentiability* Date \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Learning Goals:**

* *I can find where a function is not differentiable and distinguish between corners, cusps, discontinuities, and vertical tangents.*
* *I can approximate a derivative numerically (using a calculator).*

In Lesson 3-1, we learned that a function will not have a derivative at a point where the slopes of the secant lines fail to approach a limit as *h* approaches 0 (for the standard definition of a derivative) or as *x* approaches *a* (for the alternative definition of a derivative). So when exactly is it that a function does not have a derivative? Let’s explore graphically.

Open the Lesson 3-2 Notes Google Slides document. Take notes on the four times that a function whose graph is otherwise smooth will fail to have a derivative at a point on the graph.

**Practice #1**

When is the function not differentiable? Explain.

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 Page 2

Graphically, differentiable functions are **locally linear**; that is, a function that is differentiable at *a* closely resembles its own tangent line very close to *a*. This fact means that a differentiable function will “straighten out” when zoomed in on at the point of differentiability.

**Exploration**

I. Graph the functions and on the standard viewing window. What do you notice?

II. We know that  is not differentiable at . Graph only in your viewing window. Zoom in multiple times, centered at the point . Does the graph of appear to “straighten out”? In more technical terms, does  have local linearity at the point ?

III. Repeat the same process as in problem (II), but this time graph only in your viewing window (reset to the standard window before you start). Is differentiable at ? Explain in terms of what we have learned thus far in this lesson.

**Numerical Derivatives**

While some functions have simple derivatives, others are much more complicated. Throughout this chapter

we will learn how to take derivatives of more complicated functions. That being said, as we have discussed

about 1/3 of the AP Exam is calculator active, so let’s look at how we can use our calculators to find the

derivative at a point.

 Calculator Pointers:

 Page 3

**Practice #2**

**Use your calculator to evaluate the following derivatives.**

1. The function finds the dosage (in mg) of acetaminophen for children ages 1 to 12 years old, where *t* is age in years. What is the rate of change in mg/years for a child that is 4 years old? That is, find .
2. Let . Find the value of the derivative evaluated at.
3. Let . Find .

**Two Important Theorems**

Explain the following two theorems in your own words.

**Theorem:** If *f* has a derivative at , then *f* is continuous at .

 \* *Note that this theorem is NOT an “if and only if” statement.*

Explanation:

**Theorem:** If *a* and *b* are any two points in an interval on which *f* is differentiable, then takes on every value between and.

Explanation:

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 Page 4

**Actual 2014 AP Problem Revisited . . .**

We tried this problem earlier in Chapter 2, but I said that I had adjusted one important word. I have now changed the word back to what it was in the original problem. See if you notice which word was changed, and how that would make your answer need just a bit more explanation.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| t | 0 | 2 | 5 | 8 | 12 |
|  $v\_{A}(t)$ | 0  | 100 | 40 | $$-120$$ | $-150$  |

Train A runs back and forth on an east-west section of railroad track. Train A’s velocity, measured in meters per minute, is given by differentiable function $v\_{A}(t)$, where time *t* is measured in minutes. Selected values for $v\_{A}(t)$ are given in the table above.

Do the data in the table support the conclusion that train A’s velocity is $-100$ meters per minute at some time *t* ? Give a reason for your answer.



**Practice #3**

**** for ****

(A) 1 only (B) 2 only (C) 4 only (D) 1 and 4 (E) 2 and 6

does not exist for ****

(A) 1 only (B) 2 only (C) 1 and 2 (D) 2 and 6 (E) 1, 2, and 6

**Practice #4**

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