Chapter 1
Prerequisites for Calculus

Section 1.1 Lines (pp. 2–11)

Quick Review 1.1

1. $y = -2 + 4(3-3) = -2 + 4(0) = -2 + 0 = -2$

2. $3 = 3 - 2(x+1)$
$3 = 3 - 2x - 2$
$2x = -2$
$x = -1$

3. $m = \dfrac{2-3}{5-4} = \dfrac{-1}{1} = -1$

4. $m = \dfrac{2-(-3)}{3-(-1)} = \dfrac{5}{4}$

5. **(a)** $3(2) - 4\left(\dfrac{1}{4}\right) \overset{?}{=} 5$
$6 - 1 = 5$ Yes

(b) $3(3) - 4(-1) \overset{?}{=} 5$
$13 \neq 5$ No

6. **(a)** $7 \overset{?}{=} -2(-1) + 5$
$7 = 2 + 5$ Yes

(b) $1 = -2(-2) + 5$
$1 \neq 9$ No

7. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(0-1)^2 + (1-0)^2}$
$= \sqrt{2}$

8. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(1-2)^2 + \left(-\dfrac{1}{3} - 1\right)^2}$
$= \sqrt{(-1)^2 + \left(-\dfrac{4}{3}\right)^2}$
$= \sqrt{1 + \dfrac{16}{9}}$
$= \sqrt{\dfrac{25}{9}}$
$= \dfrac{5}{3}$

9. $4x - 3y = 7$
$-3y = -4x + 7$
$y = \dfrac{4}{3}x - \dfrac{7}{3}$

10. $-2x + 5y = -3$
$5y = 2x - 3$
$y = \dfrac{2}{5}x - \dfrac{3}{5}$

Section 1.1 Exercises

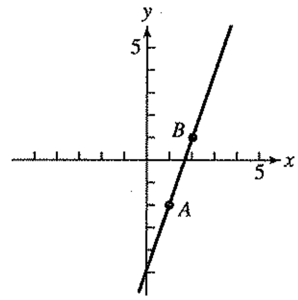
1. $\Delta x = -1 - 1 = -2$
$\Delta y = -1 - 2 = -3$

2. $\Delta x = -1 - (-3) = 2$
$\Delta y = -2 - 2 = -4$

3. $\Delta x = -8 - (-3) = -5$
$\Delta y = 1 - 1 = 0$

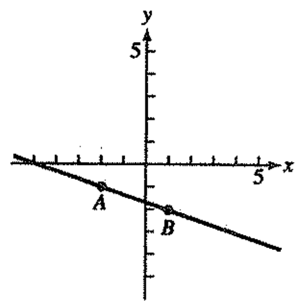
4. $\Delta x = 0 - 0 = 0$
$\Delta y = -2 - 4 = -6$

5. **(a, c)**



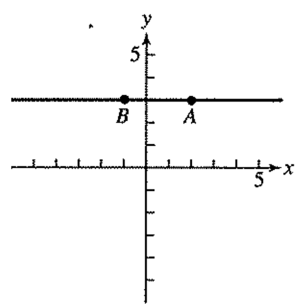
(b) $m = \dfrac{1-(-2)}{2-1} = \dfrac{3}{1} = 3$

6. **(a, c)**



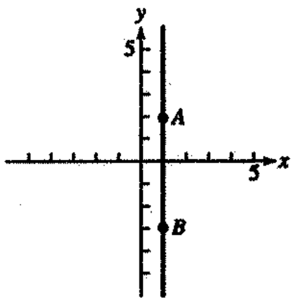
(b) $m = \dfrac{-2-(-1)}{1-(-2)} = \dfrac{-1}{3} = -\dfrac{1}{3}$

7. **(a, c)**



(b) $m = \dfrac{3-3}{-1-2} = \dfrac{0}{-3} = 0$

8. (a, c)



(b) $m=\frac{-3-2}{1-1}=\frac{-5}{0}$ (undefined)

This line has no slope.

