Math 4 Honors Name \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Lesson 7-4: *Related Rates* Date \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Learning Goal:**

* *I can use implicit differentiation to solve related rates problems.*

In differential calculus, **related rates** problems involve finding a rate at which a quantity changes by relating that quantity to other quantities whose rates of change are known.

* The rate of change is usually with respect to \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.
* Because problems to which related rate methods apply often have nested functions, solving these problems often requires *implicit differentiation*.

**Process for solving a related rates problem:**

1. Identify the known variables, including rates of change and the rate of change that is to be found. (Drawing a picture or representation of the problem can help to keep everything in order)
2. Construct an equation relating the quantities whose rates of change are known to the quantity whose rate of change is to be found.
3. Differentiate both sides of the equation with respect to time (or other rate of change). *Implicit differentiation* is employed at this step.
4. Substitute the known rates of change and the known quantities into the equation.
5. Solve for the wanted rate of change.

**Day 1 Examples:**

1. A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of ¼ ft/sec, how fast is the top of the ladder sliding when the foot of the ladder is 6ft from the wall?

2. A spherical balloon is inflated at the rate of 10 in3/sec. At what rate is the radius increasing at the instant which the radius is 3"?

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3. A box is 6 feet long, 3 feet across the top, and 2 feet deep. If water flows in at the rate of 9 ft3/min, how fast is the surface rising when the water is 1 foot deep?

**Day 2 Examples:**

4. Two airplanes in flight cross above a town at 1:00 PM. One plane travels east at 400 mph, the other

north at 300 mph. At what rate does the distance between the planes change at 3:00 PM?

5. A 6' tall man walks at the rate of 4'/sec towards a lamp that is 18' above the ground. At what rate is the length of his shadow changing when he is 10' from the lamppost?

6. Sand falls onto a conical pile at the rate of 10 ft3/min. The radius of the base of the pile is always equal to one-half of its altitude. How fast is the altitude of the pile increasing when it is 5 feet deep?

**Homework Problems**

**Day 1: 1 – 5**

**Day 2: 6 – 10**

*SHOW ALL WORK ON ANOTHER SHEET OF PAPER.*

*Keep all answers exact!*

1. A rectangular trough is 8 feet long, 2 feet across the top, and 4 feet deep. If water flows in at the rate

of 2 ft3/min., how fast is the surface rising when the water is 1 foot deep?

2. A liquid is flowing into a vertical cylindrical tank of radius 6 feet at the rate of 8 ft3/min. How fast is the surface rising?

3. A woman 5 feet tall walks at the rate of 4 ft/sec directly away from a streetlight, which is 20 feet above the street. (a) At what rate is the tip of her shadow changing? (b) At what rate is the length of her shadow changing?

4. A balloon is rising vertically over a point A on the ground at the rate 15 ft/sec. A point B on the ground is level with and 30 feet away from A. When the balloon is 40 feet from A, at what rate is its distance from B changing?

5. A ladder 20 feet long leans against a house. Find the rate at which (a) the top of the ladder is moving downward if its foot is 12 feet from the house and moving away at the rate of 2 ft/sec, (b) the slope of the ladder decreases.

6. Water is being withdrawn from a conical reservoir 3 feet is radius and 10 feet deep at 4 ft3/min. (a) How fast is the surface falling when the depth of the water is 6 feet? (b) How fast is the radius of this surface diminishing?

7. A barge whose deck is 10 feet below the level of a dock is being drawn in by means of a cable attached to the deck and passing through a ring on the dock. When the barge is 24 feet away and approaching the dock at 3/4 ft/sec, how fast is the cable being pulled in? (Neglect any sag in the cable.)

8. A girl is flying a kite at a height of 150 feet. If the kite moves horizontally away from the girl at the rate of 20 ft/sec, how fast is the string being let out when the kite is 250 feet away from her?

9. A train, starting at 11 AM travels east at 45 mi/hr while another, starting at noon from the same point, travels south at 60 mi/hr. How fast are they separating at 3:00 PM?

10. A solution is passing through a conical filter 24 inches deep and 16 inches across the top into a cylindrical vessel of diameter 12inches. At what rate is the level of the solution in the cylinder rising if when the depth of the solution in the filter is 12 inches, its level is falling at the rate of 1 in/min?

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Examples Solutions Homework Solutions

1.  ft/sec 1. ft/min

2.  in/sec 2. ft/min

3.  ft/min 3. (a)  ft/sec

4. 500 mi/hr (b)  ft/sec

5. 2 ft/sec 4. 12 ft/sec

6.  ft/min 5. (a)  ft/sec

(b) -ft/ sec

6. (a)  ft/min

(b)  ft/min

7.  ft/sec

8. 16 ft/sec

9.  10.  in/min