

# ALGEBRAIC EXPRESSIONS



$$\begin{array}{c} -2x + 5x - 9 + 10 \\ \text{is} \\ 3x + 1 \end{array}$$

$$\begin{array}{c} 3(x - 8) \\ \text{is} \\ 3x + (-24) \text{ or } 3x - 24 \end{array}$$

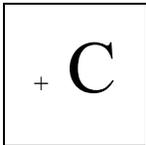
$$\begin{array}{c} 4x + 12 \\ \text{is} \\ 4(x + 3) \end{array}$$

S	Survey
P	Parentheses
C	Catch & Combine
=	...in equations...
A	Clear Add/Subtract
D/M	Clear Division/Multiplication

Name \_\_\_\_\_

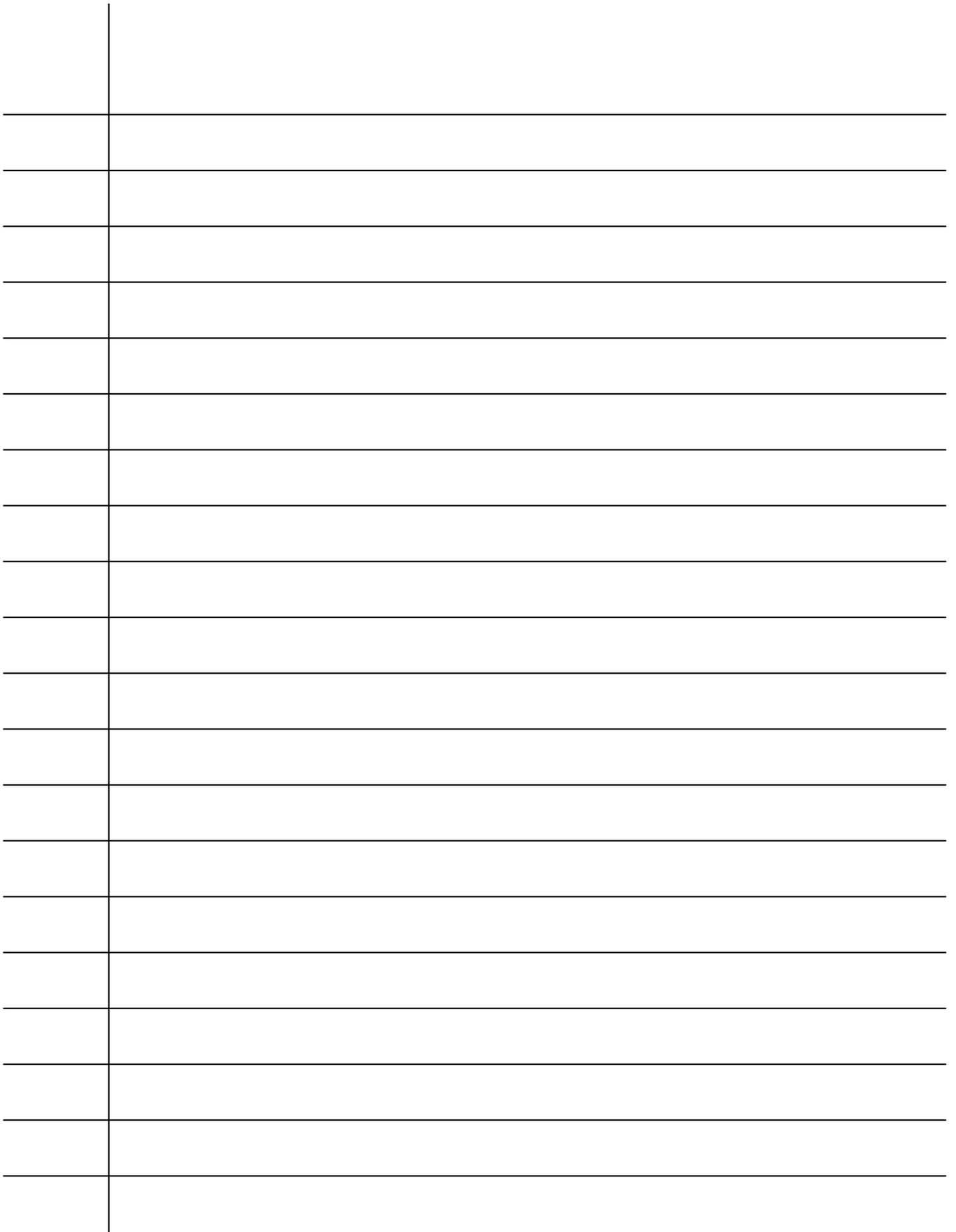
Period \_\_\_\_\_

# Simplifying Algebraic Expressions by Combining Like Terms



**Objective:** Students will identify like terms.  
 Students will simplify algebraic expressions by combining like terms.

Term	Definition	Picture/Example
<p><b>Terms</b></p>	<p>Quantities that you <b>ADD</b> to form an algebraic expression are called terms.</p>	<p>There are 3 terms in</p> $4n + 6b - 8$ <p>The terms are:</p>
<p><b>Like Terms</b></p> <p>You can <b>COMBINE</b> Like Terms  <b>**COMBINE</b> means add, so use the addition rules (SSS, DSD)</p>	<p>terms with the <b>same variable</b> raised to the <b>same power</b></p> <p>You <b>CAN</b> add/subtract like terms.</p>	
<p><b>Unlike Terms</b></p>	<p>terms whose variables are not the same, or who have the same variable, but it's raised to a different power</p> <p>You <b>CANNOT</b> add/subtract unlike terms.</p>	



For each algebraic expression, identify the number of terms. Then list the coefficients and any constant terms.

<b>Expression</b>	$6a + 3$	$6a - 3$	$0.2x - y + 8z$	$\frac{1}{2}n$
<b>Number of Terms</b>				
<b>Coefficient(s)</b>				
<b>Constant(s)</b>				

Identify the number of terms, the coefficients, and the constant term of the expressions below.

1.  $7p - 6pc + 3c - 2$

Number of terms: \_\_\_\_\_

Coefficients: \_\_\_\_\_

