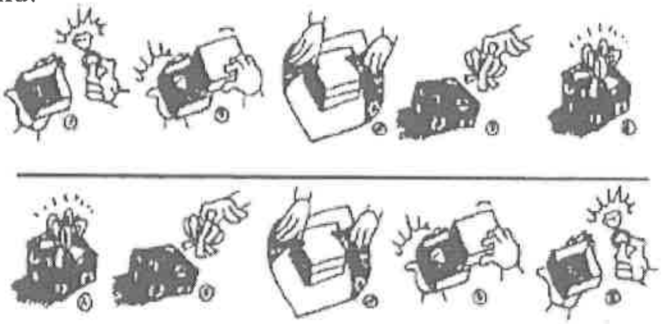
Math 3 Name Key

6-1 *Inverse Functions*

- *I can explain, write, and apply the inverse of a function.*
- *I can identify and explain the inverse of a function algebraically and graphically.*
- *I can determine if a function is a one-to-one function.*

Have you ever heard the expression "she knows it forward and backward" to describe someone who fully grasps a concept? Often, being able to reverse a process is a way to show how thoroughly you understand it. In this lesson, you will reverse mathematical processes, including functions. As you work on this lesson, keep these questions in mind:

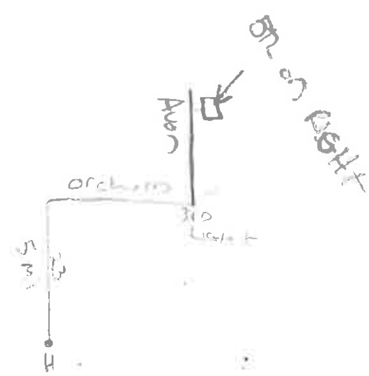
- *How can I "undo" each step?*
- *How can I justify each step?*



Introduction

Suppose you are given the following directions:

- From home, go north on Rt 23 for 5 miles
- Turn east (right) onto Orchard Street
- Go to the 3rd traffic light and turn north (left) onto Avon Drive
- Tracy's house is the 5th house on the right.



If you start from Tracy's house, write down the directions to get home.

- Pull out of the Driveway and go left on Avon.
- Go RIGHT on orchard (WEST)
- at the 3rd LIGHT, go left on 23 (SOUTH)
- Drive 5 miles home.

How did you come up with the directions to get home from Tracy's?

I Drew a Picture and worked Backwards.

Reverse the order and Do the opposite.

Suppose you are given the following algorithm:

- Starting with a number, add 5 to it
- Divide the result by 3
- Subtract 4 from that quantity
- Double your result

The final result is 10. Working backwards knowing this result, find the original number. Show your work.

$\frac{10}{2} = 5 \rightarrow 5+4 = 9 \rightarrow 9 \cdot 3 = 27 \rightarrow 27-5 = 22$

Write a function $f(x)$, which when given a number x (the original number) will model the operations given above.

$f(x) = 2\left(\frac{(x+5)}{3} - 4\right)$ x = Starting number

Write a function $g(x)$, which when given a number x (the final result), will model the backward algorithm that you came up with above.

x = final answer

$g(x) = 3\left(\frac{x}{2} + 4\right) - 5$

Fill in the following table:

x	$y = f(x)$	$z = g(y)$
22	10	22
1	−4	1
7	0	7
−8	−10	−8

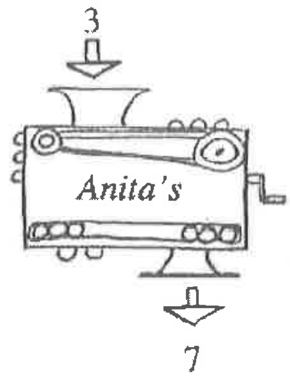
What do you notice about the above table?

f(x) and g(y) undo each other

In this scenario, $y = f(x)$ and $z = g(y)$ are **inverses** of each other because $g(y)$ will undo the actions of $f(x)$. Of course standard notation requires that the letter "x" be used as the input for a function, so we say that $f(x)$ and $g(x)$ are inverses, knowing that the "x" in $f(x)$ and $g(x)$ are not the same, much as they were not the same in the "starting number" and "ending number". In fact, the "x" in $g(x)$ is actually the *output* (the y) from $f(x)$.

