AP Calculus AB Name\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Lesson 6-4: *The Fundamental Theorem of Calculus, Part 2* Date \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Learning Goals:**

* *I can apply the Fundamental Theorem of Calculus.*
* *I understand the relationship between the derivative and definite integral as expressed in both parts of the Fundamental Theorem of Calculus.*



The graph above is of function , which represents the download speed (in kB/s) as a function of time (in seconds) for a particular file. The file took a total of 3 minutes to download, so the graph represents .

1. While we have no way to evaluate it exactly, explain what represents in the context of this

problem. Include units in your answer.

2. What would , the antiderivative of , represent in the context of this problem.

3. If we knew that and , how many kB of data were downloaded over the

two minutes pictured in the graph?

4. How does your answer to problem (1) relate to your answer to problem (3)??

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**Fundamental Theorem of Calculus, Part 2**

If *f* is continuous at every point of , and *F* is any antiderivative of *f* on , then



This part of the FTC is also called the **Integration Evaluation Theorem**

*Proof:*

Let be *any* antiderivative of *f*.

(1) Then 

(2) We know that 

(3) But 

(4) So 🡪 

(5) Substituting the conclusion from (4) into (1) 🡪

(6) Since *x* is an arbitrary value, we can conclude 

What this theorem tells us is that the definite integral of any continuous function *f* can be calculated without taking limits, calculating Riemann Sums, or any of the other tedious methods – as long as an antiderivative of *f* can be found.

**Example 1 – Evaluating Integrals**

*Evaluate the following integrals using antiderivatives*

  

\* Note that in the above examples we are *evaluating integrals using antiderivatives*, which is different than *finding the antiderivative* . . .

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**Area**

We already know that the definite integral can be interpreted as the *net* (signed) area between the graph of a function and the *x*-axis. We can therefore compute areas using antiderivatives, but we must again be careful to distinguish net (signed) area from *total* area. In our textbook, the word “area” will be taken to mean total area.

*How to Find Total Area Analytically*

To find the area between the graph of ** and the *x*-axis over the interval analytically:

1. Partition with the zeros of *f.*

2. Integrate *f* over each subinterval.

3. Add the absolute values of the integrals.

*How to Find Total Area Numerically (with your calculator)*

To find the area between the graph of ** and the *x*-axis over the interval numerically

 \*\*\*\*\*\* Evaluate . (Use the absolute value template!)

**Example 2**

Find the area of the region between the curve , , and the *x*-axis analytically

NO CALCULATOR!

**Example 3**

Find the area of the region between the curve , , and the *x*-axis using your calculator.

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**Practice 1**

*Evaluate each integral using FTC part 2 without using your calculator. Check your answer using your calculator.*

1.  2. 

**Example 4** *(two ways)*

If *F* is the antiderivative of such that, find .

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**Practice 2 (***use the FTC!***)**

1.If *F* is the antiderivative of such that, find . ***Calculator active*!**

2. If *f* is the antiderivative ofsuch that , find . ***NO CALCULATOR!***

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**Practice 3 *NO CALCULATOR***

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1.

****

2.

****

3.

4. Find the average value of on .

 (A)  (B)  (C)  (D)  (E) correct answer not given

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**Practice 4 (NO CALCULATOR)**

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**Practice 5** (*note how you need FTC in part d!!*) – **CALCULATOR ACTIVE!**

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