Constant terms: \_\_\_\_\_

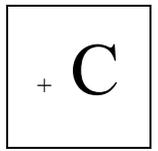
2.  $8 + 4ab - 5b$

Number of terms: \_\_\_\_\_

Coefficients: \_\_\_\_\_

Constant terms: \_\_\_\_\_

To simplify by combining like terms:



1. Search for like terms (**same variable** raised to the **same power**; and **constants with other constants**).
2. Catch the first term and any like terms.
3. Combine them using the addition rules. (SSS, DSD)
4. Continue with other like terms.

\*Remember that an "invisible 1" lurks in front of variables that appear to have no coefficient attached to them.

1)  $4x + 5x + 7 + x + 2$

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2)  $2n + 3 - 5n + 6$

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3)  $-9b + 2n - 4 + 2b$

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4)  $-7g + 3 - 8 - 3g + 7h$

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5)  $-8 + 2d - 7 - 5d + 12$

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6)  $5b + 7 - 3b - 4$

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# HOMEWORK

Identify the number of terms, the coefficient(s), and the constant term(s) of the expressions below.

1.  $6p - 7pc + 9c - 4$

Number of terms: \_\_\_\_\_

Coefficients: \_\_\_\_\_

Constant terms: \_\_\_\_\_

2.  $3 + 4ab - 5b + m$

Number of terms: \_\_\_\_\_

Coefficients: \_\_\_\_\_

Constant terms: \_\_\_\_\_

+ C

3. Sarah was asked to identify all coefficients and constants of the expression  $4 + n + 7m$ . She said that 4 is a constant, and 7 is a coefficient.

What is her error?

- a. She did not include the constant 1.
- b. She said 4 is a constant. It is actually a coefficient.
- c. She did not include the coefficient 1.
- d. She said 7 is a coefficient. It is actually a constant.

