Chapter 4: Simultaneous Linear Equations (3 weeks)

Utah Core Standard(s):

- Analyze and solve pairs of simultaneous linear equations. (8.EE.8)
  a) Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
  b) Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6.*
  c) Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.

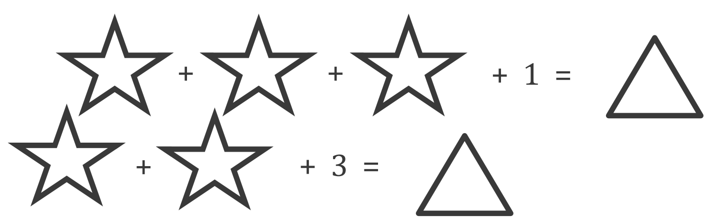
Academic Vocabulary: system of linear equations in two variables, simultaneous linear equations, solution, intersection, ordered pair, elimination, substitution, parallel, no solution, infinitely many solutions

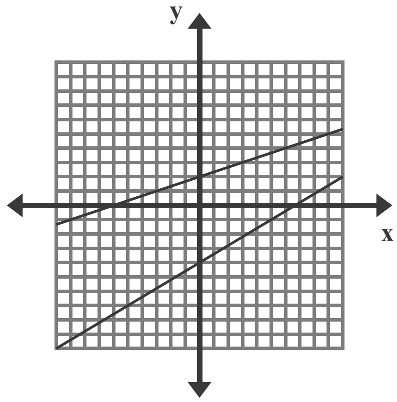
Chapter Overview:
In this chapter we discuss intuitive, graphical, and algebraic methods of solving simultaneous linear equations; that is, finding all pairs (if any) of numbers (x, y) that are solutions of both equations. We will use these understandings and skills to solve real world problems leading to two linear equations in two variables.

Connections to Content:
Prior Knowledge: In chapter 1, students learned to solve one-variable equations using the laws of algebra to write expressions in equivalent forms and the properties of equality to solve for an unknown. They solved equations with one, no, and infinitely many solutions and studied the structure of an equation that resulted in each of these outcomes. In chapter 3, students learned to graph and write linear equations in two-variables. Throughout, students have been creating equations to model relationships between numbers and quantities.

Future Knowledge: In subsequent coursework, students will gain a conceptual understanding of the process of elimination, examining what is happening graphically when we manipulate the equations of a linear system. They will also solve systems that include additional types of functions.

MATHEMATICAL PRACTICE STANDARDS

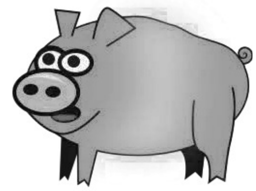
	Make sense of problems and persevere in solving them.	Kevin and Nina are competing in a bike race. When Kevin is ninety miles into the race, he is in first place. Nina is in second place and is 15 miles behind Kevin. a. From this point, Kevin continues the race at a constant rate of 25 mph and Nina continues the race at a constant rate of 30 mph. When will Nina catch Kevin? Solve this problem using any method you wish. b. If the race is 150 miles long, who will win? Assume Nina and Kevin bike at the speeds given in part a). c. Now suppose the following: Ninety miles into the race, Kevin is still in first place and Nina is still in second place, 15 miles behind Kevin. But now Kevin and Nina both finish out the race at a speed of 30 mph. When will Nina catch Kevin? If the race is 150 miles long, who will win? *The goal of this problem is that students will have the opportunity to explore a problem that can be solved using simultaneous linear equations from an intuitive standpoint, providing insight into graphical and algebraic methods that will be explored in the chapter. Students also gain insight into the meaning of the solution(s) to a system of linear equations. This problem requires students to analyze givens, constraints, relationships, and goals. Students may approach this problem using several different methods: picture, bar model, guess and check, table, equation, graph, etc.*
	Reason abstractly and quantitatively.	Write a system of equations for the model below and solve the system using substitution.  *This chapter utilizes a pictorial approach in order to help students grasp the concepts of substitution and elimination. Students work with this concrete model and then transition into an abstract model as they begin to manipulate the equations in order to solve the system.*

	Construct viable arguments and critique the reasoning of others.	How many solutions does the system of linear equations graphed below have? How do you know?  *In order to answer this question students must understand that the graph of an equation shows all of the ordered pairs that satisfy the equation and that when we graph the equation of a line we see a limited view of that line. They must also understand what the solution to a system of linear equations is and how the solution is determined graphically. Students will use this information, along with additional supporting statements, in order to make an argument as to the number of solutions to this system of equations.*
	Model with mathematics.	The student officers are buying packs of streamers and balloons to decorate for a school dance. Packs of balloons cost \$3.50 and packs of streamers cost \$2. If the student officers bought a total of 12 packs of decorations and spent \$31.50, how many packs of balloons did they buy? How many packs of streamers did they buy? Write the solution in a complete sentence. *The ability to create and solve equations gives students the power to solve many real world problems. They will apply the strategies learned in this chapter to solve problems arising in everyday life that can be modeled and solved using simultaneous linear equations.*
	Use appropriate tools strategically.	A farmer saw some chickens and pigs in a field. He counted 60 heads and 176 legs. Determine exactly how many chickens and pigs he saw. a. Solve the problem using the methods and strategies studied in this chapter. b. Which method do you prefer using to solve this problem? Use your preferred method to determine the number of chickens and pigs in a field with 45 heads and 146 legs. *While solving this problem, students should be familiar with and consider all possible tools available: graphing calculator, graph paper, concrete models, tables, equations, etc. Students may gravitate toward the use of a graphing calculator given the size of the numbers. This technological tool may help them to explore this problem in greater depth.*

	Attend to precision.	Consider the equations $-2x + y = -1$ and $y = 2x + 4$. Make sure both equations are written in slope-intercept form, then graph both equations on the coordinate plane below and solve the system of linear equations. *Solving systems of equations both graphically and algebraically requires students to attend to precision while executing many skills including using the properties of equality and laws of algebra in order to simplify and rearrange equations, producing graphs of equations, and simplifying and evaluating algebraic expressions in order to find and verify the solution to a system of linear equations.*
	Look for and make use of structure.	One equation in a system of linear equations is $6x + 4y = -12$. a. Write a second equation for the system so that the system has only **one solution**. b. Write a second equation for the system so that the system has **no solution**. c. Write a second equation for the system so that the system has **infinitely many solutions**. *In this problem, students must analyze the structure of the first equation in order to discern possible second equations that will result in one, infinitely many, or no solution.*
	Look for and express regularity in repeated reasoning.	Gabriela and Camila like to race each other. Gabriela can run 10 feet/second while Camila can run 12 feet/second. Being a good sport, Camila gives Gabriela a 20-foot head start. How long will it take Camila to catch Gabriela? *Students can use repeated reasoning in order to solve this problem. Realizing that each second Camila closes the gap between her and Gabriela by 2 feet, students may determine that it will take 10 seconds in order for Camila to catch Gabriela.*

4.0 Anchor Problem: Chickens and Pigs

A farmer saw some chickens and pigs in a field. He counted 30 heads and 84 legs. Determine exactly how many chickens and pigs he saw. There are many different ways to solve this problem, and several strategies have been listed below. Solve the problem in as many different ways as you can and show your strategies below.



Strategies for Problem Solving

- **Make a List or Table**
- **Draw a Picture or Diagram**
- **Guess, Check, and Revise**
- **Write an Equation or Number Sentence**
- **Find a Pattern**
- **Work Backwards**
- **Create a Graph**
- **Use Logic and Reasoning**

Section 4.1: Understand Solutions of Simultaneous Linear Equations

Section Overview:
In this section, students are solving simultaneous linear equations that have one, no, or infinitely many solutions using intuitive and graphical methods. In order to access the problems initially students may use logic, and create pictures, bar models, and tables. They will solve simultaneous linear equations using a graphical approach, understanding that the solution is the point of intersection of the two graphs. Students will understand what it means to solve two linear equations, that is, finding all pairs (if any) of numbers (x, y) that are solutions to both equations and they will interpret the solution in a context.

Concepts and Skills to Master:
By the end of this section, students should be able to:

1. Solve simultaneous linear equations by graphing.
2. Understand what it means to solve a system of equations.
3. Identify and provide examples of systems of equations that have one solution, infinitely many solutions, or no solution.
4. Interpret the solution to a system in a context.

4.1a Class Activity: The Bake Sale

1. The student council is planning a bake sale to raise money for a local food pantry. They are going to be making apple and peach pies. They have decided to make 10 pies. Each pie requires 2 pounds of fruit; therefore they need a total of 20 pounds of fruit.

   a. In the table below, fill out the **first two columns only** with **8** possible combinations that will yield 20 pounds of fruit.

# of Pounds of Apples	# of Pounds of Peaches	Cost of Apples	Cost of Peaches	Total Cost

   b. One pound of apples costs $2 and one pound of peaches cost $1. Fill out the rest of the table above to determine how much the student council will spend for each of the combinations.

   c. Mrs. Harper, the student council advisor, tells the students they have exactly $28 to spend on fruit. How many pounds of each type of fruit should they buy so that they have the required 20 pounds of fruit **and** spend exactly $28?

d. If p represents the number of pounds of peaches purchased and a represents the number of pounds of apples purchased, the situation above can be modeled by the following equations:

$p + a = 20$
$2a + p = 28$

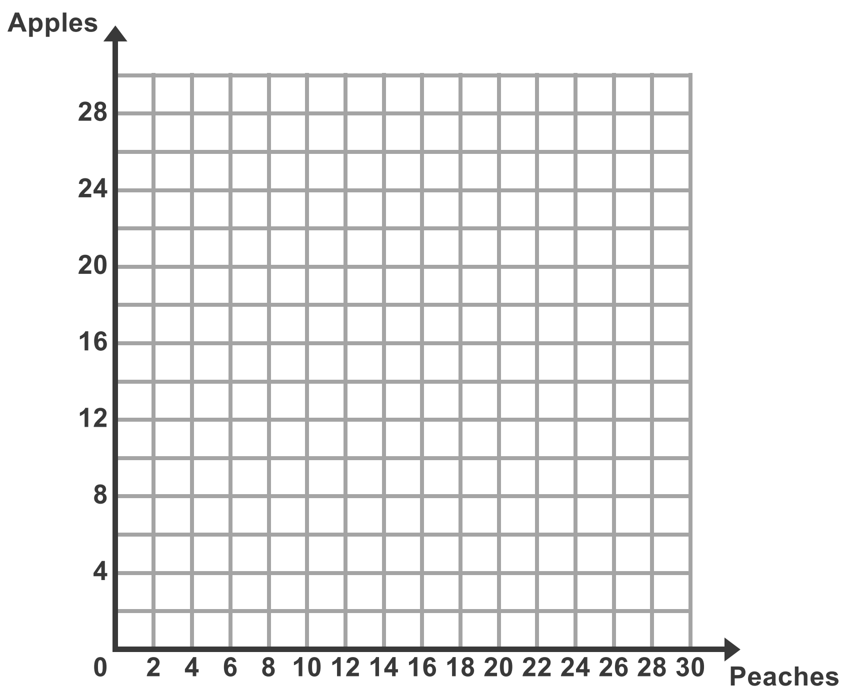
Write in words what each of these equations represents in the context.