In order to simplify the notation, mathematicians refer to the inverse of $f(x)$ as $f^{-1}(x)$, read "*f* inverse of *x*"

1. Your friend Anita built a function machine. Remember, a function is a mathematical relation in which every input value (represented by *x*) is paired with exactly one output value (represented by *y*). In more technical terms, every value in the domain is paired with exactly one value in the range. A picture of Anita's function machine is shown at the right. When she put in the number 3, the number 7 came out. When she put in a 4, a 9 came out. When she put in a -3, a -5 came out.



1a. Make a table to organize the inputs and outputs from Anita's function machine. Explain in words what this machine is doing to the input to generate the output.

Input (*x*)	−3	3	4
Output (*y*)	−5	7	9

Double the input and ADD 1.

1b. Anita's function machine suddenly started working backwards: it began pulling outputs back up into the machine, reversing the machine's process, and returning the original input. If 7 is pulled back into this machine, what value do you think will come out the top? Anita sets up her new backwards function machine and enters the other outputs. What would you expect to come out the top if 9 is entered? If -5 is entered? Explain.

$7 \to 3$
$9 \to 4$
$-5 \to -3$

Take the value Being pulled Back in and subtract 1 Then take half.

1c. Record the inputs and outputs of the backwards function machine in a table. Record the numbers going in as *x*, and the numbers coming out as *y*. Explain in words what Anita's backwards function machine is doing.

Being Pulled Back in

Input (*x*)	−5	7	9
Output (*y*)	−3	3	4

1d. How are the tables for the two machines related?

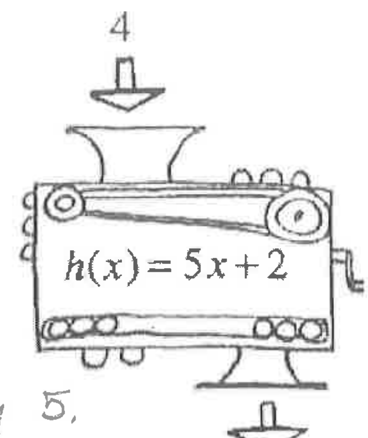
The tables are inverses. X's and Y's are switched

1e. Write equations for Anita's original function machine and for her backwards machine. How are the two equations related?

$f(x) = 2x+1$ input

$f'(x) = \frac{(x-1)}{2}$ output

2. The function machine at right follows the equation $h(x) = 5x + 2$.



2a. If the crank is turned backwards, what number should be pulled up into the machine in order to have a 4 come out of the top?

$h(4) = 5(4) + 2$ 22

$h(4) = 22$

2b. If you want to build a new machine that will undo what $h(x)$ does to an input. What must your machine do to 17 to undo it and return a value of 3? Subtract 2 and Divide By 5.

2c. As we learned above, an "undo" function is called an inverse and has the notation $h^{-1}(x)$. Write an equation for $h^{-1}(x)$, the "undo" function machine.

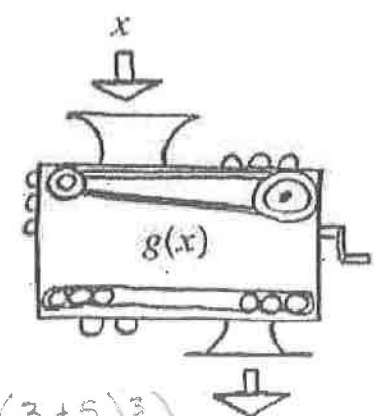
$$h^{-1}(x) = \frac{x-2}{5}$$

2d. Choose a value for *x*. Then show that your function, $h^{-1}(x)$, undoes the effects of the function machine $h(x)$. let $x = 10$

$h(10) = 5(10) + 2 = 52$ $\qquad$ $h^{-1}(52) = \frac{52-2}{5} = \frac{50}{5} = 10$

$h(10) = 52$ $\qquad$ $h^{-1}(52) = 10$

3. Mr. Sheppard is now working with a new function, $g(x)$. He wrote down the following steps for $g(x)$:



- Add 5
- Divide by 2
- Cube it (find the third power)
- Multiply by 6

3a. What is the equation for $g(x)$? What is the output when 3 is the input? $g(x) = 6\left(\left(\frac{x+5}{2}\right)^3\right)$ $\qquad$ $g(3) = 6\left(\left(\frac{3+5}{2}\right)^3\right)$

$g(3) = 384$

3b. Help Mr. Sheppard write down the steps (in words) of the inverse machine, $g^{-1}(x)$, and then write its equation.