4. Add.  $2a + 8 + 4b + 5$

5. Add.  $8x - 7 + 6x + 8$

6. Find the sum.  $8x + 2 - 9x + 7$

7. Find the sum.  $3n + 4 - 8n - 1$

<b>Variable</b>	<p>A symbol used to represent an unknown amount.</p> <p>The symbol is usually a letter of the alphabet.</p>	
<b>Coefficient</b>	<p>The number being multiplied by a variable.</p> <p>It is the number attached to the variable, and is <i>usually</i> in front.</p>	
<b>*Special note!</b>	<p>A variable with <u>no</u> coefficient has an <u>"INVISIBLE 1"</u> attached to it!</p>	
<b>Constant</b>	<p>A number that doesn't change. There is no variable attached to a constant.</p>	
<b>Algebraic Expression</b>	<p>An expression that contains variables.</p>	

## Expanding Algebraic Expressions (The Distributive Property) day 1

Objective: Students will simplify algebraic expression using the distributive property.

Term	Definition	Example
Distributive Property	The distributive property combines multiplication with addition and subtraction	

You can **multiply constants and algebraic terms** simply by **multiplying the constant and the coefficient**.

The variable remains the same.

Remember, if the variable has **no coefficient**, it's an **invisible 1**.

a.  $2(3x) = \underline{\hspace{2cm}}$

b.  $-2(3d) = \underline{\hspace{2cm}}$

c.  $5(n) = \underline{\hspace{2cm}}$

d.  $-3y(4) = \underline{\hspace{2cm}}$

You can also **multiply variables** by one another.

e.  $a \cdot t = \underline{\hspace{2cm}}$

f.  $b(y) = \underline{\hspace{2cm}}$

g.  $3c(b) = \underline{\hspace{2cm}}$

h.  $2n(4x) = \underline{\hspace{2cm}}$

But what happens when you have more than one term inside the parentheses?

Examples:  $2(n + 4)$

$3(x - 8)$

## The Distributive Property

x **P** (clear Parentheses)

# KISS



YOUR PARENTHESSES GOODBYE!

Step 1: Catch the number touching the parentheses (on the outside) and any number inside that has a sign.

Step 2: Multiply the number on the *outside* of the parentheses by the **FIRST** number *inside* the parentheses.



I “times” over the rainbow 😊

Step 3: Multiply the number on the *outside* by the **SECOND** number that’s inside.

Examples:

1.  $3(4x + 2)$

2.  $-3(4x + 2)$

3.  $-3(4x - 2)$

1.  $5(x + 3)$

b.  $2(x + 1)$

c.  $4(x + 5)$

2.  $-3(x + 4)$

b.  $-6(x + 5)$

c.  $-1(x + 4)$

3.  $-4(x - 4)$

b.  $-8(x - 3)$

c.  $-1(x - 7)$

4.  $-3(2x - 5)$

b.  $-2(4x - 7)$

c.  $-2(6x - 8)$

5.  $a(b - 4)$

b.  $n(d + 1)$

c.  $y(5 - z)$

6.  $2a(3p - 5)$

b.  $4n(6d + k)$

c.  $5y(6h - w)$

a.  $3(x + 2)$

b.  $5(2y - 7)$

c.  $-2(n + 9)$

d.  $-3(k - 1)$

e.  $-4(1 + a)$

f.  $3(d - 4)$

g.  $-1(3x + 4)$

h.  $-3(b - 9 + 2y)$

i.  $-5(2 - m)$

j.  $3(n - 4 + 5y)$

k.  $-6(j - 2 + 3k)$

l.  $-1(3 - h)$

## Expanding Algebraic Expressions (The Distributive Property) day 2

Let's review using the distributive property:

x P

1.  $3(x - 4)$

2.  $4(n + 1)$

3.  $-5(x - 5)$

4.  $\frac{1}{2}(10)$

5.  $\frac{1}{2}(12)$

6.  $\frac{1}{3}(9)$

7.  $\frac{1}{4}(16)$

8.  $\frac{1}{2}(6x + 10)$

9.  $\frac{1}{2}(8x - 4)$

10.  $\frac{1}{3}(12x + 9)$

11.  $\frac{1}{3}(15x - 3)$

12.  $5(2x + 1 - n)$

**Objective:** Students will simplify algebraic expression using the distributive property. Students will recognize that a problem can be written in different forms. Students will recognize that some problems that have parentheses, may NOT require the dist. property.