$p + a = 20$ ____

$2a + p = 28$ ____

e. Does the solution you found in part c) make both equations true?

f. Graph the equations from part d) on the coordinate plane below. Label the lines according to what they represent in the context.



g. Find the point of intersection in the graph above. What do you notice?

h. The Bake Sale problem can be modeled and solved using a **system of linear equations**. Write in your own words what a **system of linear equations** is.

i. Explain, in your own words, what the **solution** to a system of linear equations is. How can you find the solution in the different representations (table, graph, equation)?

j. Josh really likes apple pie so he wants to donate enough money so that there are an equal number of pounds of peaches and apples. How much does he need to donate?

k. What if the students had to spend exactly $25? Exactly $20?

   How would the equations change?

   How would the graphs change?

   What would the new solutions be?

l. What if the students wanted to make 20 pies and had exactly $64 to spend? Write the system of equations that models this problem. Find a combination that works.

4.1b Class Activity: Who Will Win the Race

1. Kevin and Nina are competing in a bike race. When Kevin is ninety miles into the race, he is in first place. Nina is in second place and is 15 miles behind Kevin.
   a. From this point, Kevin continues the race at a constant rate of 25 mph and Nina continues the race at a constant rate of 30 mph. When will Nina catch Kevin? Solve this problem using any method you wish.

Picture:

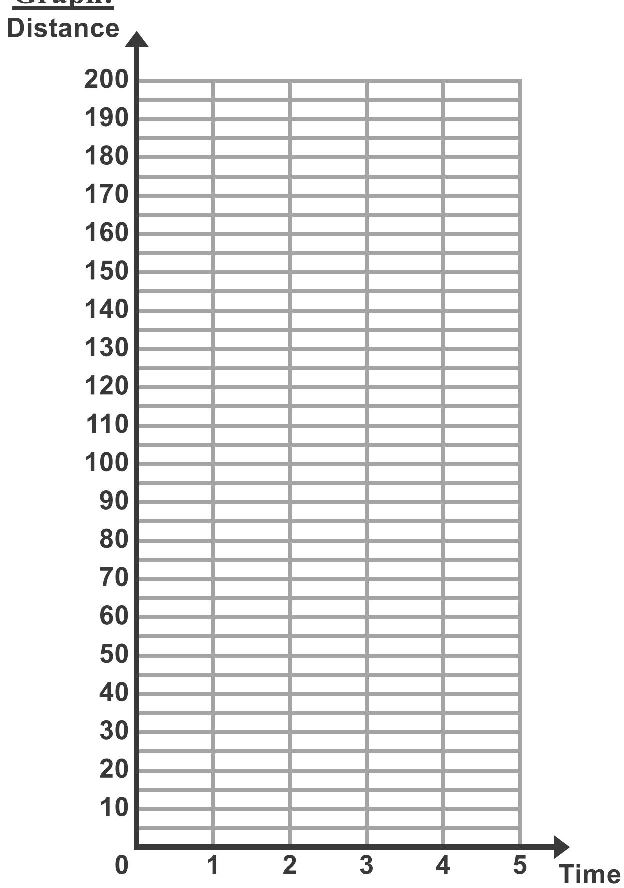
Kevin

Nina

Table:

Time (hours)					
Kevin					
Nina					

Graph:



Other Methods:

Write an Equation, Use Logic and Reasoning, Guess, Check, and Revise, Find a Pattern, Work Backwards.

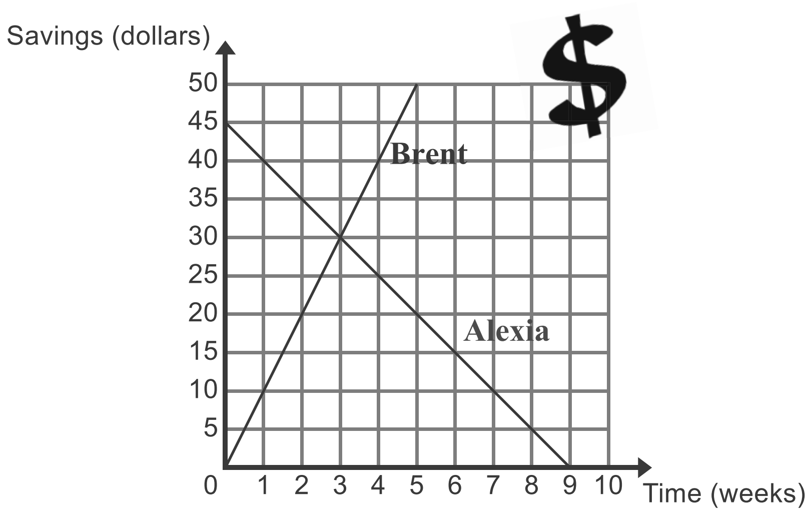
b. If the race is 150 miles long, who will win? Assume Nina and Kevin bike at the speeds given in part a).

c. Now suppose the following: Ninety miles into the race, Kevin is still in first place and Nina is still in second place, 15 miles behind Kevin. But now Kevin and Nina both finish out the race at a constant speed of 30 mph. When will Nina catch Kevin? If the race is 150 miles long, who will win?

2. The graph below shows the amount of money Alexia and Brent have in savings.

a. Write an equation to represent the amount y that each person has in savings after x weeks:

Alexia: ________________________ Brent: ________________________



b. Tell the story of the graph. Be sure to include what the point of intersection means in the context.

4.1b Homework: Who Will Win the Race

1. Gabriela and Camila like to race each other. Gabriela can run 10 feet/second while Camila can run 12 feet/second. Being a good sport, Camila gives Gabriela a 20-foot head start.
   a. How long will it take Camila to catch Gabriela? (For ideas on how to solve this problem, see the strategies used in the classwork)

   b. If the girls are racing to a tree that is 30 yards away, who will win the race? (*Remember there are 3 feet in 1 yard)*.

2. Darnell and Lance are both saving money. Darnell currently has \$40 and is saving \$5 each week. Lance has \$25 and is saving \$8 each week.
   a. When will Darnell and Lance have the same amount of money?

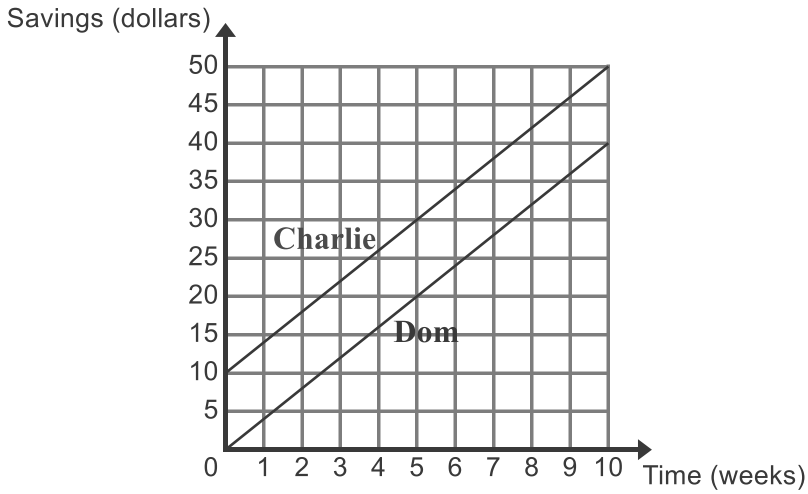
   b. How much will each boy have when they have the same amount of money?

   c. If both boys continue saving at this rate, who will have \$100 first?

3. The graph below shows the amount of money Charlie and Dom have in savings.

   a. Write an equation to represent the amount y that each person has in savings after x weeks:

   Charlie: ______________________ Dom: __________________________

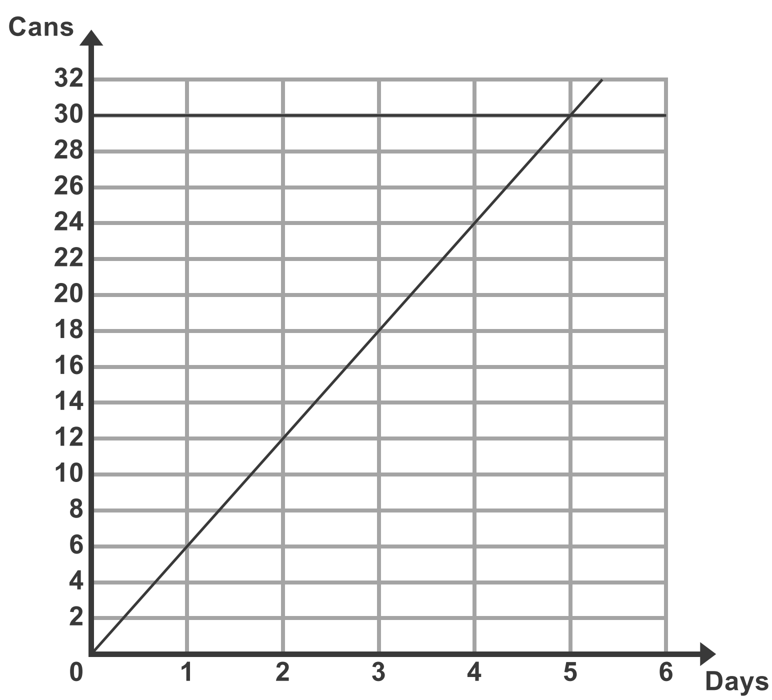


   b. Tell the story of the graph.

4. Lakeview Middle School is having a food drive. The graph below shows the number of cans each class has collected for the food drive with time 0 being the start of week 3 of the food drive.

   a. Write an equation to represent the number of cans y that each class has collected after x days.

   Mrs. Lake's Class: ______________________ Mr. Luke's Class: ______________________



   b. Tell the story of the graph.

4.1c Class Activity: Solving Simultaneous Linear Equations by Graphing

One method for solving simultaneous linear equations is graphing. In this method, both equations are graphed on the same coordinate grid, and the solution is found at the point where the two lines intersect.

Consider the simultaneous linear equations shown below and answer the questions that follow:

$$2x + y = 4$$
$$y = 4x - 2$$

1. What problems might you encounter as you try to graph these two equations?

2. What form of linear equations do we typically use when graphing?

As we have seen, it is possible to rearrange an equation that is not in slope-intercept form using the same rules we used when solving equations. We can rearrange this equation to put it in slope-intercept form. Remember, slope-intercept form is the form $y = mx + b$, so our goal here will be to isolate y on the left side of the equation, then arrange the right side so that our slope comes first, followed by the y-intercept.

$2x + y = 4$	Subtract $2x$ from both sides to isolate y
$y = 4 - 2x$	(Remember that 4 and $-2x$ are not like terms and cannot be combined)
$y = -2x + 4$	Rearrange the right side so that the equation is truly in slope-intercept form

3. Let's look at an example that is a little more challenging. With your teacher's help, write in the steps you complete as you go.