- Divide By 6
- Take the cube root
- Multiply By 2
- Subtract 5

$g^{-1}(x) = 2\left(\sqrt[3]{\frac{x}{6}}\right) - 5$

3c. Verify that your equation in part (b) correctly "undoes" the output of $g(x)$ in part (a).

$g^{-1}(384) = 2\left(\sqrt[3]{\frac{384}{6}}\right) - 5$

$g^{-1}(384) = 3$

Finding an inverse of a function algebraically.

Notes

- Swap input and output variables
- Solve for y.

Example $f(x)=\frac{2}{x-3}$

$y=\frac{2}{x-3}$

$\frac{x}{1}=\frac{2}{y-3}$

$x(y-3)=2$

$y-3=\frac{2}{x}$

$y=\frac{2}{x}+3$

$f^{-1}(x)=\frac{2}{x}+3$

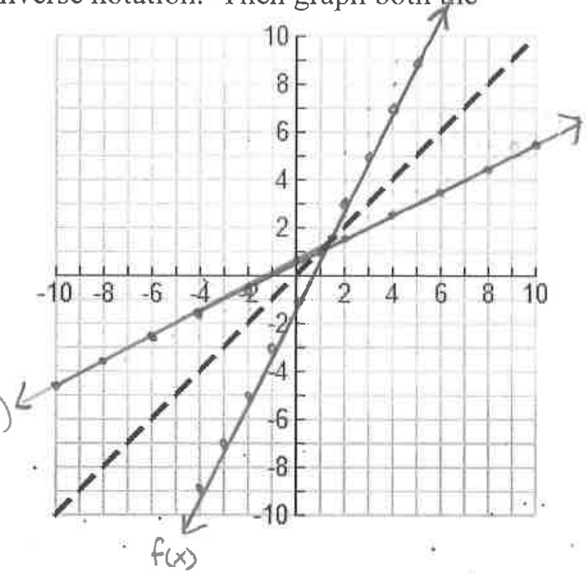
4. Find the inverse of each of the below functions. Use inverse notation. Then graph both the function and its inverse on the given coordinate plane.

4a. $f(x)=2x-1$

$f^{-1}(x)=\frac{x+1}{2} = \frac{1}{2}x+\frac{1}{2}$

x	-4	-2	0	2	4
f(x)	-9	-5	-1	3	7

x	-9	-5	-1	3	7
$f^{-1}(x)$	-4	-2	0	2	4

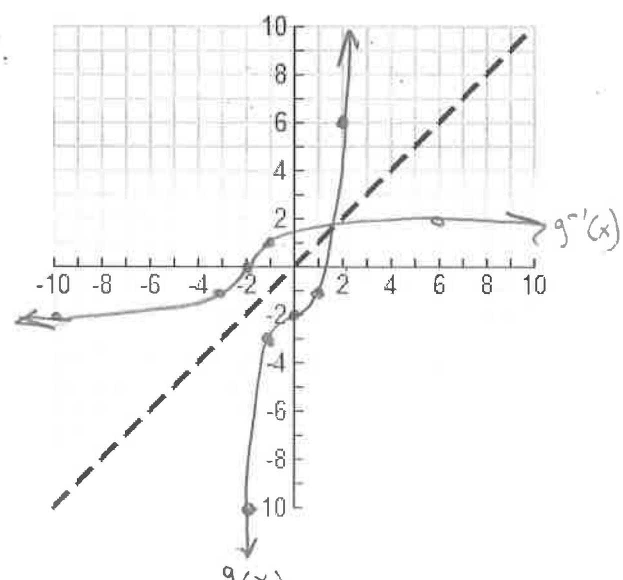


4b. $g(x)=x^3-2$

$g^{-1}(x)=\sqrt[3]{x+2}$

x	-2	-1	0	1	2
g(x)	-10	-3	-2	-1	6

x	-10	-3	-2	-1	6
$g^{-1}(x)$	-2	-1	0	1	2



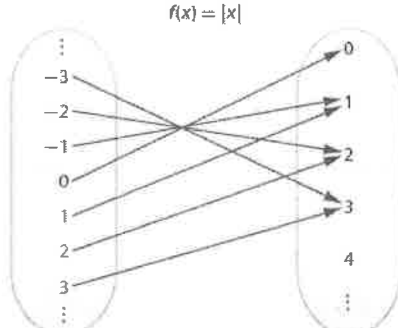
4c. Why was the line $y=x$ drawn on each coordinate plane? How is the line $y=x$ related to a function and its inverse?