### Remember!

The distributive property is **multiplication** over **addition** or **subtraction**. This means that in order to distribute, you must have a term (constant or variable) that is touching parentheses that contain more than one term. Those terms must be separated by addition or subtraction signs, NOT multiplication or division signs.

## ERROR ALERT!

Some addition and subtraction problems *look very similar* to distributive property problems.

Ex.  $-3(x + 7)$  This expression requires the distributive property.

$(5 - x)(-2)$  This expression requires the distributive property.

$(5 - x) - 8$  and  $2(n \cdot 4)$  These expressions do not!

1. Circle the problems that require the distributive property. Put an X through those that do NOT require the distributive property.

$-a(3 + b)$

$(-a)(3) + (b)$

$(3 + b) - a$

$(3 + b)(-a)$

$4(2 \cdot n)$

$(3 - g)(-5)$

$(-4)(6) + n$

$(8 + h) - 2$

SHOULD I USE THE DISTRIBUTIVE PROPERTY??

YES	NO
$-3(x + 1)$	$-3 + (x + 1)$
$(2x - 4)(3)$	$(2x - 4) + 3$
$(3 + y)(-2)$	$(3 \cdot y)(-2)$

Use the Distributive Property to expand each expression.

x P

Ex.  $-4(2n + 5)$

a.  $3(4x + 2)$

b.  $-5(y - 7)$

c.  $-2(n + 9)$

d.  $3(-1 - 5c)$

e.  $x(3 + 4y)$

f.  $-1(a - 1)$

g.  $\frac{1}{2}(4x - 6)$

h.  $-\frac{1}{4}(8x + 12)$

i.  $\frac{3}{4}(8x - 12)$

HOMWORK
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Expand the expressions that require the distributive property.

Put an X through the expressions that do not require the distributive property.

REMEMBER—you can only distribute (multiply) over addition or subtraction!

1.  $-6(a + 8)$

2.  $4(1 + 8x + a)$

3.  $(-5n + 7)6$

4.  $2 + (4k - 3)$

5.  $(9m + 10) \cdot 2$

6.  $-8(-b - 4)$

7.  $(3)(y)(-2)$

8.  $5(3 \cdot y)$

9.  $-4(-6p + 7)$

10.  $-4(n \div 5)$

11.  $\frac{1}{3}(9n + 15)$

12.  $\frac{1}{2}(6b - 10)$



## Simplifying Algebraic Expressions by Distributing and Combining Like Terms

S
P
C
=
A
D/M

**Objective:** Students will simplify algebraic expressions by combining like terms.

Students will recognize when the distributive property is required to simplify an expression and when it is not.

Before simplifying an expression that contains parentheses, you must determine whether or not you need to use the distributive property. If so, **DO IT FIRST!**

A. Simplify the expressions below (Hint: **ONE** of them needs dist. property).

a.  $2x + 5 + (6x + 1)$

b.  $2x + 5 + 6(x + 1)$

B. Sometimes you will need to **CATCH** the term (including a subtraction sign) before you distribute.

1.  $x - 2(x + 3)$

2.  $3n - 2(n + 5)$

3.  $4h - 3(2h + 5)$

Simplify. (Ask... Do I need to distribute? If so, do it FIRST!)

1.  $3(x + 6y - 7)$

S
P
C
=
A
D/M

2.  $-7(2x - 4)$

3.  $4(3x + 7) - 5x$

4.  $3x - 2(-4x + 5)$

### Extra Practice

Rewrite the expressions without parentheses.

(Hint: check for dist. property first!) Then simplify.

S
P
C
=
A
D/M

1.  $8 + (2x - 1)7$

2.  $-3(3x - 5) + 8x$

3.  $9x + 2 + (3x - 7)$

4.  $12x + (-4x + 1)3$

5.  $2y + 7(2y - 3)$

6.  $-8y + 4(3y + 3) - 5$

7.  $-4 + (3n - 5) - 7n$

Notice the difference between #7 and #8.

