

Answer Key

PROBABILITY

#24

PROBABILITY is the likelihood that a specific outcome will occur, represented by a number between 0 and 1.

There are two categories of probability.

THEORETICAL PROBABILITY is calculated probability. If every outcome is equally likely, it is the ratio of outcomes in an event to all possible outcomes.

$$\text{Theoretical probability} = \frac{\text{number of outcomes in the specified event}}{\text{total number of possible outcomes}}$$

EXPERIMENTAL PROBABILITY is the probability based on data collected in experiments.

$$\text{Experimental probability} = \frac{\text{number of outcomes in the specified event}}{\text{total number of possible outcomes}}$$

Example 1

There are three pink pencils, two blue pencils, and one green pencil. If one pencil is picked randomly, what is the theoretical probability it will be blue?

- Find the total number of possible outcomes, that is, the total number of pencils. $3 + 2 + 1 = 6$
- Find the number of specified outcomes, that is, how many pencils are blue? 2
- Find the theoretical probability. $P(\text{blue pencil}) = \frac{2}{6} = \frac{1}{3}$ (You may reduce your answer.)

Example 2

Jayson rolled a die twelve times. He noticed that three of his rolls were fours.

- a. What is the theoretical probability of rolling a four?

Because the six sides are equally likely and there is only one four, $P(4) = \frac{1}{6}$.

- b. What is the experimental probability of rolling a four?

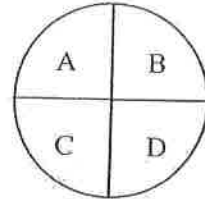
There were three fours in twelve rolls. The experimental probability is: $P(4) = \frac{3}{12} = \frac{1}{4}$.

Example 3

On the spinner, what is the probability of spinning an A or a B?

The probability of an A is $\frac{1}{4}$. The probability of a B is $\frac{1}{4}$. Add the two probabilities for the combined total.

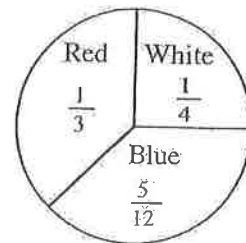
$$\frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2} \quad P(\text{A or B}) = \frac{1}{2}$$



Example 4

What is the probability of spinning red or white?

- We know that $P(R) = \frac{1}{3}$ and $P(W) = \frac{1}{4}$.
 - Add the probabilities together. $\frac{1}{3} + \frac{1}{4} = \frac{7}{12}$
- $P(R \text{ or } W) = \frac{7}{12}$



Problems *★ Answer these problems! ★*

- There are five balls in a bag: 2 red, 2 blue, and 1 white. What is the probability of randomly choosing a red ball? *2/5*
- In a standard deck of cards, what is the probability of drawing an ace? *4/52 = 1/13*
- A fair die numbered 1, 2, 3, 4, 5, 6 is rolled. What is the probability of rolling an odd number? *3/6 = 1/2*
- In the word "probability," what is the probability of selecting a vowel? *4/11*
- Anna has some coins in her purse: 5 quarters, 3 dimes, 2 nickels, and 4 pennies. *= total = 14*
 - What is the probability of selecting a quarter? *5/14*
 - What is the probability she will select a dime or a penny? ** add them together 7/14 = 1/2*
- Tim has some gum drops in a bag: 20 red, 5 green, and 12 yellow. *total = 37*
 - What is the probability of selecting a green? *5/37*
 - What is the probability of not selecting a red? *17/37*

7. What is the probability of spinning:

a. Pink or blue? $13/40$

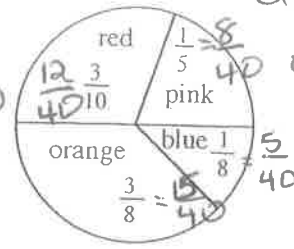
b. Orange or pink? $23/40$

c. Red or orange?

$$27/40$$

d. Red or blue?

$$17/40$$



* common denominator

$$80/40$$

8. What is the probability of spinning:

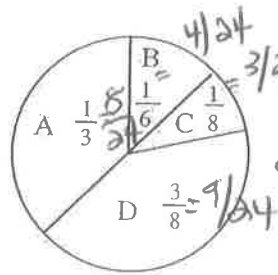
a. A or C? $11/24$

b. B or C? $7/24$

c. A or D? $17/24$

d. B or D? $13/24$

e. A, B, or C? $\frac{15}{24}$ or $\frac{5}{8}$



* common denominator

$$80/24$$

COMPOUND PROBABILITY

When multiple outcomes are specified, and either or several of them may happen but not both, find the probability of each specified outcome and add their probabilities.