$4x - 8y = 16$	______________________
$-8y = 16 - 4x$	______________________
$y = -2 + \frac{1}{2}x$	______________________
$y = \frac{1}{2}x - 2$	______________________

4. ***Skill Review:*** Put the following equations into slope-intercept form.

a. $5x + y = 9$

c. $4y - x = 16$

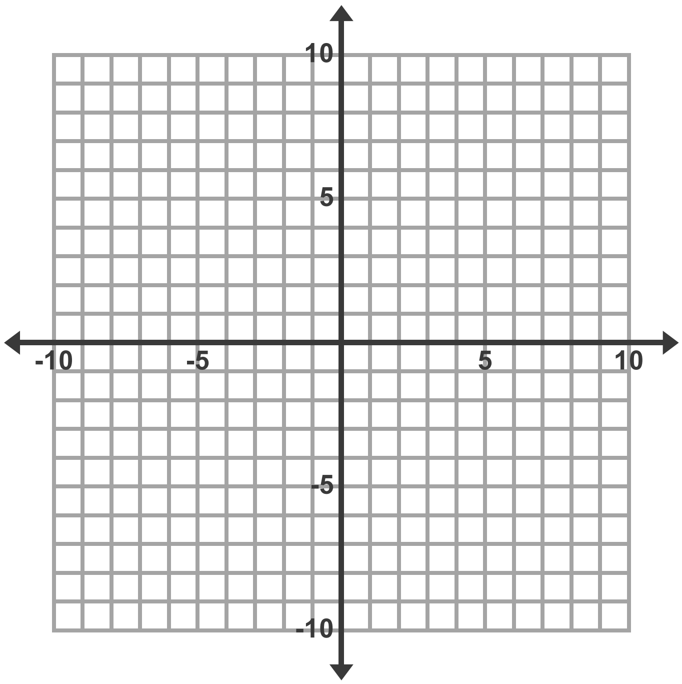
e. $-y = x - 2$

b. $4x + 2y = -12$

d. $4x - 2y = -24$

f. $-2x + 5y = 3$

5. Consider the linear equations $2x + y = 4$ and $y = 4x - 2$ from the previous page. Graph both equations on the coordinate plane below.



a. Find the coordinates (x, y) of the point of intersection.

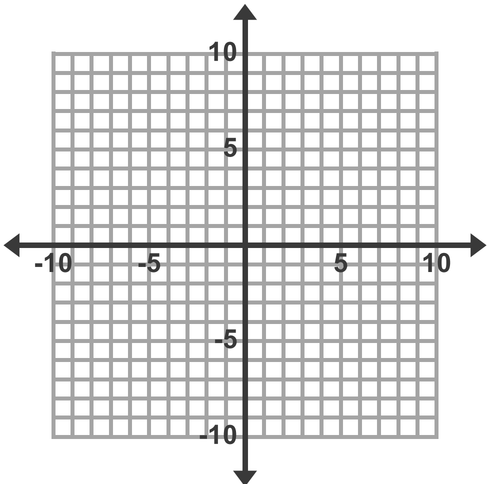
b. Verify that the point of intersection you found satisfies both equations.

The **solution(s) to a pair of simultaneous linear equations** is all pairs (if any) of numbers (x, y) that are solutions of both equations, that is (x, y) satisfy both equations. When solved graphically, the solution is the point or points of intersection (if there is one).

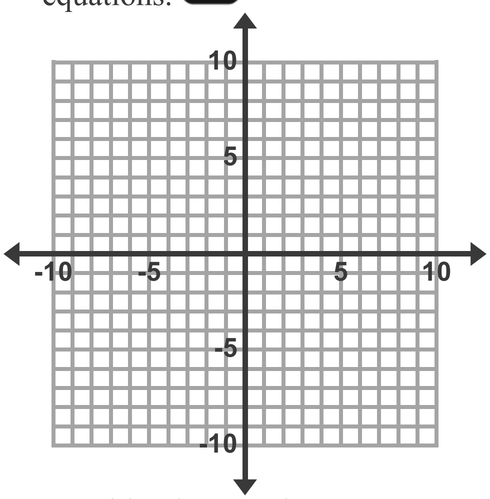
6. Determine whether (3, 8) is a solution to the following system of linear equations:
$2x + y = 14$
$x + y = 11$

7. Determine whether $(0, -5)$ is a solution to the following system of linear equations:
$y = 2x - 5$
$4x + 5y = 25$

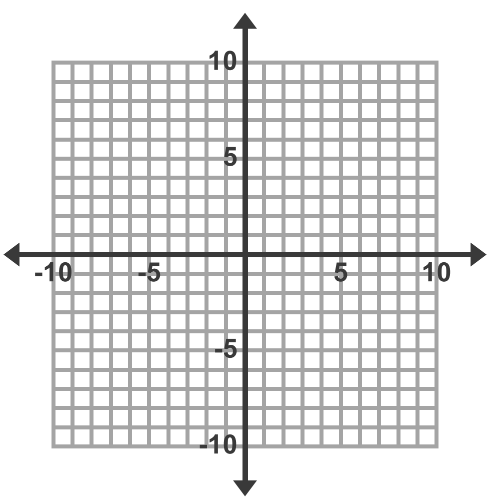
8. Consider the equations $y = -2x$ and $y = -\frac{1}{2}x - 3$. Make sure both equations are written in slope-intercept form, then graph both equations on the coordinate plane below and find the solution. Verify that the solution satisfies both equations.



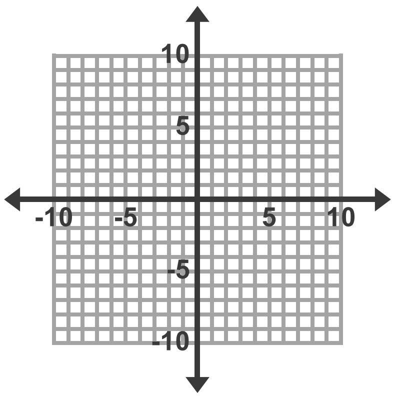
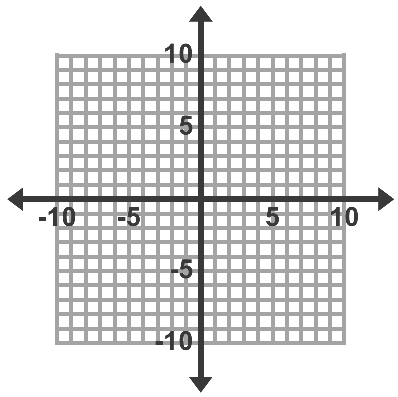
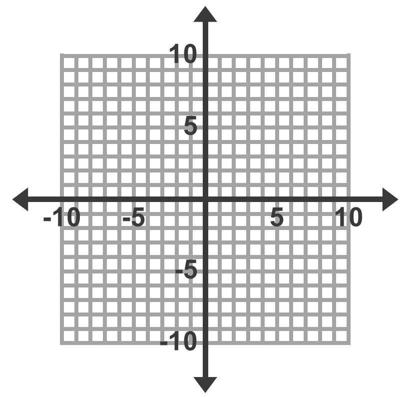
9. Consider the equations $-2x + y = -1$ and $y = 2x + 4$. Make sure both equations are written in slope-intercept form, then graph both equations on the coordinate plane below and solve the system of linear equations.



10. Consider the equations $x + y = 3$ and $3x + 3y = 9$. Graph both equations on the coordinate plane below and solve the system of linear equations.



11. In the table below, draw an example of a graph that represents the different solving outcomes of a system of linear equations:

One Solution	**No Solution**	**Infinitely Many Solutions**
		

12. Without graphing, determine whether the following systems of linear equations will have one solution, no solution, or infinitely many solutions.

a. $y = 8x + 2$ and $y = -4x$	b. $y = -\frac{2}{3}x - 5$ and $y + \frac{2}{3}x = 1$
c. $2x + y = 8$ and $y = 2x - 2$	d. $x + y = 5$ and $-2x - 2y = -10$
e. $3x + 2y = 5$ and $3x + 2y = 6$	f. $y = 2x + 5$ and $4x - 2y = -10$

13. One equation in a system of linear equations is $6x + 4y = -12$.
   a. Write a second equation for the system so that the system has only **one solution**.

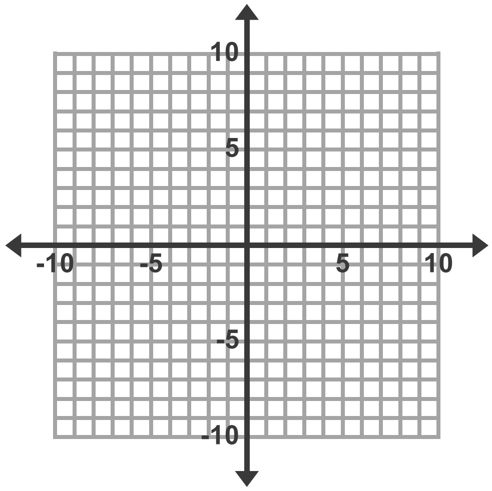
   b. Write a second equation for the system so that the system has **no solution**.

   c. Write a second equation for the system so that the system has **infinitely many solutions**.

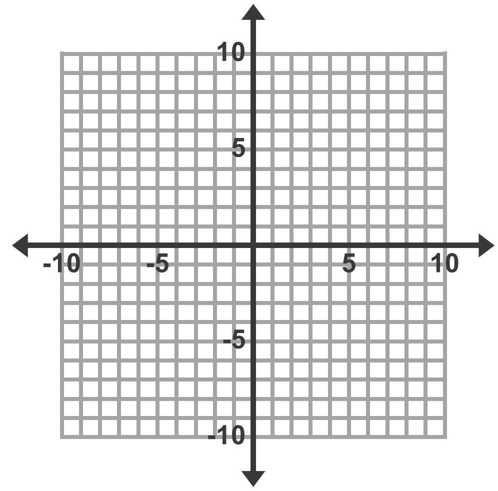
4.1c Homework: Solving Simultaneous Linear Equations by Graphing

1. Solve the system of linear equations graphically. If there is one solution, verify that your solution satisfies both equations.

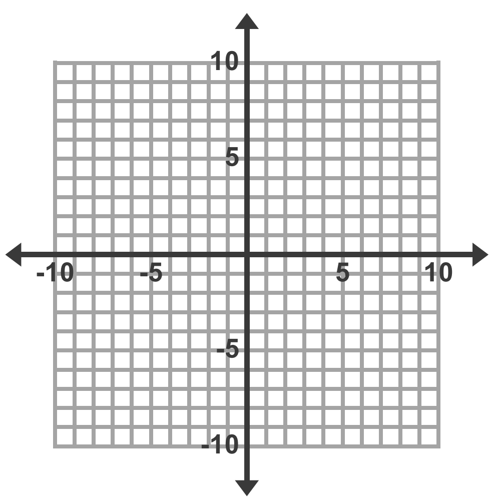
a. $y = 3x + 1$ and $x + y = 5$



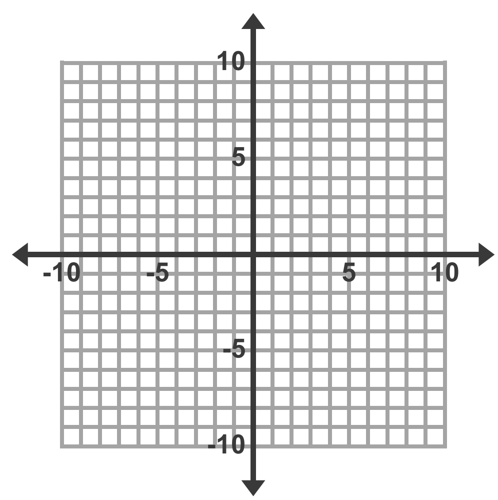
b. $y = -5$ and $2x + y = -3$



c. $y = -3x + 4$ and $y = \frac{1}{2}x - 3$

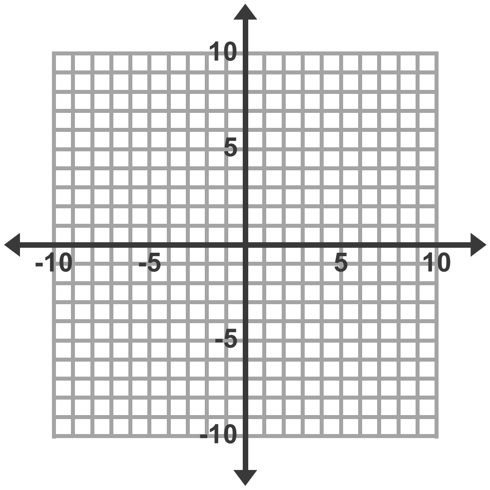


d. $x - y = -2$ and $-x + y = 2$

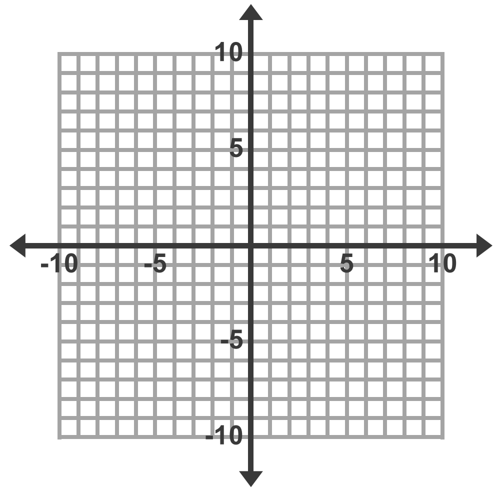


List 2 points that are solutions to this system.