8.  $-4(3n - 5) - 7n$

## HOMEWORK

**Simplify.**

1a.  $-3(2x - 3y - 5)$

1b.  $-4(2y - 3) + 7y$

S
P
C
=
A
D/M

2a.  $(b - 7) + (3b - 9)$

2b.  $(-6x - 11) + (5x + 12)$

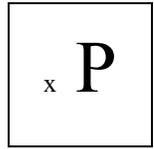
3a.  $-2(-4x + 5) + 6x$

3b.  $7x + 3(-2x + 3)$

4. For each algebraic expression, identify the number of terms. Then list the coefficients and any constant terms.

Expression	$8x - 3$	$4x + y + 11$
Number of Terms		
Coefficient(s)		
Constant(s)		

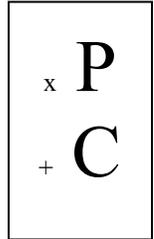
## Simplifying Algebraic Expressions by Distributing and Combining Like Terms



1.a  $3(x + 6y - 7)$

1.b  $-7(2x - 4)$

Distribute first.... then catch and combine like terms.



2a.  $4(3x + 7) - 5x$

2.b.  $-2(-4x + 5) + 3x$

Ask... Do I need to distribute? If not, Catch and Combine Like Terms.

3a.  $5x + 6y - 4x + 3y - 9$

3.b.  $-9x + 7 - x + 5x - 4y$

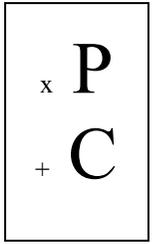
Ask... Do I need to distribute? If so, do it first. Then "Bring Down the Junk." If not, drop the parenthesis and then catch and combine like terms.

4a.  $(x - 5) + (4x - 2)$

4b.  $6x + 4(-2x + 3)$

# HOMWORK

Rewrite the expression without parentheses. Ask...  
Do I need to distribute? If so, do it first. Then  
"Bring Down the Junk." If not, drop the parenthesis and  
then catch and combine like terms.



1.  $7(2x - 1) + 8$

2.  $-3(3x - 5) + 8x$

3.  $9x + 2(2x - 7)$

4.  $12x + 3(-4x + 1)$

5.  $12y + 7(2y - 3)$

## Adding and Subtracting Algebraic Expressions

**Objective:** Students will add and subtract algebraic expressions.

Recall that only *like terms* can be added or subtracted.

Simplify the following problem by combining like terms.

Ex 1:  $(2n + 3) + (4n + 5)$

Ex 2:  $(-3h + 2) + 3(4h - 2)$

**\*\*\*Subtraction of expressions can be especially difficult!**

Note the difference between the two problems below.

Which problem requires the distributive property to simplify it? \_\_\_\_\_

Rewrite using the distributive property where necessary.

$$(8x - 3) + 2(3x + 1) \quad \text{and} \quad (8x - 3) - 2(3x + 1)$$

Note the difference between the two problems below.

Which problem requires the distributive property to simplify it? \_\_\_\_\_

Rewrite using the distributive property where necessary.

$$(7x - 3) + 1(4x + 1) \quad \text{and} \quad (7x - 3) - 1(4x + 1)$$

Recall that often times in math the 1 is "invisible", as is a -1.

Here are the same problems rewritten with an "invisible" 1.

$$(7x - 3) + (4x + 1) \quad \text{and} \quad (7x - 3) - (4x + 1)$$

HOWEVER, that means *when there is a subtraction sign between expressions*, you must think of it as *distributing a -1*.

a. Distribute the -1

b. Distribute the -1

$$(6n - 5) - (2n + 1)$$

$$(8a + 1) - (2a + 4)$$

c. Distribute the -1

$$(5h - 4) - (2h + 1)$$

d. Distribute the -1

$$(2n + 4) - (n - 1)$$

e. Distribute the -1

$$(-2x - 5) - (6x + 3)$$

f. Distribute the -1

$$(-3y + 1) - (4y + 8)$$

# HOMework

1. Distribute the -1

$$(6h + 4) - (2h + 3)$$

x P  
+ C

2. Distribute the -1

$$(6h + 4) - (2h - 3)$$

3. Distribute the -1

$$(6h - 4) - (2h - 3)$$

# Factoring Algebraic Expressions

Objective:

Students will factor algebraic expressions by **REVERSING** the distributive property.