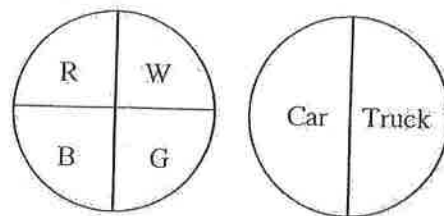
If the desired outcome is a compound event, that is, it has more than one characteristic (as in Example 1, a red car), find the probability of each outcome (in Example 1, the probability of "red," then the probability of "car") and multiply the probabilities.

Example 1

To find the $P(\text{Red and Car})$:

- Find the probability of Red: $\frac{1}{4}$.
- Find the probability of Car: $\frac{1}{2}$.
- Multiply them together.

$$\frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8} \quad P(\text{Red and Car}) = \frac{1}{8}$$



The solution can also be shown with a probability rectangle.

- There are four equally likely choices of color on the first spinner. The rectangle is divided vertically into four equal parts, each labeled with its probability and color.
- There are two equally likely choices of vehicle on the second spinner. The rectangle is divided horizontally into two equal parts, each labeled with its probability and vehicle.
- Write the probability of spinning each combination in its section of the rectangle, multiplying the probability to get the area of the rectangular subproblem as in a multiplication table.
- The $P(\text{Red and Car}) = \frac{1}{8}$.

	Red $\frac{1}{4}$	White $\frac{1}{4}$	Blue $\frac{1}{4}$	Green $\frac{1}{4}$
Car $\frac{1}{2}$	RC $\frac{1}{8}$	WC $\frac{1}{8}$	BC $\frac{1}{8}$	GC $\frac{1}{8}$
Truck $\frac{1}{2}$	RC $\frac{1}{8}$	WC $\frac{1}{8}$	BC $\frac{1}{8}$	GC $\frac{1}{8}$

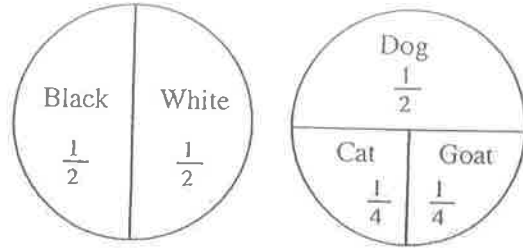
Example 2

What is the probability of spinning a black dog or a black cat?

$$P(\text{BCat}) = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$$

$$P(\text{BDog}) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

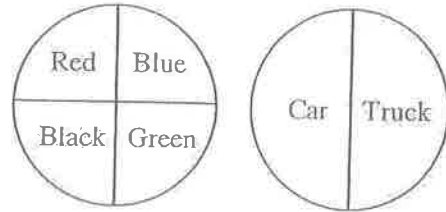
$$P(\text{BCat or BDog}) = \frac{1}{8} + \frac{1}{4} = \frac{1}{8} + \frac{2}{8} = \frac{3}{8}$$



Problems *★ Answer these problems*

1. If each section in each spinner is the same size, what is the probability of getting a Black Truck?

$$\frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}$$



2. Bipasha loves purple, pink, turquoise and black, and has a blouse in each color. She has two pairs of black pants and a pair of khaki pants. If she randomly chooses one blouse and one pair of pants, what is the probability she will wear a purple blouse with black pants?

3 pairs of pants →

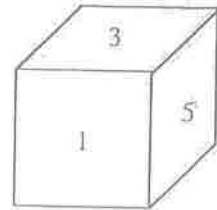
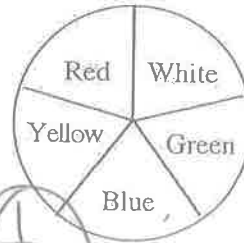
$$\frac{1}{4} \cdot \frac{2}{3} = \frac{2}{12} = \frac{1}{6}$$

4 blouses

3. The spinner at right is divided into five equal regions. The die is numbered from 1 to 6. What is the probability of:

a. Rolling a red 5? $\frac{1}{5} \cdot \frac{1}{6} = \frac{1}{30}$

b. A white or blue even number? $\frac{1}{5} + \frac{1}{5} = \frac{2}{5} \cdot \frac{2}{6} = \frac{4}{30} = \frac{2}{15}$



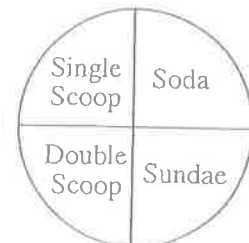
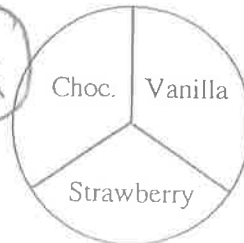
4. What is the probability Joanne will win a:

a. Chocolate double scoop? $\frac{1}{3} \cdot \frac{1}{4} = \frac{1}{12}$

b. Chocolate or strawberry sundae? $\frac{1}{3} + \frac{1}{3} = \frac{2}{3} \cdot \frac{1}{4} = \frac{2}{12} = \frac{1}{6}$

c. Chocolate double scoop or chocolate sundae?

