Math 4 Honors Name \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Lesson 4-7: *DeMoivre’s Theorem & nth Roots Theorem* Date \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Learning Goals:

* *I use can use DeMoivre’s Theorem to calculate powers of complex numbers.*
* *I can find nth roots of complex numbers using the Complex nth Roots Theorem.*

I. **Powers of Complex Numbers**

Review of finding the product of two complex numbers written in trig. form from Lesson 4-6:

If then  = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Let *z = r*(cos *θ* + *i* sin *θ*). Use the product formula to expand the following. After the first couple powers, you should notice a pattern . . . .

*z*2 = *z ·z =*

*z*3 = *z2 ·z =*

*z*4 =

*z*5 =

Generalize the pattern:

*zn* =

The pattern you generalized is known as **DeMoivre’s Theorem**, which enables us to calculate powers of complex numbers written in trigonometric form fairly effortlessly.

**Examples:**

***Use DeMoivre’s Theorem to calculate the exact values of the following. Write your answers in the form of the original number.***

1.  2. 

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II. **Roots of Complex Numbers**

DeMoivre’s Theorem provides an efficient way to calculate powers of any complex number when written in trigonometric form.. Reversing the reasoning suggested by that result provides a way of finding all complex number roots of polynomial equations in the form *zn* – *a* = 0, or equivalently

*zn* = *a*, for any positive integer *n*.



Consider the following equation:

1. How many solutions in the real number system? What are they?
2. How many solutions in the complex number system? What are they?

*I*

Each of the solutions above is called a **4th root of 9**.

Plot all 4th roots of 9 on the graph to the right.

*R*

**Finding All *n*th Roots of a Complex Number**

***Complex nth Roots Theorem:***



**Polar Form*:*** The *n*th roots of  are where *k* = 0, 1, 2, . . . , *n –* 1.



**Trig. From:**The *n*th roots of are , where *k* = 0, 1, 2, . . . , *n –* 1.

**Example:** Use the theorem to find the **4th roots of 9**.

*n = \_\_\_\_\_\_ r = \_\_\_\_\_\_ θ = \_\_\_\_\_\_ k = \_\_\_\_\_\_\_\_\_\_\_\_\_*

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**Examples:**

1. Find the fifth roots of 243 and plot them in the complex plane.

*n = \_\_\_\_\_\_ r = \_\_\_\_\_\_ θ = \_\_\_\_\_\_ k = \_\_\_\_\_\_\_\_\_\_\_\_\_*

*I*

*R*

2. Find the sixth roots of  and plot them in the complex plane.

*n = \_\_\_\_\_\_ r = \_\_\_\_\_\_ θ = \_\_\_\_\_\_ k = \_\_\_\_\_\_\_\_\_\_\_\_\_*

*I*

*R*

**Lesson 4-7 HOMEWORK: *Show all work on another piece of paper.***



