Term	Definition	Example
<p><b>Factor</b> (as a noun)</p>	<p>A number that is multiplied by another number to get a product.</p>	<p>List the factors of 18 and of 12.</p>
<p><b>Factor</b> (as a verb)</p>	<p>When you factor an expression, you reverse the distributive property.</p> <p>The number on the outside of the parentheses is the <i>GCF</i>.</p>	<p>Factor: <math>18n + 12</math></p> <p>Skeleton:  <math>\underline{\hspace{1cm}} (\underline{\hspace{1cm}} + \underline{\hspace{1cm}})</math></p> <ol style="list-style-type: none"> <li>1. Find the <i>GCF</i> of the numbers.</li> <li>2. Find the <i>GCF</i> of the variables.</li> <li>3. The <i>GCF</i>'s go on the outside.</li> <li>4. The leftovers go on the inside.</li> </ol>
<p><b>Greatest Common Factor</b></p>	<p>The largest factors that is shared by two or more numbers.</p>	<p>The <i>GCF</i> of 18 and 12 is <math>\underline{\hspace{1cm}}</math>.</p> <p>(See top example.)</p>
<p><b>Factored Form</b></p>	<p>An expression in factored form has the <i>GCF</i> on the outside of the parentheses, and a sum or difference on the inside.</p>	<p>The factored form of <math>18n + 12</math> is <math>\underline{\hspace{2cm}}</math>.</p> <p>(See 2nd example.)</p>

Ex 1. Factor.  $14xy + 21x$

To factor an algebraic expression you use division to undo the distributive property.

1. Make the "skeleton" of a distributive property problem under the given one. It will look like this:  $\_\_ ( \_\_ + \_\_ )$  or  $\_\_ ( \_\_ - \_\_ )$  this.
2. Look for a variable that is shared by the terms (they may share more than one); circle it and then place the shared variable on the outside, next to the parentheses.
3. Bring down any variables not circled.
4. Find the *GCF* of the coefficients. (Check to see if the smallest # you see is a factor of the others. If so, it's the *GCF*. Otherwise, use the "rainbow" method.)
5. Place the *GCF* on the outside of the parentheses. If there are any letters already there, the *GCF* will become their coefficient.

To fill the inside:

6. Divide the 1<sup>st</sup> term by the *GCF* you found. Place the quotient on the line.
7. Divide the 2<sup>nd</sup> term by the *GCF* you found. Place the quotient on the line.
8. Repeat until all terms have been divided by the *GCF*.
9. Check your work by applying the distributive property to your answer to see if it matches the original expression.

Ex 2.  $12x + 6$

Ex. 3       $5b - 15n$

Ex. 4       $3n - nj$

Use the distributive property to check.

1.       $10n + 15$

Check:

2.       $8y - 12y$

Check:

3.       $8n - 2$

Check:

4.       $14z + 21$

Check:

5.       $4h - 12$

Check:

6.  $8a + 4$

Check:

7.  $12a + 16b - 10c$

Check:

8.  $12x - 24y - 3$

Check:

9.  $9n + 7n$

Check:

10.  $-5xy + 25x$

Check:

11.  $18b + 3$

Check:

## HOMWORK

1. Write the factored form of each expression.
2. Check using distributive property.

a.  $18a + 3$

Check:

b.  $-2c + 6d$

Check:

c.  $20xy + 10x$

Check:

**Error analysis:** Ali factored the above problem, c, and got  $2x(10y + 5)$ . When she checked it using the distributive property, she got the original problem! Since her check worked, she thinks she has the correct answer. How can that be? Explain her mistake.

3. **Error analysis:** Jamie incorrectly factored  $15x - 20xy$ . She got  $5x(3 - 4xy)$ . Factor the expression correctly.

What error did she likely make?

- a. She did not have the correct operation inside parentheses.
- b. She did not factor the variable from the first term.
- c. She did not factor the variable from the second term.
- d. She did not simplify the terms inside the parentheses.

More factoring practice.

1.  $24xy + 10x$

2.  $15m - 18mn$

3.  $4x - xy$

4.  $42x + 7y$

5.  $4x - 2xy$

6.  $32xyz + 12xy$