$$\left(\frac{1}{3} \cdot \frac{1}{4} \right) + \left(\frac{1}{3} \cdot \frac{1}{4} \right) = \frac{1}{12} + \frac{1}{12} = \frac{2}{12} = \frac{1}{6}$$



7 shirts
5 pants

5. Jay is looking in his closet, trying to decide what to wear. He has 2 red t-shirts, 2 black t-shirts, and 3 white t-shirts. He has 3 pairs of blue jeans and 2 pairs of black pants.

a. What is the probability of his randomly choosing a red shirt with jeans? $\frac{2}{7} \cdot \frac{3}{5} = \frac{6}{35}$

b. What is the probability of his randomly choosing an all black outfit? $\frac{2}{7} \cdot \frac{2}{5} = \frac{4}{35}$

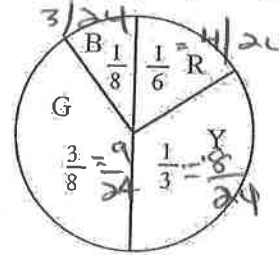
c. Which combination out of all his possible choices has the greatest probability of being randomly picked? How can you tell? *White shirt, + blue jeans. most selections in both categories*

6. What is the probability of spinning a:

a. Red or Green? $\frac{4}{24} + \frac{9}{24} = \frac{13}{24}$

b. Blue or Yellow? $\frac{3}{24} + \frac{8}{24} = \frac{11}{24}$

c. Yellow or Green? $\frac{8}{24} + \frac{9}{24} = \frac{17}{24}$



Common denominator 24

PROBABILITY: DEPENDENT AND INDEPENDENT EVENTS

Two events are **DEPENDENT** if the outcome of the first event affects the outcome of the second event. For example, if you draw a card from a deck and do not replace it for the next draw, the two events – drawing one card without replacing it, then drawing a second card – are dependent.

Two events are **INDEPENDENT** if the outcome of the first event does not affect the outcome of the second event. For example, if you draw a card from a deck but replace it before you draw again, the two events are independent.

Example 1

Aiden pulls an ace from a deck of regular playing cards. He does not replace the card. What is the probability of pulling out a second ace?

First draw: $\frac{4}{52}$ Second draw: $\frac{3}{51}$ aces left
cards left to pull from

This is an example of a dependent event – the probability of the second draw has changed.

Example 2

Jayson was tossing coins. He tossed a head. What is the probability of tossing a second head?

It is still $\frac{1}{2}$. The probability for the second event has not changed. This is an independent event.

Problems ~~★~~ Answer these Problems

1. You throw a die twice. What is the probability of throwing a six and then a second six? Is this an independent or dependent event? $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36} \Rightarrow$ independent, die does not change.
2. You have a bag of candy filled with pieces which are all the same size and shape. Four are gumballs and six are sweet and sour. You draw a gumball out, decide you don't like it, put it back, and select another piece of candy. What is the probability of selecting another gumball? Are these independent or dependent events? $\frac{4}{10} = \frac{2}{5} \Rightarrow$ independent, put the first one back.
3. Joey has a box of blocks with eight alphabet blocks and four plain red blocks. He gave an alphabet block to his sister. What is the probability his next selection will be another alphabet block? Are these independent or dependent events?

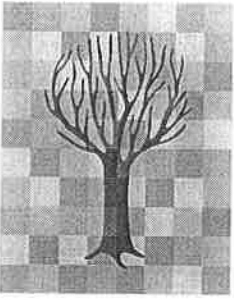


$\frac{7}{11}$, dependent because

did not put the first block back in the bag, so total changes to 11 + Alphabet is now 7.

4. In your pocket you have three nickels and two dimes.
 - a. What is the probability of selecting a nickel? $\frac{3}{5}$
 - b. What is the probability of selecting a dime? $\frac{2}{5}$
 - c. If you select a nickel and place it on a table, what is the probability the next coin selected is a dime? Is this an independent or dependent event? $\frac{2}{4} = \frac{1}{2}$ dependent, did not put nickel back in the pocket.
 - d. If all the coins are back in your pocket, what is the probability that the next coin you take out is a dime? Is this an independent or dependent event? $\frac{2}{5}$, independent, all back in
5. How do you tell the difference between dependent and independent events?

with ~~the~~ dependent events, the second probability changes because you did not replace the first pick. Independent events are not affected by the first pull because the object was replaced.

