

Half-Life

Complete the following half-life problems be sure to show your thinking when solving problems.

PROBLEM

If 100.0 g of carbon-14 decays until only 25.0 g of carbon is left after 11,460 y, what is the half-life of carbon-14?

SOLUTION

Step 1: Write down the equation relating half-life, the number of half-lives, and the decay time, and rearrange it to solve for half-life.

$$\text{total time of decay} = \text{number of half-lives} \times \frac{\text{number of years}}{\text{half-life}}$$

$$\frac{\text{number of years}}{\text{half-life}} = \frac{\text{total time of decay}}{\text{number of half-lives}}$$

Step 2: Calculate how many half-lives have passed during the decay of the 100.0 g sample.

$$\text{fraction of sample remaining} = \frac{\text{final mass of sample}}{\text{initial mass of sample}} = \frac{25.0 \text{ g}}{100.0 \text{ g}} = \frac{1}{4}$$

$$\text{after one half-life} = \frac{1}{2}; \text{ after two half-lives} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \text{ of sample}$$

Two half-lives have passed.

Step 4: Calculate the half-life.

$$\frac{\text{number of years}}{\text{half-life}} = \frac{11,460 \text{ y}}{2 \text{ half-lives}} = \frac{5,730 \text{ y}}{\text{half-life}}$$

$$\text{half-life of carbon-14} = 5,730 \text{ y}$$

Practice

1. What is the half-life of a 100.0 g sample of nitrogen-16 that decays to 12.5 g of nitrogen-16 in 21.6 s?
2. All isotopes of technetium are radioactive, but they have widely varying half-lives. If an 800.0 g sample of technetium-99 decays to 100.0 g of technetium-99 in 639,000 y, what is its half-life?
3. A 208 g sample of sodium-24 decays to 13.0 g of sodium-24 within 60.0 h. What is the half-life of this radioactive isotope?

4. If the half-life of iodine-131 is 8.10 days, how long will it take a 50.00 g sample to decay to 6.25 g?
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5. The half-life of hafnium-156 is 0.025 s. How long will it take a 560 g sample to decay to one-fourth its original mass?
6. Chromium-48 has a short half-life of 21.6 h. How long will it take 360.00 g of chromium-48 to decay to 11.25 g?
7. Potassium-42 has a half-life of 12.4 hours. How much of an 848 g sample of potassium-42 will be left after 62.0 hours?
8. Carbon-14 has a half-life of 5,730 y. How much of a 144 g sample of carbon-14 will remain after 1.719×10^4 y?
9. If the half-life of uranium-235 is 7.04×10^8 y and 12.5 g of uranium-235 remain after 2.82×10^9 y, how much of the radioactive isotope was in the original sample?

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$$\frac{\text{number of years}}{\text{half-life}} = \frac{\text{total time of decay}}{\text{number of half-lives}}$$

Step 2: Calculate how many half-lives have passed during the decay of the 100.0 g sample.

$$\text{fraction of sample remaining} = \frac{\text{final mass of sample}}{\text{initial mass of sample}} = \frac{25.0 \text{ g}}{100.0 \text{ g}} = \frac{1}{4}$$

$$\text{after one half-life} = \frac{1}{2}; \text{ after two half-lives} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \text{ of sample}$$

Two half-lives have passed.

Step 4: Calculate the half-life.

$$\frac{\text{number of years}}{\text{half-life}} = \frac{11,460 \text{ y}}{2 \text{ half-lives}} = \frac{5,730 \text{ y}}{\text{half-life}}$$

$$\text{half-life of carbon-14} = 5,730 \text{ y}$$

Practice

1. What is the half-life of a 100.0 g sample of nitrogen-16 that decays to 12.5 g of nitrogen-16 in 21.6 s?

$$\frac{12.5 \text{ g}}{100 \text{ g}} = .125 = \frac{1}{8}$$

$$\frac{21.6 \text{ s}}{3 \text{ half-lives}} = 7.2 \text{ s / half life}$$

$$100 \xrightarrow[1st]{\frac{1}{2}} 50 \xrightarrow[2nd]{\frac{1}{4}} 25 \xrightarrow[3rd]{\frac{1}{8}} 12.5 \text{ g}$$

