AP Calculus AB Name\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Lesson 4-4: *Derivatives of Exponential and Log. Functions* Date \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Learning Goal:**

* *I can find the derivatives of exponential and logarithmic functions.*

I. How Fast Does the Flu Spread?

The spread of a flu at a certain school is modeled by the equation , where is the total number of students infected *t* days after the flu was first noticed [note that many of them may already be well again at time *t*].

1. Estimate the initial number of students infected with the flu.
2. How fast is the flu spreading after 3 days? Include units in your answer. *No Calculator!!*

II. Leaving Milk on the Counter . . .

A glass of cold milk from the refrigerator is left on the counter on a warm summer day. Its temperature *y* (in degrees Fahrenheit) after sitting on the counter *t* minutes is .

1. What is the temperature of the room? Explain mathematically, using limits.
2. What is the temperature of the refrigerator?
3. Determine algebraically when the temperature of the milk reaches 55 degrees.
4. At what rate is the milk warming when its temperature is 55 degrees? Include units.

You will need your calculator for this problem.

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To answer question (c) in “Leaving Milk on the Counter” part (d), you should have used your calculator in order to find the numerical derivative. However, doing so should have peaked your curiosity as to how to find the derivative of exponential functions that are not base *e*. Let’s explore.

As you may or may not have learned in the past, ALL exponential functions can be written as powers of *e*. The formula for doing so is . Remembering that are inverses, this formula can be proven as follows: . Let’s use this equivalence (and the chain rule) to find the derivative of . Complete the following proof of the derivative of . *Remember,* **a** *is a constant!*



**Another Derivative Rule to Memorize!!**



**Example 1**

Find  

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**Practice #1**

1. Algebraically verify your answer to “Leaving Milk on the Counter . . .”, part d.
2. At what point on the graph of the function does the tangent line have a slope of 21?
3. Review: Find if 

**Derivative of **

Another item you may or may not have learned in the past is the *change of base* formula for logarithms. This formula has been rendered somewhat obsolete by the fact that our calculators now can handle bases other than 10, but before that time (which was not that long ago), the change of base formula was necessary. The change of base formula says , where *c* can be any number. Generally, we would choose 10 or *e* because those two bases were built into calculators (or, in times that pre-date even me, log tables in the back of textbooks). Why are we even discussing this fact? The change of base formula helps us develop the formula for the derivative of ****.

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Use the given start below (and the quotient rule) to find the formula for the derivative of ****.



**Yet Another Derivative Rule to Memorize!**



**Practice #2**

1. Find if . b. Find if .

**Logarithmic Differentiation**

Findfor .

Try on another sheet of paper: Findfor .