9. (a) $x=3$

(b) $y=2$

10. (a) $x=-1$

(b) $y=\frac{4}{3}$

11. (a) $x=0$

(b) $y=-\sqrt{2}$

12. (a) $x=-\pi$

(b) $y=0$

13. $y=1(x-1)+1$

14. $y=-1[x-(-1)]+1$
$y=-1(x+1)+1$

15. $y=2(x-0)+3$

16. $y=-2[x-(-4)]+0$
$y=-2(x+4)+0$

17. $y=3x-2$

18. $y=-1x+2$ or $y=-x+2$

19. $y=-\frac{1}{2}x-3$

20. $y=\frac{1}{3}x-1$

21. $m=\frac{3-0}{2-0}=\frac{3}{2}$

$y=\frac{3}{2}(x-0)+0$

$y=\frac{3}{2}x$

$2y=3x$

$3x-2y=0$

22. $m=\frac{1-1}{2-1}=\frac{0}{1}=0$

$y=0(x-1)+1$

$y=1$

23. $m=\frac{-2-0}{-2-(-2)}=\frac{-2}{0}$ (undefined)

Vertical line: $x=-2$

24. $m=\frac{-2-1}{2-(-2)}=\frac{-3}{4}=-\frac{3}{4}$

$y=-\frac{3}{4}[x-(-2)]+1$
$4y=-3(x+2)+4$
$4y=-3x-2$
$3x+4y=-2$

25. The line contains (0, 0) and (10, 25).

$m=\frac{25-0}{10-0}=\frac{25}{10}=\frac{5}{2}$

$y=\frac{5}{2}x$

26. The line contains (0, 0) and (5, 2).

$m=\frac{2-0}{5-0}=\frac{2}{5}$

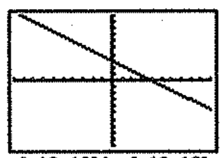
$y=\frac{2}{5}x$

27. $3x+4y=12$
$4y=-3x+12$
$y=-\frac{3}{4}x+3$

(a) Slope: $-\frac{3}{4}$

(b) y-intercept: 3

(c)



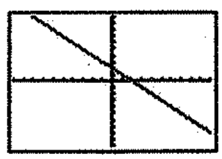
[−10, 10] by [−10, 10]

28. $x+y=2$
$y=-x+2$

(a) Slope: −1

(b) y-intercept: 2

(c)



[−10, 10] by [−10, 10]

29. $\frac{x}{3}+\frac{y}{4}=1$

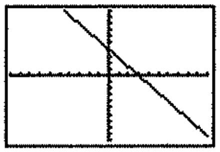
$\frac{y}{4}=-\frac{x}{3}+1$

$y=-\frac{4}{3}x+4$

(a) Slope: $-\frac{4}{3}$

(b) y-intercept: 4

(c)



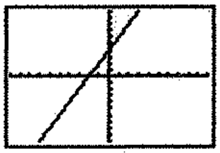
[−10, 10] by [−10, 10]

30. $y=2x+4$

(a) Slope: 2

(b) y-intercept: 4

(c)



[−10, 10] by [−10, 10]

31. (a) The desired line has slope −1 and passes through (0, 0):

$y=-1(x-0)+0$ or $y=-x$.

(b) The desired line has slope $\frac{-1}{-1}=1$ and passes through (0, 0):

$y=1(x-0)+0$ or $y=x$.

32. (a) The given equation is equivalent to $y=-2x+4$. The desired line has slope −2 and passes through (−2, 2):

$y=-2(x+2)+2$ or $y=-2x-2$.

(b) The desired line has slope $\frac{-1}{-2}=\frac{1}{2}$ and passes through (−2, 2):

$y=\frac{1}{2}(x+2)+2$ or $y=\frac{1}{2}x+3$.

33. (a) The given line is vertical, so we seek a vertical line through (−2, 4): $x=-2$.

(b) We seek a horizontal line through (−2, 4): $y=4$.

34. (a) The given line is horizontal, so we seek a horizontal line through $\left(-1, \frac{1}{2}\right)$: $y=\frac{1}{2}$.

(b) We seek a vertical line through $\left(-1, \frac{1}{2}\right)$: $x=-1$.

35. $m=\frac{9-2}{3-1}=\frac{7}{2}$

$f(x)=\frac{7}{2}(x-1)+2=\frac{7}{2}x-\frac{3}{2}$

Check: $f(5)=\frac{7}{2}(5)-\frac{3}{2}=16$, as expected.

Since $f(x)=\frac{7}{2}x-\frac{3}{2}$, we have $m=\frac{7}{2}$ and $b=-\frac{3}{2}$.

36. $m=\frac{-4-(-1)}{4-2}=\frac{-3}{2}=-\frac{3}{2}$

$f(x)=-\frac{3}{2}(x-2)+(-1)=-\frac{3}{2}x+2$

Check: $f(6)=-\frac{3}{2}(6)+2=-7$, as expected.