e. $y = \frac{1}{2}x - 2$ and $y = \frac{1}{2}x + 4$



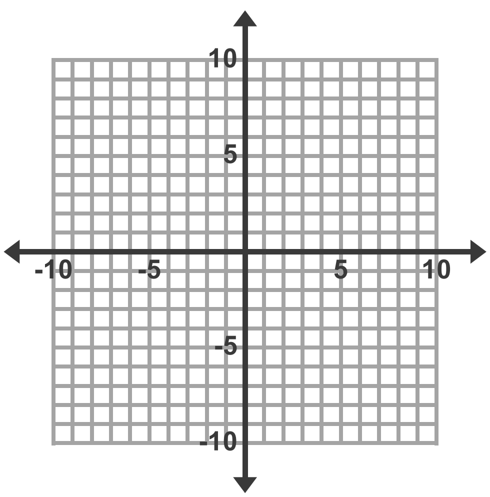
f. $2x - 8y = 6$ and $x - 4y = 3$



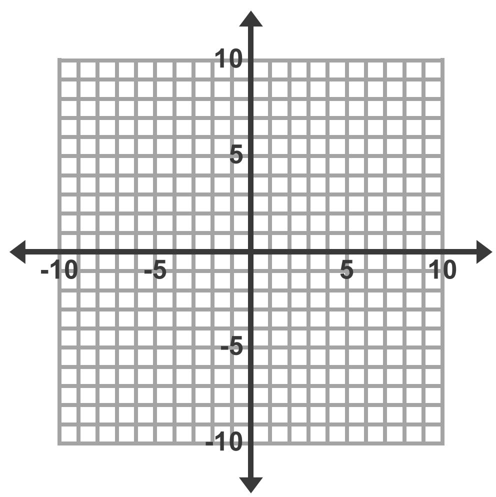
Circle the ordered pair(s) that are solutions to this system.

$(0, 0)$ $(0, -1)$ $(3, 0)$ $(9, 3)$

g. $y = 6x - 6$ and $y = 3x - 6$



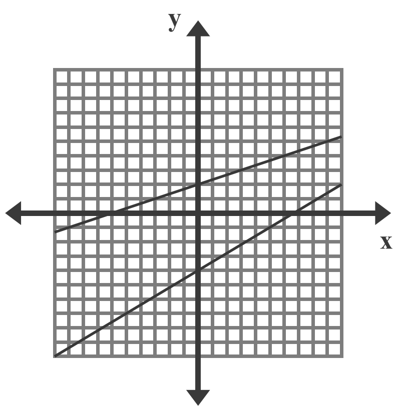
h. $2x + y = -4$ and $y + 2x = 3$



2. Without graphing, determine whether the following systems of linear equations will have one solution, no solution, or infinitely many solutions.

a. $x + y = 5$ and $x + y = 6$	b. $-3x + 9y = 15$ and $y = \frac{1}{3}x + \frac{5}{3}$
c. $y = 6$ and $y = 2x + 1$	d. $x - y = 5$ and $x + y = 5$

3. How many solutions does the system of linear equations graphed below have? How do you know?

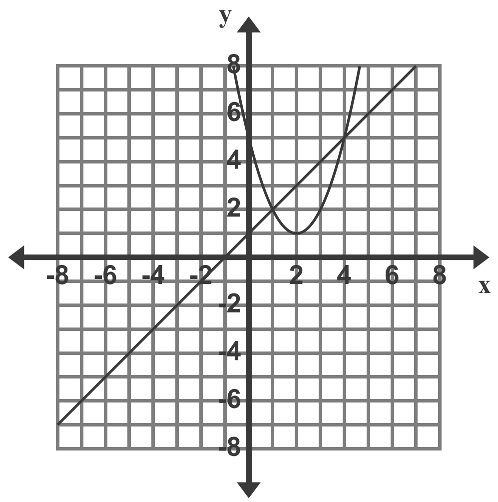


4. One equation in a system of linear equations is $y = x - 4$.
   a. Write a second equation for the system so that the system has only **one solution**.

   b. Write a second equation for the system so that the system has **no solution**.

   c. Write a second equation for the system so that the system has **infinitely many solutions**.

5. The grid below shows the graph of a line and a parabola (the curved graph).



a. How many solutions do you think there are to this system of equations? Explain your answer.

b. Estimate the solution(s) to this system of equations.

c. The following is the system of equations graphed above.
$y = x + 1$
$y = (x - 2)^2 + 1$

How can you verify whether the solution(s) you estimated in part b) are correct?

d. Verify the solution(s) from part b).

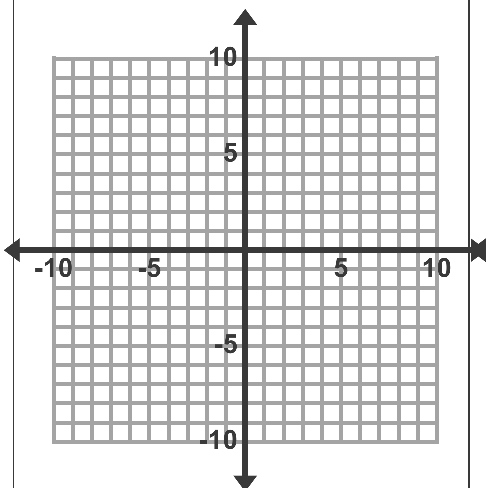
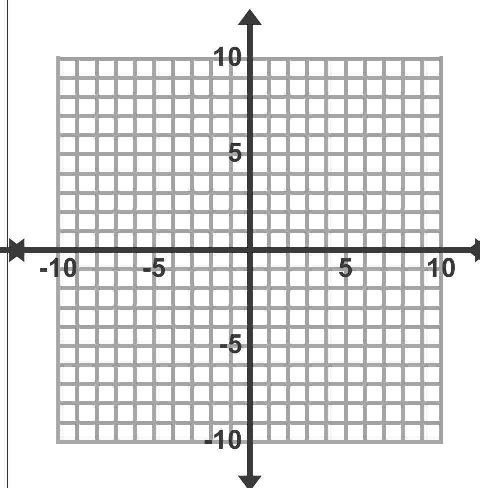
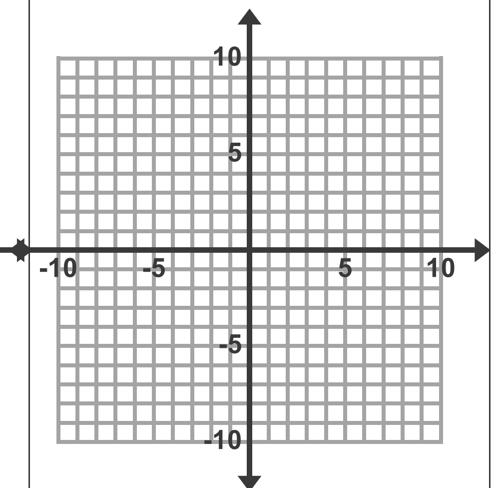
4.1d Self-Assessment: Section 4.1

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept.

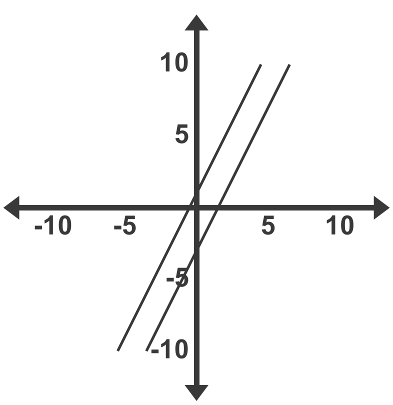
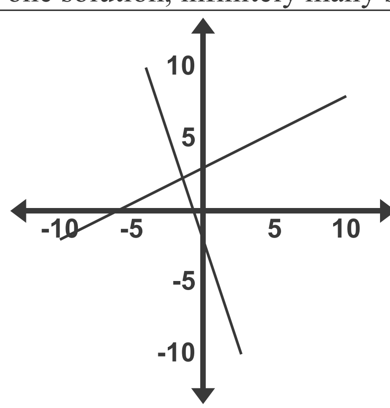
Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Mastery 3	Substantial Mastery 4
1. Solve simultaneous linear equations by graphing. Sample Problem #1	I can identify the solution to a system of linear equations when given the graphs of both equations.	I know that I can find a solution to a system of equations by graphing, but I often mess up with the graphing or getting the equations in slope-intercept form.	I can graph to find the solution to a system of equations, but I am not sure how to verify using algebra that the solution is correct.	I can re-write equations in slope-intercept form, graph them to find the solution, and plug the solution back in to verify my answer with very few mistakes.
2. Understand what it means to solve a system of equations. Sample Problem #1	I know that when I graph a system of equations, the answer is where the lines cross.	I know that when I graph a system of equations, the answer is the point where the two lines intersect, and is written as an ordered pair (x, y).	I know that the solution to a system of equations is the point where the lines intersect, and that if you plug this point into the equations they should both be true.	I understand that the solution to a system of equations is the point on the coordinate plane where two lines intersect and because of this, it is also an ordered pair that satisfies both equations at the same time.
3. Identify and provide examples of systems of equations that have one solution, infinitely many solutions, or no solution. Sample Problems #2, #3	I can look at the graph of a system of equations and tell if it has one, no, or infinitely many solutions, but I sometimes get them mixed up.	I can look at the graph of a system of equations and tell if it has one, no, or infinitely many solutions. I can sometimes tell just by looking at the equations as well.	I know how to tell how many solutions a system of equations has by looking at a graph. I can also tell how many solutions a system of equations has by looking at the equations. When given an equation, I can write another equation that would give the system of equations one, no, or infinitely many solutions.	I know how to tell how many solutions a system of equations has by looking at a graph and by looking at just the equations. I understand what it is about the structure of the equations that makes the graphs look the way they do. I can write a system of equations that would have one solution, no solution, or infinitely many solutions.
4. Interpret the solution to a system in a context. Sample Problem #4	When I solve a story problem involving a system of equations I struggle to explain what the solution represents in the context.	When I solve a story problem involving a system of equations, I understand what the solution means, and I can explain to someone what my answer means most of the time.	When given a story problem involving a system of equations, I can write a sentence explaining what the answer means in the context.	When given a story problem involving a system of equations, I can write a sentence describing what the answer means in the context. I can also answer additional questions about the situation.