2. All isotopes of technetium are radioactive, but they have widely varying half-lives. If an 800.0 g sample of technetium-99 decays to 100.0 g of technetium-99 in 639,000 y, what is its half-life?

$$\frac{100 \text{ g}}{800 \text{ g}} = \frac{1}{8} = 3 \text{ Half-life}$$

$$\frac{639,000 \text{ yrs}}{3 \text{ half lives}} = 213,000 \text{ yr}$$

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3. A 208 g sample of sodium-24 decays to 13.0 g of sodium-24 within 60.0 h. What is the half-life of this radioactive isotope?

$$\frac{13 \text{ g}}{208 \text{ g}} = .0625 = 4 \text{ half-life}$$

$$\frac{60.0 \text{ hours}}{4 \text{ half-lives}} = 15 \text{ hrs}$$

$$208 \xrightarrow[1st]{\frac{1}{2}} 104 \xrightarrow[2nd]{\frac{1}{4}} 52 \xrightarrow[3rd]{\frac{1}{8}} 26 \xrightarrow[4th]{\frac{1}{16}} 13$$

4. If the half-life of iodine-131 is 8.10 days, how long will it take a 50.00 g sample to decay to 6.25 g?

$$\frac{6.25 \text{ g}}{50.0 \text{ g}} = .125 = \frac{1}{8} = 3 \text{ Half-lives}$$

$$8.10 \text{ days} \times 3 \text{ Half-lives} = 24.3 \text{ days}$$

5. The half-life of hafnium-156 is 0.025 s. How long will it take a 560 g sample to decay to one-fourth its original mass?

$$\frac{1}{4} = 2 \text{ half-lives}$$

$$= .025 \text{ s} \times 2 \text{ half-lives} = .050 \text{ sec.}$$

6. Chromium-48 has a short half-life of 21.6 h. How long will it take 360.00 g of chromium-48 to decay to 11.25 g?

$$\frac{11.25 \text{ g}}{360.00 \text{ g}} = .03125$$

$$.03125 = \frac{1}{32} = 5 \text{ Half-lives}$$

$$21.6 \text{ hours} \times 5 \text{ Half-lives}$$

$$= 108 \text{ hours}$$

7. Potassium-42 has a half-life of 12.4 hours. How much of an 848 g sample of potassium-42 will be left after 62.0 hours?

$$\frac{62 \text{ h}}{12.4 \text{ h}} = 5 \text{ Half-lives}$$
$$= \frac{1}{32}$$

$$\frac{1}{2} \quad \frac{1}{4} \quad \frac{1}{8} \quad \frac{1}{16} \quad \frac{1}{32}$$

$$848 \text{ g} \times \frac{1}{32} = 26.5 \text{ g left}$$

8. Carbon-14 has a half-life of 5,730 y. How much of a 144 g sample of carbon-14 will remain after 1.719×10^4 y?

$$\frac{1.719 \times 10^4 \text{ yr}}{5,730 \text{ yr}} = 3 \text{ half-lives}$$
$$= \frac{1}{8}$$

$$144 \text{ g} \times \frac{1}{8} = 18 \text{ grams left}$$

9. If the half-life of uranium-235 is 7.04×10^8 y and 12.5 g of uranium-235 remain after 2.82×10^9 y, how much of the radioactive isotope was in the original sample?

$$\frac{2.82 \times 10^9 \text{ yr}}{7.04 \times 10^8 \text{ yr}} = 4 \text{ half-lives}$$
$$= \frac{1}{16}$$

$$12.5 \text{ g} \times 16 = 200 \text{ g}$$