Since $f(x)=-\frac{3}{2}x+2$, we have $m=-\frac{3}{2}$ and $b=2$.

37. $-\frac{2}{3}=\frac{y-3}{4-(-2)}$

$-\frac{2}{3}(6)=y-3$

$-4=y-3$

$-1=y$

38. $2=\frac{2-(-2)}{x-(-8)}$

$2(x+8)=4$

$x+8=2$

$x=-6$

39. $y=1\cdot(x-3)+4$

$y=x-3+4$

$y=x+1$

This is the same as the equation obtained in Example 5.

40. (a) When $y=0$, we have $\frac{x}{c}=1$, so $x=c$.

When $x=0$, we have $\frac{y}{d}=1$, so $y=d$.

(b) When $y=0$, we have $\frac{x}{c}=2$, so $x=2c$.

When $x=0$, we have $\frac{y}{d}=2$, so $y=2d$.

The x-intercept is $2c$ and the y-intercept is $2d$.

41. (a) The given equations are equivalent to $y = -\frac{2}{k}x + \frac{3}{k}$ and $y = -x + 1$, respectively, so the slopes are $-\frac{2}{k}$ and -1. The lines are parallel when $-\frac{2}{k} = -1$, so $k = 2$.

(b) The lines are perpendicular when $-\frac{2}{k} = \frac{-1}{-1}$, so $k = -2$.

42. (a) $m \approx \frac{68 - 69.5}{0.4 - 0} = \frac{-1.5}{0.4} = -3.75$ degrees/inch

(b) $m \approx \frac{9 - 68}{4 - 0.4} = \frac{-59}{3.6} \approx -16.1$ degrees/inch

(c) $m \approx \frac{4 - 9}{4.7 - 4} = \frac{-5}{0.7} \approx -7.1$ degrees/inch

(d) Best insulator: Fiberglass insulation

Poorest insulator: Gypsum wallboard

The best insulator will have the largest temperature change per inch, because that will allow larger temperature differences on opposite sides of thinner layers.

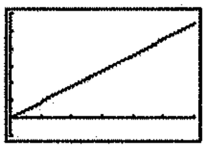
43. Slope: $k = \frac{\Delta p}{\Delta d} = \frac{10.94 - 1}{100 - 0} = \frac{9.94}{100}$
$= 0.0994$ atmospheres per meter

At 50 meters, the pressure is

$p = 0.0994(50) + 1 = 5.97$ atmospheres.

44. (a) $d(t) = 45t$

(b)



[0, 6] by [−50, 300]

(c) The slope is 45, which is the speed in miles per hour.

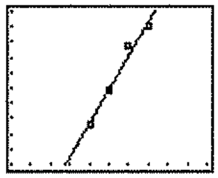
(d) Suppose the car has been traveling 45 mph for several hours when it is first observed at point P at time $t = 0$.

(e) The car starts at time $t = 0$ at a point 30 miles past P.

45. (a) $y = 1{,}060.4233x - 2{,}077{,}548.669$

(b) The slope is 1,060.4233. It represents the approximate rate of increase in earnings in dollars per year.

(c)



[1995, 2005] by [40000, 50000]

(d) When $x = 2000$,

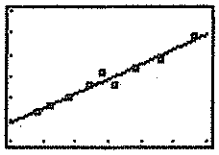
$y \approx 1{,}060.4233(2000) - 2{,}077{,}548.669 \approx 43{,}298$.

In 2000, the construction workers' average annual compensation will be about $43,298.

46. (a) $y = 0.680x + 9.013$

(b) The slope is 0.68. It represents the approximate average weight gain in pounds per month.

(c)



[15, 45] by [15, 45]

(d) When $x = 30$, $y \approx 0.680(30) + 9.013 = 29.413$.

She weighs about 29 pounds.