Section 4.1 Sample Problems (For use with self-assessment)

1. Graph the following systems of equations to find the solution. After you have found your solution, verify that it is correct.

$\begin{cases} y = 3x - 5 \\ y = \frac{1}{2}x \end{cases}$	$\begin{cases} y = -2x + 7 \\ x + 3y = -9 \end{cases}$	$\begin{cases} -4x + 6y = 6 \\ x + y = 6 \end{cases}$
		
Verify:	Verify:	Verify:

2. Tell whether the system of equations has one solution, infinitely many solutions, or no solutions.

		$\begin{cases} y = 3x + 4 \\ 2y = 6x + 8 \end{cases}$ $\begin{cases} y = -\frac{1}{4}x + 6 \\ y = -\frac{1}{4}x - 4 \end{cases}$

3. One equation in a system of linear equations is $y = -2x + 4$.
   a. Write a second equation for the system so that the system has only **one solution**.

   b. Write a second equation for the system so that the system has **no solution**.

   c. Write a second equation for the system so that the system has **infinitely many solutions**.

4. At the county fair, you and your little sister play a game called Honey Money. In this game she covers herself in honey and you dig through some sawdust to find hidden money and stick as much of it to her as you can in 30 seconds. The fair directors have hid only $1 bills and $5 bills in the sawdust. During the game your little sister counts as you put the bills on her. She doesn't know the difference between $1 bills and $5 bills, but she knows that you put 16 bills on her total. You were busy counting up how much money you were going to make, and you came up with a total of $40. After the activity you put the all the money into a bag and your little sister takes it to show her friends and loses it. The fair directors find a bag of money, but say they can only give it to you if you can tell them how many $1 bills you had, and how many $5 bills you had. What will you tell the fair directors so you can get your money back?

   a. Solve this problem using any method you wish. Show your work in the space below.

   b. Write your response to the fair directors in a complete sentence on the lines provided.

Section 4.2: Solve Simultaneous Linear Equations Algebraically

Section Overview:
In this section, students are solving simultaneous linear equations that have one, no, or infinitely many solutions using algebraic methods. The section utilizes concrete models and real world problems in order to help students grasp the concepts of substitution and elimination. Students then solve systems of linear equations abstractly by manipulating the equations. Students then apply the skills they have learned in order to solve real world problems that can be modeled and solved using simultaneous linear equations.

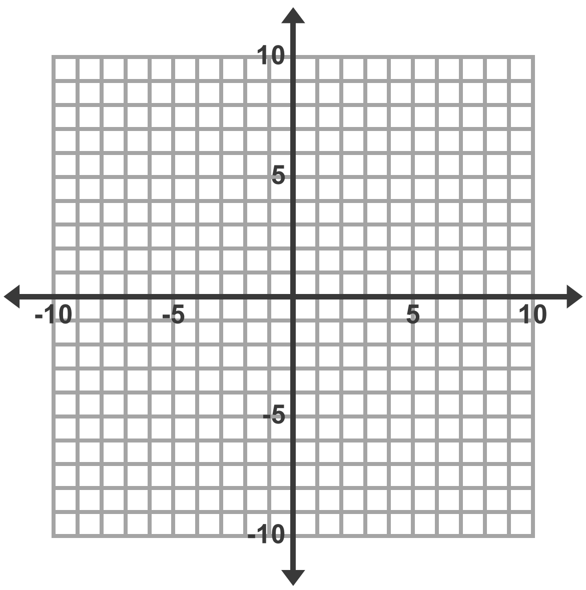
Concepts and Skills to Master:
By the end of this section, students should be able to:

1. Determine which method of solving a system of linear equations may be easier depending on the problem.
2. Solve simultaneous linear equations algebraically.
3. Create a system of linear equations to model a real world problem, solve the system, and interpret the solution in the context.

4.2a Class Activity: Introduction to Substitution

In the previous section, you learned how to solve a system of linear equations by graphing. In this section, we will learn another way to solve a system of linear equations. Solve the following system by graphing.

$y = 3x + 2$
$y = -5x$



Give some reasons as to why graphing is not always the best method for solving a system of linear equations.

In this section, we will learn about algebraic methods for solving systems of linear equations. These methods are called substitution and elimination.

Directions: Find the value of each shape. Verify your answers.

1.
◯ + ◯ + □ =25
□ =5

◯ = ________

How did you determine the circle's value?

2.

$$\bigcirc + \bigcirc + \square = 18$$
$$\bigcirc + \square = 10$$

$\square$ = ______

$\bigcirc$ = ______

How did you determine the value of each shape?

3.

$$\bigcirc + \bigcirc + \square + \square = 20$$
$$\bigcirc + \bigcirc + \square = 17$$

$\square$ = ______

$\bigcirc$ = ______

How did you determine the value of each shape?

4.

$$\triangle + \triangle + \triangle = 27$$
$$\triangle + \square = 8$$

$\triangle$ = ______

$\square$ = ______

How did you determine the value of each shape?

5.

$$\star + \star = 16$$
$$\star + \star + \triangle + \triangle = 26$$

$\star$ = ______

$\triangle$ = ______

How did you determine the value of each shape?

6.

☆ + ☆ + ○ + ○ + ○ = 19

☆ + ☆ + ○ = 13

☆ = ______

○ = ______

How did you determine the value of each shape?

7.

○ + □ + □ = 30

□ = ○ + ○

□ = ______

○ = ______

How did you determine the value of each shape?

8.

○ + ○ + △ + △ = 16

○ + ○ = △ + △ + 4

○ = ______

△ = ______

How did you determine the value of each shape?

9.

□ + □ + ☆ = 21

□ + □ + □ + □ + ☆ + ☆ = 42

□ = ______

☆ = ______

Directions: Draw a picture of each equation with shapes and then find the value of each shape.

10. $3x + 2y = 41$

    $2y = 8$

11. $2x + y = 9$

    $x + y = 5$

12. $x + 3y = 41$

    $x + 2y = 32$

13. $2x + 2y = 18$

    $2x = y$

Challenge Questions: Find the value of each variable using shapes.

14. $x + 2y = 46$

    $y + 3z = 41$

    $3z = 27$

15. $2x + z = 46$

    $3z = 18$

    $2y + z = 40$

16. $2x + 2y = 50$

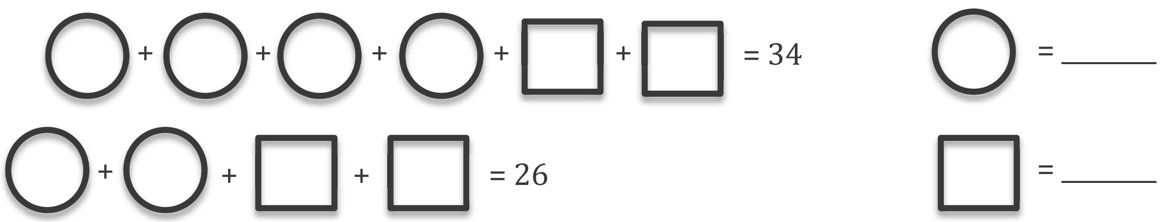
    $2x + y = 42$

    $y + 2z = 18$

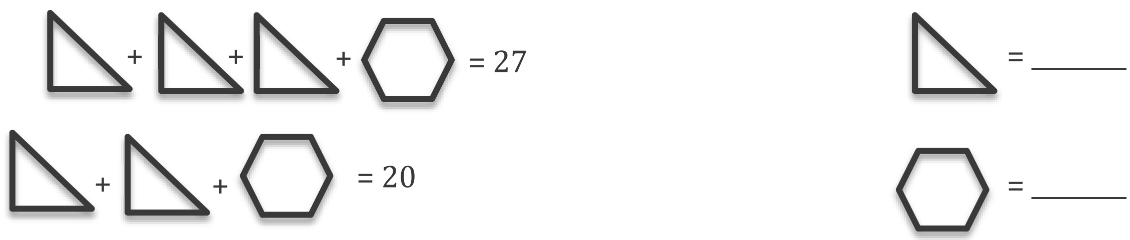
4.2a Homework: Introduction to Substitution

Directions: Find the value of each shape. Explain how you determined each. Verify your answers.

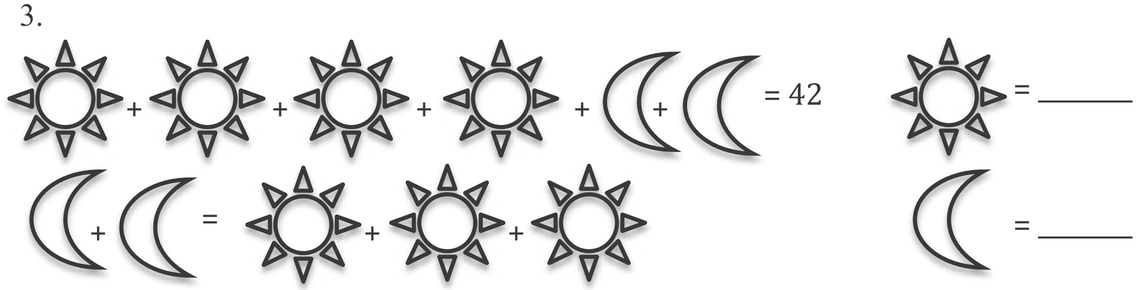
1.



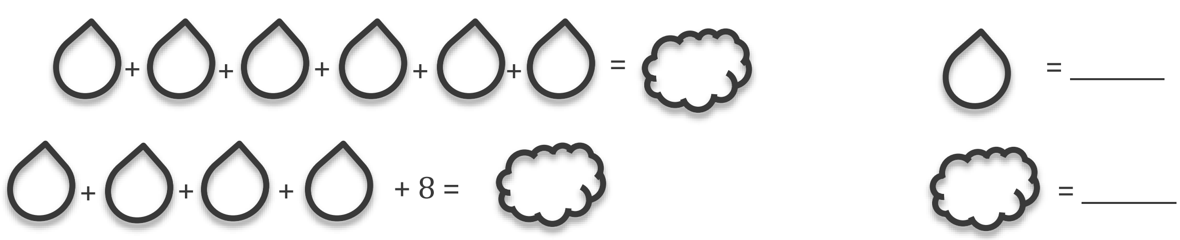
2.



3.



4.



Directions: Draw a picture of each equation with shapes and then find the value of each shape.

5. $x + y = 15$

   $y = x + 10$

6. $y + x = 5$

   $x = y - 3$

7. $y = 4x$

   $x + y = 5$

8. $2x + y = 7$

   $x + y = 1$

9. $3x + 4y = 19$

$3x + 6y = 33$

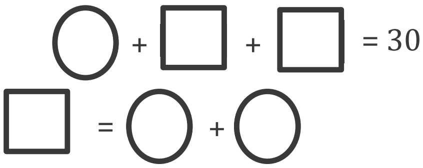
10. $5x + 6y = 100$

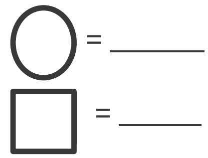
$4x + 6y = 92$

4.2b Class Activity: Substitution Method for Solving Systems of Equations

Directions: Write a system of equations from the shapes. Find the value of each shape.

1.

