AP Calculus AB
Lessons 3-3 & **4-4** *Rules for Differentiation, Part 1* (Extension)

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1. Let u and v be differentiable functions. If $\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{13}{(2x+3)^2}$ and $u'=3$, find u and v.

$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{vu'-uv'}{v^2}$

$v=2x+3$
$v'=2$
$u=?$
$u'=3$

$\frac{13}{(2x+3)^2}=\frac{(2x+3)(3)-u(2)}{(2x+3)^2}$

$13=(2x+3)(3)-2u$
$13=6x+9-2u$

$\frac{4}{-2}-\frac{6x}{-2}=-\frac{2u}{-2}$

$-2+3x=u$

$\boxed{u=3x-2 \quad v=2x+3}$

Check: $(2x+3)(3)-(3x-2)(2)=$
$6x+9-6x+4=13$ ✓

2. Which of the following could be true if $f''(t)=t^{-\frac{2}{3}}$

I. $f(t)=\frac{9}{4}t^{\frac{4}{3}}$ ✓

II. $f'(t)=7-3t^{\frac{1}{3}}$ X

III. $f'''(t)=-\frac{2}{3}t^{-\frac{5}{3}}$ ✓

(A) I and II only (B) I and III (C) I only

(D) III only (E) I, II, and III

I. $f'(t)=3t^{\frac{1}{3}}$
$f''(t)=t^{-\frac{2}{3}}$

II. $f'(t)=7-3t^{\frac{1}{3}}$
$f''(t)=0-t^{-\frac{2}{3}}$
$=-t^{-\frac{2}{3}}$

III. $f'''(t)=-\frac{2}{3}t^{-\frac{5}{3}}$

$=-\frac{2}{3}\cdot\frac{-2}{3}t^{-\frac{5}{3}+1}$

$=1\cdot t^{-\frac{2}{3}}$

$=t^{-\frac{2}{3}}$

or $f''(t)=t^{-\frac{2}{3}}$
$f'''(t)=-\frac{2}{3}t^{-\frac{2}{3}-1}$
$=-\frac{2}{3}t^{-\frac{5}{3}}$

OVER →

3. If $y = \frac{\overset{u}{\ln x}}{x^2 e^x}$, verify that $\frac{dy}{dx} = \frac{1 - x\ln x - 2\ln x}{x^3 e^x}$.

$$\frac{dy}{dx} = \frac{x^2 e^x \cdot \frac{1}{x} - \ln x\left(x^2 e^x + 2x e^x\right)}{(x^2 e^x)^2}$$

$$= \frac{x e^x - \ln x\,(x^2 e^x) - \ln x\,(2x e^x)}{(x^2 e^x)^2}$$

☆ GCF of num: xe^x

$$= \frac{\cancel{x}\cancel{e^x}\left(1 - \ln x\,(x) - \ln x\,(2)\right)}{x^{3+1} e^{2x} e^x}$$

$$= \frac{1 - x\ln x - 2\ln x}{x^3 e^x}$$

4. Use the following table to answer the below questions:

x	f	f'	g	g'
0	2	1	5	−4
1	3	2	3	−3
2	5	3	1	−2
3	10	4	0	−1

$A = f + 2g$

$A' = f' + 2g'$

If $A = f + 2g$, then $A'(3) =$

(A) −2 (B) 2 (C) 7 (D) 8 (E) 10

$A'(3) = f'(3) + 2 \cdot g'(3)$

$= 4 + 2 \cdot -1 = 2$

If $B = f \cdot g$, then $B'(2) =$

(A) −20 (B) −7 (C) −6 (D) −1 (E) 13

$B' = f' \cdot g + g' \cdot f$

$B'(2) = f'(2) \cdot g(2) + g'(2) \cdot f(2)$

$= 3 \cdot 1 + -2 \cdot 5 = 3 - 10 = -7$