A function and its inverse are reflections of each other over the line $y=x$.

VLT = vertical line Test

HLT = Horizontal Line TEST

5. Below is an arrow drawing representing the assignment of inputs to outputs for the function $f(x)=|x|$.



5a. Explain how you know $f(x)$ is a function.

every input has exactly 1 and only 1 output.
(each x is Paired w/ exactly 1 y)

5b. Explain why $f^{-1}(x)$ does not exist.

$f^{-1}(x)$ would not be a function.
When you reverse the arrows inputs of 1, 2 and 3 would have multiple outputs making the relation not a function.

Notes

The existence of inverses

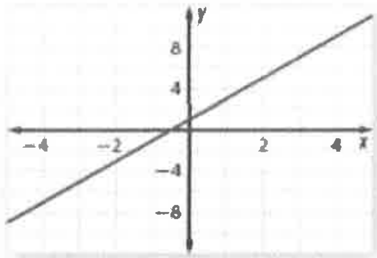
- for a relation to be a function, each x MUST be Paired with exactly one y. Graphically - it Passes the VLT.
- If a function has an inverse, each y MUST also Be Paired with exactly one x. The original function must Pass the HLT.
- If a relation is a function and its inverse exists, The function is said to be one-to-one
- examples of one to-one: linear
- examples of non one-to-one: quadratics, sine, cosine

6. Decide of the inverse of the below graphs exists. Explain your reasoning.

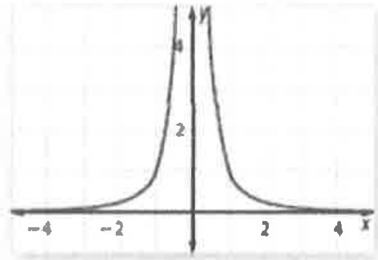
Graph I $f(x) = 2x + 1$ Graph II $f(x) = \frac{1}{x}$

6. Decide ~~of~~ if the inverse of the below graphs exists. Explain your reasoning.

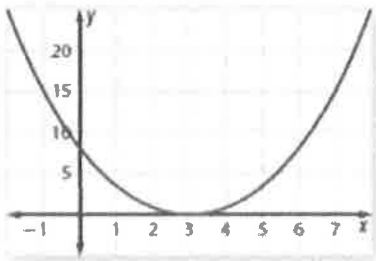
Graph I $f(x) = 2x + 1$



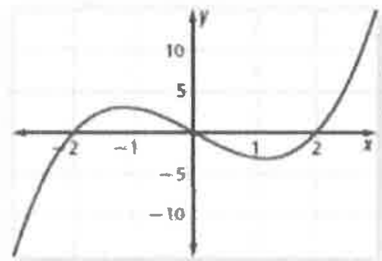
Graph II $f(x) = \frac{1}{x^2}$



Graph III $f(x) = (x - 3)^2$



Graph IV $f(x) = x^3 - 4x$



Graph	Inverse (Yes/No)	Reasoning Pass VLT = funct, Pass HLT = inv
I	Yes	- every x has 1 y, and every y has 1 x. - Passes VLT and HLT.
II	NO	a y value of 4 has multiple x values - fails HLT
III	NO	a y value of 15 has multiple x values - fails HLT
IV	NO	a y value of 0 has multiple x values - fails HLT

7a. Solve the following equation: $x^2 = 8$

$x = \pm\sqrt{8}$

7b. Explain how the fact that the above equation has a solution seems to contradict your answers to number (6). (change question)

Quadratics do not have inverses because they do not pass the HLT. There are y values (outputs) that are paired to more than 1 x (input). an output of 8 could have inputs of $\sqrt{8}$ and $-\sqrt{8}$.

Notes
Restricted domains

The function $y = x^2$ will have an inverse if the Domain is restricted (Limited) to $x \geq 0$.
This is why square roots are always positive.

* we also limit the Domains for sine, cosine, and Tangent so they can have inverses.