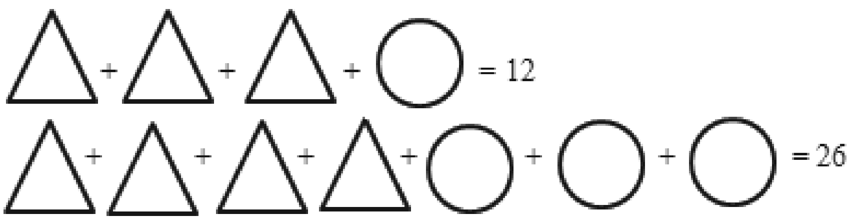


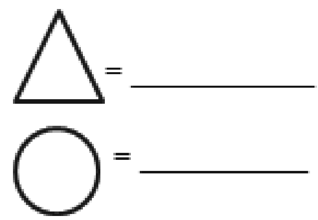


System of Equations:

How did you determine the value of each shape?

2.





System of Equations:

How did you determine the value of each shape?

To solve any system of linear equations using substitution, do the following:

1. Rewrite one of the equations so that one variable is expressed in terms of the other (solve one of the equations for one of its variables).
2. Substitute the expression from step 1 into the other equation and solve for the remaining variable.
3. Substitute the value from step 2 into the equation from step 1 and solve for the remaining variable.
4. Check the solution in each of the original equations.

Revisit problem #2 from above and use these steps to solve.

3.

$$\square + \square + 7 = \bigcirc$$
$$\square + \square + \square + \bigcirc + \bigcirc = 35$$

a. Write a system of equations for the picture above.

b. Solve this system of equations using substitution showing all steps. Check your solution.

4.

$$\star + \star + \star + 1 = \triangle$$
$$\star + \star + 3 = \triangle$$

a. Write a system of equations for the picture above.

b. Solve this system of equations using substitution showing all steps. Check your solution.

5.

○ + ○ + ○ + 5 = △

○ + ○ + ○ + 3 = △

○ = ______

△ = ______

a. Write a system of equations for the pictures above.

b. Solve this system of equations using substitution showing all steps. Check your solution.

c. Describe what you would see in a graph of this system.

6.

□ + □ + ☆ = 21

□ + □ + □ + □ + ☆ + ☆ = 42

□ = ______

☆ = ______

a. Write a system of equations for the pictures above.

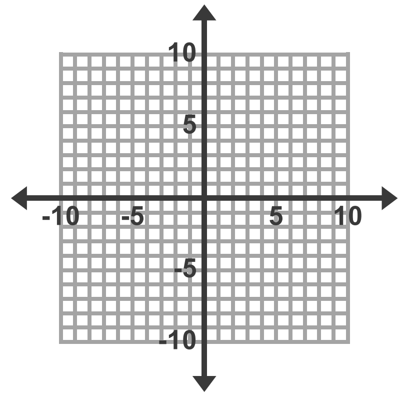
b. Solve this system of equations using substitution showing all steps. Check your solution.

c. Describe what you would see in a graph of this system.

Directions: Solve each system using the substitution method. When asked, solve the system by graphing in addition to using the substitution method.

7. $y = 5x + 4$
   $y = -3x - 12$

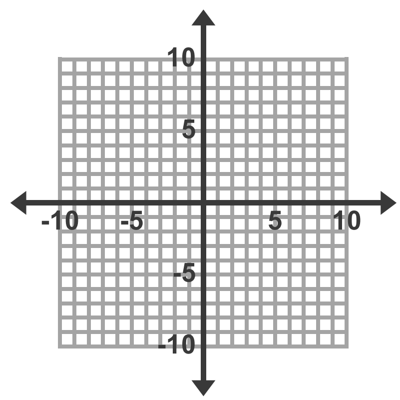
8. $y = 6x + 4$
   $y = 6x - 10$



Solve by graphing.

9. $y = x + 2$
   $x + 3y = -2$

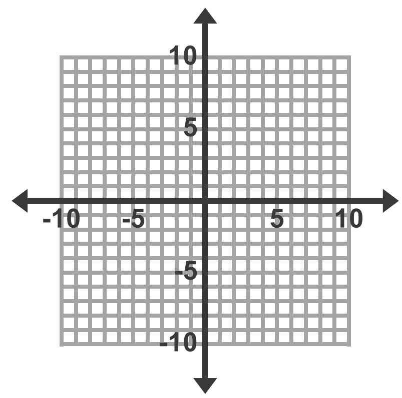
10. $x = 2y - 4$
    $x + y = 2$



Solve by graphing.

11. $y - x = 5$
    $2x + y = -10$

12. $2x + y = 5$
    $y = -5 - 2x$

13. $y + x = 5$ $x = y - 3$	14. $x + y = 4$ $y = -x + 4$ List 2 points that are solutions to this system.
15. $x + 2y = 7$ $2x + 3y = 12$	16. $6x + y = -5$ $-12x - 2y = 10$  Solve by graphing.
17. $x = 2y + 4$ $4y + x = 2$	18. $y = 3x + 2$ $y = -5x$

Directions: The following are examples of real-world problems that can be modeled and solved with systems of linear equations. Answer the questions for each problem.

19. Nettie's Bargain Clothing is having a huge sale. All shirts are $3 each and all pants are $5 each. You go to the sale and buy twice as many shirts as pants and spend $66.

    The following system of equations models this situation where s = number of shirts and p = number of pants:
    $s = 2p$
    $3s + 5p = 66$

    a. Write in words what each of the equations in the system represents in the context.

       $s = 2p$ ____

       $3s + 5p = 66$ ____

    b. Solve this system using substitution to determine how many of each item you bought. Write your answer in a complete sentence.

20. Xavier and Carlos have a bet to see who can get more "friends" on a social media site after 1 month. Carlos has 5 more friends than Xavier when they start the competition. After much work, Carlos doubles his amount of friends and Xavier triples his. In the end they have a total of 160 friends together.

    The following system of equations models this situation where c = the number of friends Carlos starts with and x = the number of friends Xavier starts with.
    $c = x + 5$
    $2c + 3x = 160$

    a. Write in words what each of the equations in the system represents in the context.

       $c = x + 5$ ____

       $2c + 3x = 160$ ____

    b. Solve this system using substitution to determine how many friends each boy started with. Write your answer in a complete sentence.

4.2b Homework: Substitution Method for Solving Systems of Equations

Directions: Solve each system of linear equations using substitution.

1. $y = 4x$ $x + y = 5$	2. $x = -4y$ $3x + 2y = 20$	3. $y = x - 1$ $x + y = 3$
4. $3x - y = 4$ $2x - 3y = -9$	5. $x - y = 6$ $2x = 12 + 2y$	6. $2x + 2y = 6$ $x + y = 0$
7. $2x + y = -15$ $y - 5x = 6$	8. $y = 3x + 4$ $y = x - 7$	9. $y = -3x + 6$ $9x + 3y = 18$

10. Niona needs \$50 to go on a school trip. She sells necklaces for \$15 each and bracelets for \$5 each. If she raises the money by selling half as many necklaces as bracelets, how many necklaces and bracelets does she sell? Write a system of linear equations that represent this problem.

Define your Unknowns:

System of Equations:

Next to each equation, write in words what the equation represents in the context.

Solve. Write your answer in a complete sentence.

11. A restaurant needs to order stools and chairs. Each stool has 3 legs and each chair has 4 legs. The manager wants to be able to seat 36 people. The restaurant has hard wood floors and the manager doesn't want to scratch them. Therefore, they have ordered 129 plastic feet covers for the bottom of the legs to ensure the stools and chairs don't scratch the floor. How many chairs and how many stools did the restaurant order?
    a. Write a system of equations that matches the verbal descriptions given below if s = number of stools and c = number of chairs.

System of Equations:

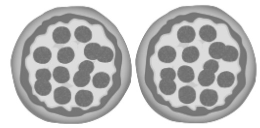
Equation 1: ______________ Each stool needs 3 plastic feet covers. Each chair needs 4 plastic feet covers. One hundred twenty-nine plastic feet covers are needed.

Equation 2: _________________ There are a total of 36 chairs and stools needed.

b. Solve the system. Write your answer in a complete sentence.

4.2c Class Activity: Elimination Method for Solving Systems of Linear Equations

1. Ariana and Emily are both standing in line at Papa Joe's Pizza. Ariana orders 4 large cheese pizzas and 1 order of breadsticks. Her total before tax is $34.46. Emily orders 2 large cheese pizzas and 1 order of breadsticks. Her total before tax is $18.48. Determine the cost of 1 large cheese pizza and 1 order of breadsticks. **Explain** the method you used for solving this problem.



2. Carter and Sani each have the same number of marbles. Sani's little sister comes in and takes some of Carter's marbles and gives them to Sani. After she has done this, Sani has 18 marbles and Carter has 10 marbles. How many marbles did each of the boys start with? How many marbles did Sani's sister take from Carter and give to Sani?

3.

$$\bigcirc + \bigcirc + \bigcirc + \bigcirc + \square = 18 \qquad \bigcirc = \underline{\qquad}$$

$$\bigcirc + \bigcirc + \square = 10 \qquad \square = \underline{\qquad}$$

a. Find the value of each shape.

b. Which of the problems from the previous page is this similar to? Compare the strategies you used to solve these problems.

4.

$$\bigcirc + \square = 15 \qquad \bigcirc = \underline{\qquad}$$

$$\bigcirc - \square = 7 \qquad \square = \underline{\qquad}$$

a. Find the value of each shape.

b. Which of the problems from the previous page is this similar to? Compare the strategies you used to solve these problems.

Directions: Write a system of equations from the shapes. Find the value of each shape. Show the solving actions algebraically.

5. ○ + ○ + □ = 14

○ + ○ − □ = 10

○ = _____

□ = _____

Equations:

$$2c + s = 14$$

$$2c - s = 10$$

$c\ (circle) =$

$s\ (square) =$

6. □ + ○ + ○ = 19

○ − □ = 11

○ = _____

□ = _____

Equations:

7. □ + □ + ○ = 27

○ − □ − □ = 15

○ = _____

□ = _____

Equations:

8. The name of the method you are using to solve the systems of linear equations above is **elimination**. Why do you think this method is called **elimination**?

Directions: Write a system of equations from the shapes. Find the value of each shape. Show the solving actions algebraically.

9.

☆ + ☆ − ○ = 8

☆ + ☆ − ○ − ○ = 4

☆ = _____

○ = _____

10.

△ + △ + △ + △ + ☆ + ☆ = 8

△ + △ + ☆ + ☆ = -6

△ = _____

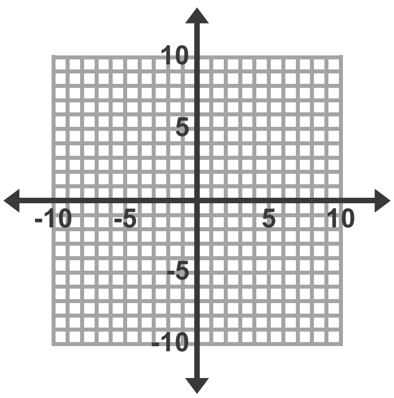
☆ = _____

11. How are problems 9 and 10 different from #5 – 7. Describe in your own words how you solved the problems in this lesson.

Directions: Solve each system of linear equations using **elimination**. Make sure the equations are in the same form first. Graph the first three problems as well as using elimination.

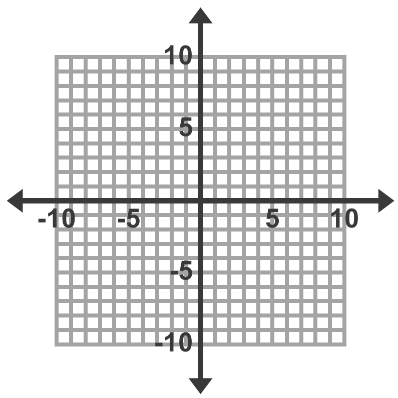
12. $x + y = -3$
$2x - y = -3$

Solve by graphing.