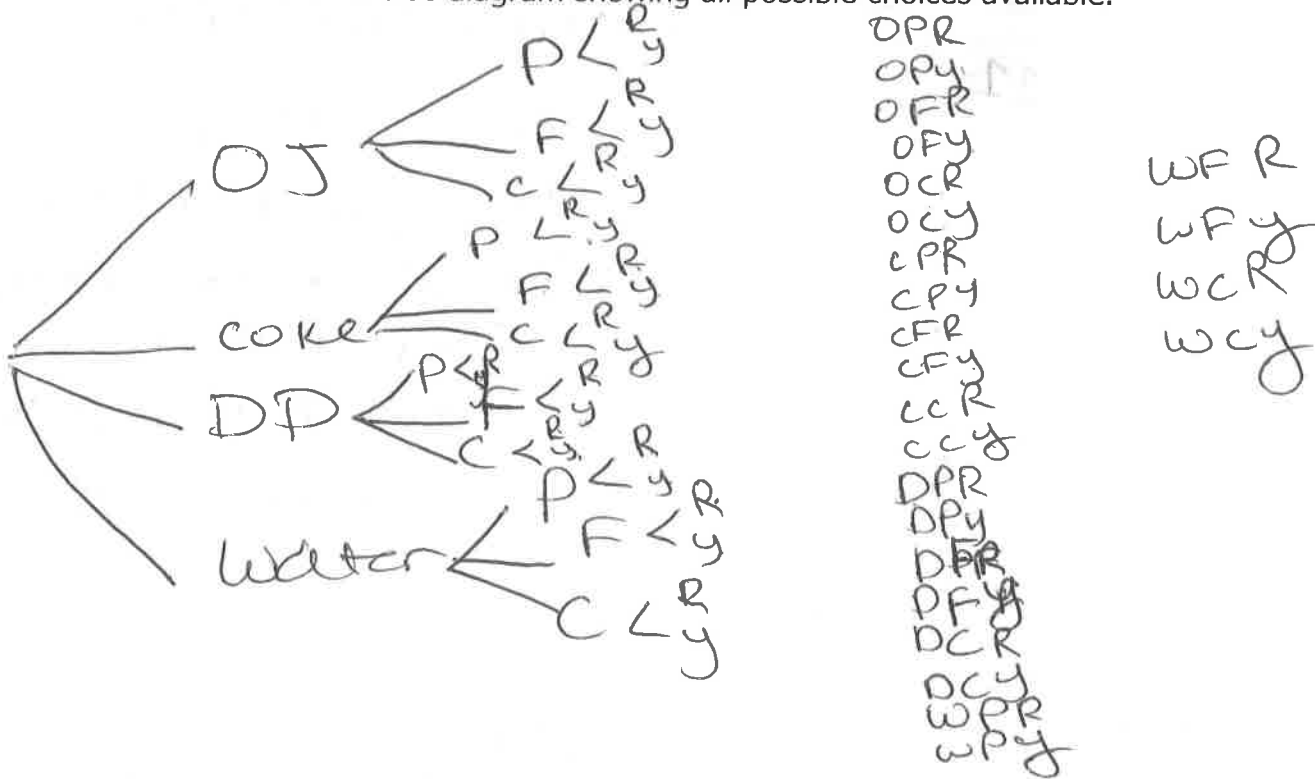


Working with Tree Diagrams Worksheet

Name _____

1. At the after school Tiger Club meeting, there were four drinks you could choose from: orange juice, Coke, Dr. Pepper, and water. There were three snacks you could choose from: peanuts, fruit, and cookies. There are two plates you can pick for your snacks: red or yellow. Each student may only have one drink, one snack and one plate.

A. Create a tree diagram showing all possible choices available.



- B. How many outcomes are there? What is another way we could have figured out the number of outcomes without making a tree diagram?

$$4 \times 3 \times 2 = 24$$

d s p

(multiply 1 by the amounts together)

- C. What is the probability of getting Coke, peanuts and a yellow plate?

$$1/24$$

- D. What is the probability of getting a yellow plate with any combination of snack and drink?

$$12/24 = 1/2$$

- E. What is the probability of getting fruit or a cookie with any plate or drink?

$$16/24 = 2/3$$

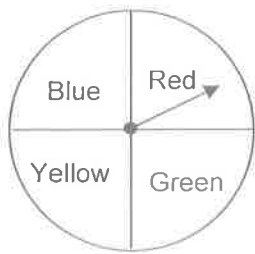
add
↓

2) Chris is rolling two number cubes and then finding the sum of the two results. What is the theoretical probability that he will have a sum greater than or equal to 5? Create an area model below to help you answer this question.

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$$\frac{30}{36} = \frac{5}{6}$$

3) Samantha is flipping a coin and then spinning a spinner with the colors red, blue, green and yellow. What is the theoretical probability that she will flip heads and then spin a blue or a green on the spinner? Create an area model below to help you answer this question.



coin

Spinner

	B	R	Y	G
H	HB	HR	HY	HG
T	TB	TR	TY	TG

$$\frac{2}{8} = \frac{1}{4}$$