8a. Given the following table of values for the function *f(x)*, describe the domain and range.

x	*-4*	*-3*	*-2*	*-1*	*0*	*1*	*2*	*3*	*4*
f(x)	2	*1*	*0*	*-1*	*-2*	*-3*	*-4*	*-5*	*-6*

Domain: $\{x \in \mathbb{Z} : -4 \leq x \leq 4\}$

Range: $\{y \in \mathbb{Z} : -6 \leq y \leq 2\}$

8b. Make a similar table that shows the values of $f^{-1}(x)$, then describe the domain and range

x	2	1	0	-1	-2	-3	-4	-5	-6
$f^{-1}(x)$	-4	-3	-2	-1	0	1	2	3	4

Domain: $\{x \in \mathbb{Z} : -6 \leq x \leq 2\}$

Range: $\{y \in \mathbb{Z} : -4 \leq y \leq 4\}$

8c. Is the relation f^{-1} a function? yes. every X is Paired with 1 y.

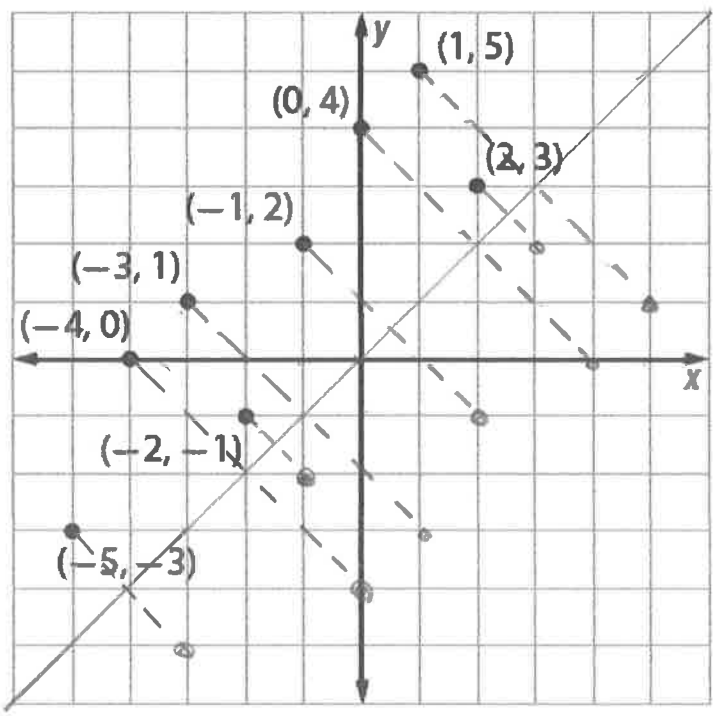
9. Make a table of values for a function g that does NOT have an inverse. Explain.

x	1	2	3	4	5
$g(x)$	7	7	7	7	7

- g(x) is a function since every x is Paired to 1 output.
- g(x) Does not have an inverse since The output of 7 is Paired with more than 1 input.

$7 \to 1$
$7 \to 2$

10. Suppose that the coordinates of the function $g(x)$ are shown on the graph below. Plot points that represent coordinates for $g^{-1}(x)$. Switch x and y

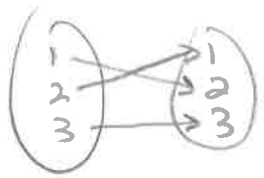


b. After drawing the line segments connecting each (a, b) point to each (b, a) point, I can see that the graphs of $g(x)$ and $g^{-1}(x)$ are reflections of each other over the line $y =$ <u>X</u>.

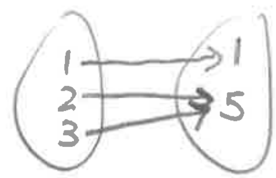
SUMMARY:

What patterns in an arrow diagram of a function indicate that the function does or does not have an inverse?

Two arrows going to the Same y value means an inverse Does not exist



Inverse exists



Inverse Does not exist

What patterns in the graph of a single coordinate graph indicate that the function does or does not have an inverse?

If the graph Passes The Horizontal line test, it has an inverse.

if it fails the HLT, it Does not have an inverse.

What geometric pattern relates graphs of functions and their inverses?

A function and its inverse are reflections of each other over the line $y=x$

What strategies do you use to algebraically find f^{-1} when you know the equation for f ?

Switch x and y and solve for y.