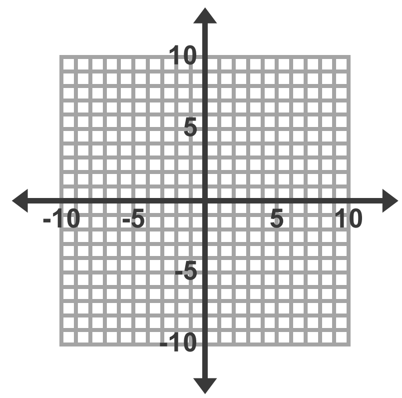
13. $x + y = 5$
$-x - y = -5$

Solve by graphing.



14. $x + y = 3$
$2x - y = -3$

Solve by graphing.



15. $2x - y = -3$
$3x - y = 1$

16. $2x - y = 9$
$x + y = 3$

17. $7x - 4y = -30$
$3x + 4y = 10$

18. $2x + y = 6$ $2x + y = -7$	19. $3x - y = 1$ $x = -y + 3$	20. $x = y + 3$ $x - 2y = 3$

21. Complete the story for the system of equations shown below if s is number of shirts and p is number of pants. Solve the system and write your solution in a complete sentence.
$s + p = 18$
$5s + 12p = 160$

Story

Jennifer is buying shirts and pants at a sale.
She buys 18...

Shirts cost $5 each and pants cost...

Jennifer spends...
How many shirts and how many pants did Jennifer purchase?

Solve:

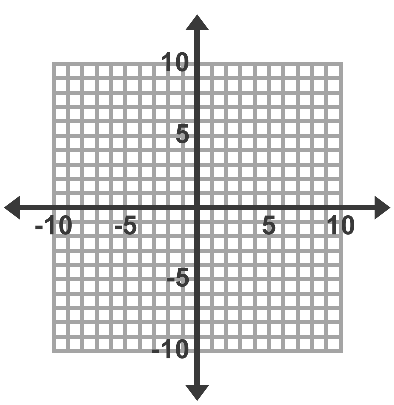
Solution (in a complete sentence):

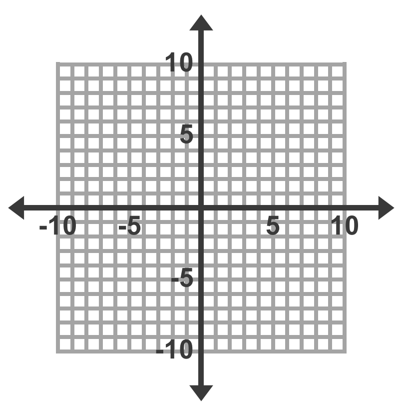
4.2c Homework: Elimination Method of Solving Linear Systems

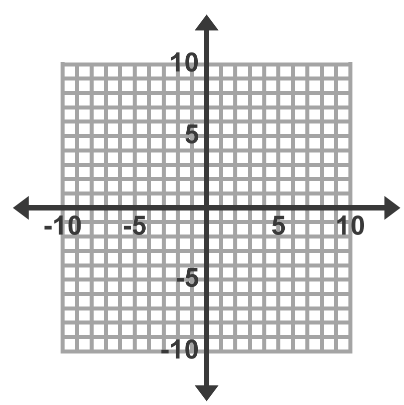
Directions: Solve each system of linear equations using **elimination**. Make sure the equations are in the same form first. Choose three problems to solve by graphing as well as using elimination to solve the system. The graphs are located after problem #9.

1. $6x - y = 5$ $3x + y = 4$	2. $x + 4y = 9$ $-x - 2y = 3$	3. $x + 5y = -8$ $-x - 2y = -13$
4. $2x + y = 7$ $x + y = 1$	5. $4x + 3y = 18$ $4x = 8 + 2y$	6. $-5x + 2y = 22$ $3x + 2y = -10$

7. $6x - 3y = 36$ $5x = 3y + 30$	8. $-4x + y = -12$ $-y + 6x = 8$	9. $x + y = 7$ $-x - y = 12$







10. An exam worth 145 points contains 50 questions. Some of the questions are worth two points and some are worth five points. How many two-point questions are on the test? How many five-point questions are on the test?

4.2d Class Activity: Elimination Method Multiply First

1. Solve the following system of linear equations using **elimination**:
   $4x + y = 7$
   $-2x - 3y = -1$

To solve <u>any</u> system of linear equations using elimination, do the following:

1. Write both equations in the same form.
2. Multiply the equations by nonzero numbers so that one of the variables will be eliminated if you take the sum or difference of the equations.
3. Take the sum or difference of the equations to obtain a new equation in just one unknown.
4. Solve for the remaining variable.
5. Substitute the value from step 4 back into one of the original equations to solve for the other unknown.
6. Check the solution in each of the original equations.

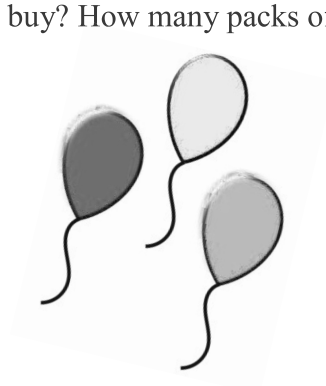
Directions: Solve each system of linear equations using **elimination**.

2. $x + 2y = 15$ $5x + y = 21$	3. $-3x + 2y = -8$ $6x - 4y = -20$
4. $2x - 3y = 5$ $-3x + 4y = -8$	5. $3x - 2y = 2$ $5x - 5y = 10$
6. $9x + 13y = 10$ $-9x - 13y = 8$	7. $-16x + 2y = -2$ $y = 8x - 1$

8. $\frac{1}{2}x + y = 0$ $x - y = -15$	9. $\frac{2}{3}x + \frac{1}{3}y = 13$ $-2x + y = -21$

Directions: Write a system of equations for each word problem below and then solve the system of equations using **elimination**. Write your answer in a complete sentence.

10. The student officers are buying packs of streamers and balloons to decorate for a school dance. Packs of balloons cost \$3.50 and packs of streamers cost \$2. If the student officers bought a total of 12 packs of decorations and spent \$31.50, how many packs of balloons did they buy? How many packs of streamers did they buy? Write the solution in a complete sentence.



Define your Unknowns:

Equation for Number of Packs of Decorations:

Equation for Cost of Decorations:

Solve:

Solution (in a complete sentence):

11. Jayda has a coin collection consisting of nickels and dimes. Write a story that matches the system of equations shown below that describes the coins in Jayda's collection where *n* is the number of nickels Jayda has and *d* is the number of dimes Jayda has.

$n + d = 28$
$0.05n + .1d = 2.25$

Story

Solve:

Solution (in a complete sentence):

4.2d Homework: Elimination Method Multiply First

Directions: Solve each system using **elimination**.

1. $x + y = 4.5$ $-2x + 4y = 6$	2. $4x + y = -8$ $3x + 3y = 3$
3. $2x + y = 7$ $4x + 2y = 14$	4. $2x + 3y = -10$ $-4x + 5y = -2$
5. $x - 2y = \frac{2}{3}$ $-3x + 5y = -2$	6. $-3x - y = -15$ $8x + 4y = 48$
7. $3x - y = 10$ $2x + 5y = 35$	8. $x + y = 15$ $-2x - 2y = 30$

Directions: Write a system of equations for each word problem below and then solve the system of equations using **elimination**. Write your answer in a complete sentence.

9. Tickets for a matinee are \$5 for children and \$8 for adults. The theater sold a total of 142 tickets for one matinee. Ticket sales were \$890. How many of each type of ticket did the theater sell? Write the solution in a complete sentence.

   Define your Unknowns:

   Equation for Number of Tickets Sold:

   Equation for Ticket Sales:

   Solve:

   Solution (in a complete sentence):

10. Jasper has a coin collection consisting of quarters and dimes. He has 50 coins worth \$8.60. How many of each coin does he have? Write the solution in a complete sentence.

    Define your Unknowns:

    Equation for Number of Coins:

    Equation for Value of Coins:

    Solve:

    Solution (in a complete sentence):

4.2e Class Activity: Revisiting Chickens and Pigs

1. A farmer saw some chickens and pigs in a field. He counted 60 heads and 176 legs. Determine exactly how many chickens and pigs he saw.
    a. Solve the problem using the methods/strategies studied in this chapter. Solve in as many different ways as you can (graph, substitution, and elimination) and make connections between the strategies.

    b. Which method do you prefer using to solve this problem? Use your preferred method to determine the number of chickens and pigs in a field with 45 heads and 146 legs.

4.2e Homework: Revisiting Chickens and Pigs

Directions: Solve each of the following problems by writing and solving a system of equations. Use any method you wish to solve. Write your answer in a complete sentence.

1. In 1982, the US Mint changed the composition of pennies from all copper to zinc with copper coating. Pennies made prior to 1982 weigh 3.1 grams. Pennies made since 1982 weigh 2.5 grams. If you have a bag of 1,254 pennies, and the bag weighs 3,508.8 grams, how many pennies from each time period are there in the bag?

2. Blake has some quarters and dimes. He has 20 coins worth a total of $2.90. How many of each type of coin does he have?

3. Ruby and Will are running a team relay race. Will runs twice as far as Ruby. Together they run 18 miles. How far did each person run?

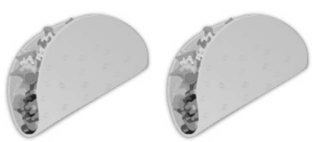
4. Sarah has \$400 in her savings account and she has to pay \$15 each month to her parents for her cell phone. Darius has \$50 and he saves \$20 each month from his job walking dogs for his neighbor. At this rate, when will Sarah and Darius have the same amount of money? How much money will they each have?

5. The admission fee at a local zoo is \$1.50 for children and \$4.00 for adults. On a certain day, 2,200 people enter the zoo and \$5,050 is collected. How many children and how many adults attended?

6. Dane goes to a fast food restaurant and orders some tacos t and burritos b. Write a story that matches the system of equations shown below that describes the number of items Dane ordered and how many calories he consumed. Solve the system to determine how many tacos and how many burritos Dane ordered and ate.

$t + b = 5$
$170t + 370b = 1250$

Story



Solve:

Solution (in a complete sentence):

4.2f Class Activity: Solving Systems of Equations Mixed Strategies

Directions: Choose the method you feel is easiest for a given problem (graphing, substitution, or elimination). Place a letter in the box (g, s, or e) for each problem to identify the method you will be using before you solve the system. Solve each system of linear equations. There are blank graphs for you to use after #12.

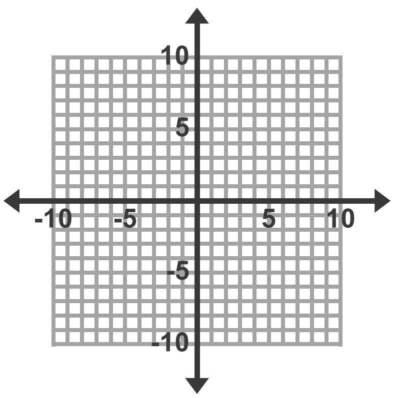
1. $2x - 3y = 12$ $x = 4y + 1$	2. $x + y = 3$ $3x - 4y = -19$
3. $y = x - 6$ $y = x + 2$	4. $y - 2x = 1$ $2x + y = 5$
5. $y = 4x - 3$ $y = x + 6$	6. $x - y = 0$ $2x + 4y = 18$
7. $3y - 9x = 1$ $y = 3x + \frac{1}{3}$	8. $x + 2y = 6$ $-7x + 3y = -8$

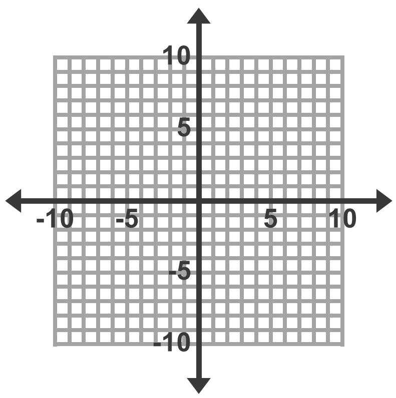
9. $y = -x + 5$
   $x - 4y = 10$

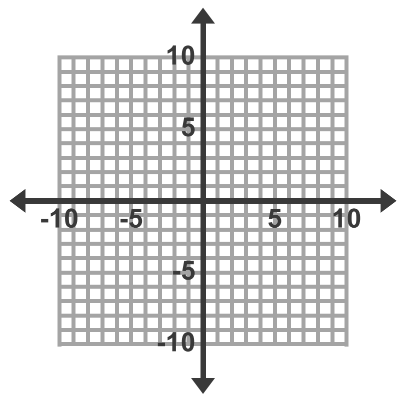
10. $y = x + 5$
    $y = 2x - 10$

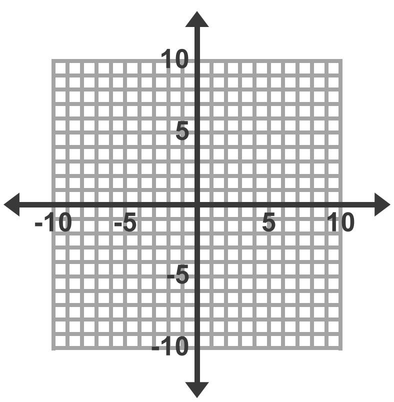
11. $3x + 2y = -5$
    $x - y = 10$

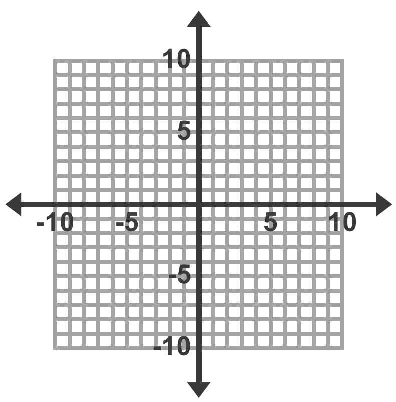
12. $2x - 5y = 6$
    $2x + 3y = -2$

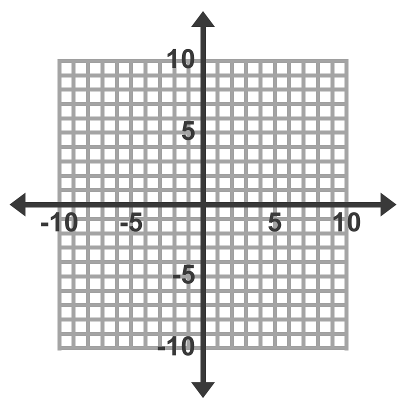












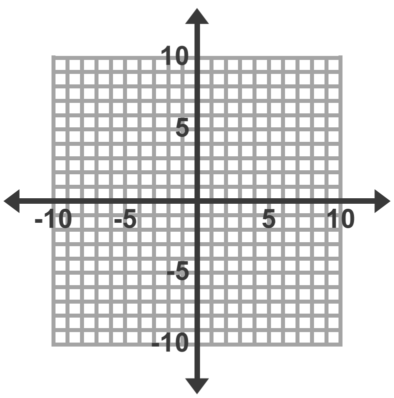
4.2f Homework: Solving Systems of Equations Mixed Strategies

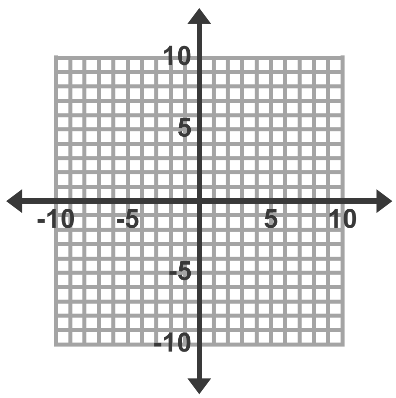
Directions: Choose the method you feel is easiest for a given problem (graphing, substitution, or elimination). Place a letter in the box (g, s, or e) for each problem to identify the method you will be using before you solve the system. Solve each system of linear equations. There are blank graphs for you to use after #8.

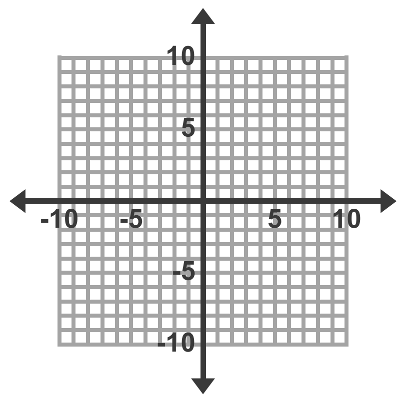
1. $y = 4x$ $x + y = 5$	2. $x = -4y$ $3x + 2y = 20$
3. $y = x - 1$ $y = -x + 3$	4. $3x - y = 4$ $2x - 3y = -9$
5. $x + 5y = 4$ $3x + y = -2$	6. $y = -x + 10$ $y = 10 - x$

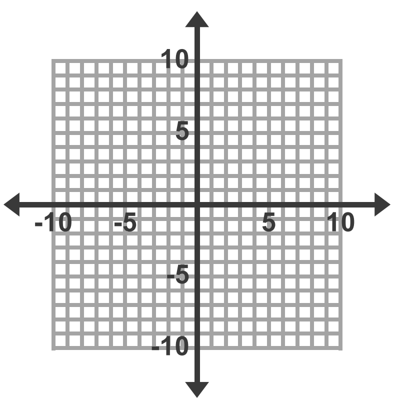
7. $y = 2x$
$x + y = 12$

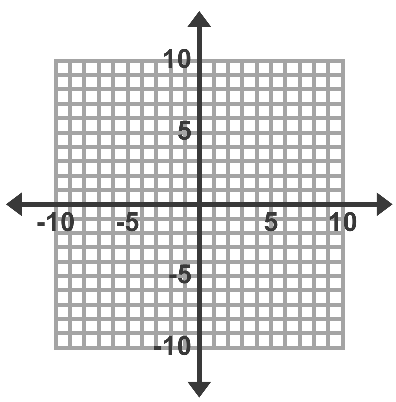
8. $y = 2x - 5$
$4x - y = 7$

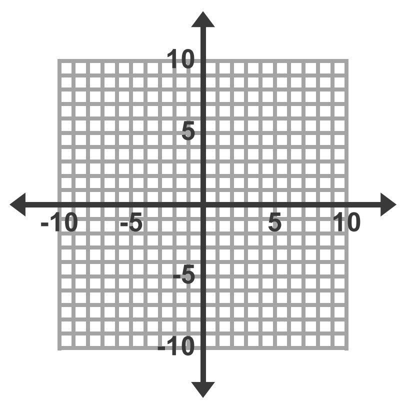


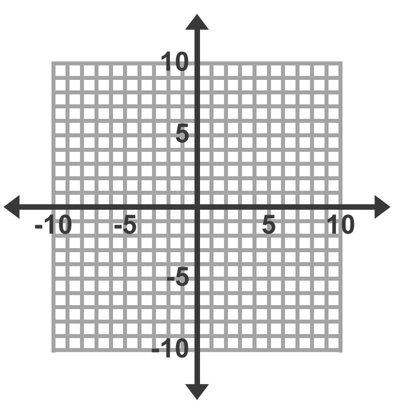


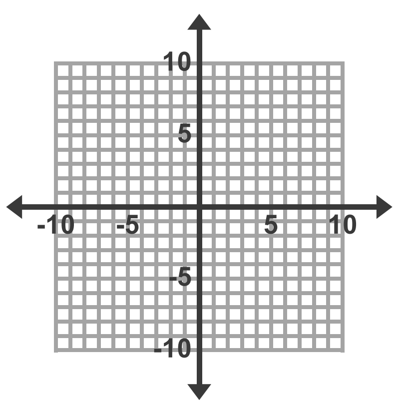


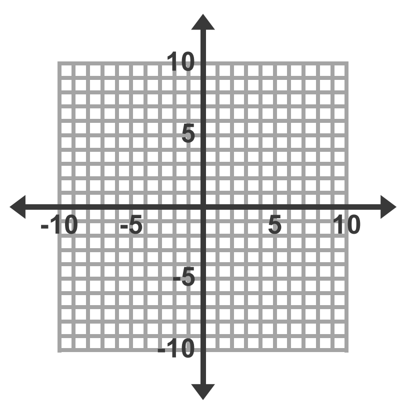












4.2g Self-Assessment: Section 4.2

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept.

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Mastery 3	Substantial Mastery 4
1. Determine which method of solving a system of linear equations may be easier depending on the problem. Sample Problems #1, #2.	I am struggling to determine the best method to use to solve a system of linear equations.	I can determine the best method to use to solve all of the equations in Set A on the following page.	I can determine the best method to use to solve all of the equations in Set A and most of the equations in Set B.	I can determine which method of solving a system of linear equations will be easier depending on the problem.
2. Solve simultaneous linear equations algebraically. Sample Problems #1, #2	I can solve the equations in Set A on the following page.	I can solve all of the equations in Set A and most of the equations in Set B.	I can solve simultaneous linear equations algebraically by using both substitution and elimination methods.	I can solve simultaneous linear equations algebraically by using both substitution and elimination methods. I can also explain why I chose a particular solution method.
3. Create a system of linear equations to model a real world problem, solve the system, and interpret the solution in the context. Sample Problem #3	When faced with word problems similar to those in this chapter, I can match the different pieces of the equations that have been given to me to the story and solve the system of equations.	When faced with word problems similar to those in this chapter, I can identify the important quantities in a practical situation, complete partial expressions and equations *that have been given to me*, solve the system, and interpret the solution in the context.	When faced with word problems similar to those in this chapter, I can identify the important quantities in a practical situation, write expressions and equations showing the relationship between the quantities, solve the equations, and interpret the solution in the context.	When faced with word problems similar to those in this chapter, I can identify the important quantities in a practical situation, *write* expressions and equations showing the relationship between the quantities, solve the equations, and interpret the solution in the context. I can explain the reasoning behind each step in the process of arriving at my answer.

Section 4.2 Sample Problems (For use with self-assessment)

1. Determine which method will be easier to use for each of the following problems by placing S (substitution), G (graphing) or E (elimination) in the small box in the corner of each problem.

2. Solve the following systems of equations algebraically.

Set A

1. $y = 2$ $y = 3x + 2$	2. $x = y + 2$ $4x - 3y = 11$	3. $x + 2y = 13$ $-x + 4y = 11$

Set B

1. $y = -4x + 8$ $5x + 2y = 13$	2. $2y = x - 5$ $2y = x + 5$	3. $3x = y - 20$ $-7x + y = 40$

3. Tickets to the local basketball arena cost \$54 for lower bowl seats and \$20 for upper bowl seats. A large group purchased 123 tickets at a cost of \$4,262. How many of each type of ticket did they purchase?