47. False: $m = \frac{\Delta y}{\Delta x}$ and $\Delta x = 0$, so it is undefined, or has no slope.

48. False: perpendicular lines satisfy the equation $m_1 m_2 = -1$, or $m_1 = -\frac{1}{m_2}$

49. A : $y = m(x - x_1) + y_1$

$$y = \frac{1}{2}(x + 3) + 4$$

or $y - 4 = \frac{1}{2}(x + 3)$

50. E.

51. D : $y = 2x - 5$

$$0 = 2x - 5$$
$$5 = 2x$$
$$x = \frac{5}{2}$$

52. B : $y = -3x - 7$

$$-1 = -3(-2) - 7$$
$$-1 = 6 - 7$$
$$-1 = -1$$

53. **(a)** $y = 5632x - 11{,}080{,}280$

(b) The rate at which the median price is increasing in dollars per year

(c) $y = 2732x - 5{,}362{,}360$

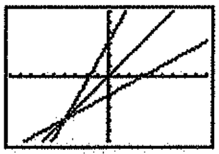
(d) The median price is increasing at a rate of about $5632 per year in the Northeast, and about $2732 per year in the Midwest. It is increasing more rapidly in the Northeast.

54. **(a)** Suppose x°F is the same as x°C.

$$x = \frac{9}{5}x + 32$$
$$\left(1 - \frac{9}{5}\right)x = 32$$
$$-\frac{4}{5}x = 32$$
$$x = -40$$

Yes, −40°F is the same as −40°C.

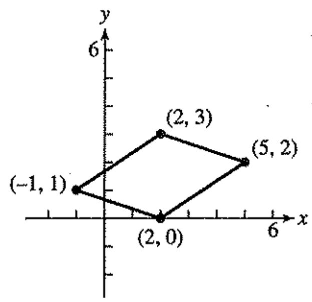
(b)

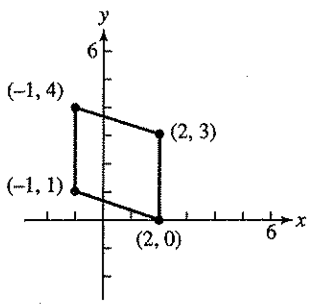


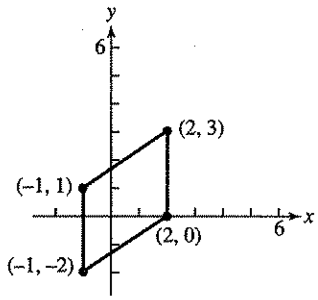
[−90, 90] by [−60, 60]

It is related because all three lines pass through the point(−40, −40) where the Fahrenheit and Celsius temperatures are the same.

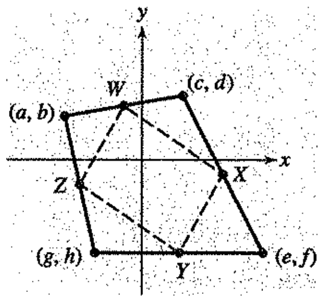
55. The coordinates of the three missing vertices are (5, 2), (−1, 4) and (−1, −2), as shown below.







56.



Suppose that the vertices of the given quadrilateral are (a, b), (c, d), (e, f), and (g, h). Then the midpoints of the consecutive sides are

$W\left(\frac{a+c}{2}, \frac{b+d}{2}\right)$, $X\left(\frac{c+e}{2}, \frac{d+f}{2}\right)$, $Y\left(\frac{e+g}{2}, \frac{f+h}{2}\right)$,

and $Z\left(\frac{g+a}{2}, \frac{h+b}{2}\right)$. When these four points are connected, the slopes of the sides of the resulting figure are:

$$WX: \frac{\frac{d+f}{2} - \frac{b+d}{2}}{\frac{c+e}{2} - \frac{a+c}{2}} = \frac{f-b}{e-a}$$

$$XY: \frac{\frac{f+h}{2} - \frac{d+f}{2}}{\frac{e+g}{2} - \frac{c+e}{2}} = \frac{h-d}{g-c}$$

$$ZY: \frac{\frac{f+h}{2} - \frac{h+b}{2}}{\frac{e+g}{2} - \frac{g+a}{2}} = \frac{f-b}{e-a}$$

$$WZ: \frac{\frac{h+b}{2} - \frac{b+d}{2}}{\frac{g+a}{2} - \frac{a+c}{2}} = \frac{h-d}{g-c}$$

Opposite sides have the same slope and are parallel.

57. The radius through (3, 4) has slope $\frac{4-0}{3-0}=\frac{4}{3}$.

The tangent line is tangent to this radius, so its slope is $\frac{-1}{4/3}=-\frac{3}{4}$. We seek the line of slope $-\frac{3}{4}$ that passes through (3, 4).

$$y=-\frac{3}{4}(x-3)+4$$
$$y=-\frac{3}{4}x+\frac{9}{4}+4$$
$$y=-\frac{3}{4}x+\frac{25}{4}$$

58. (a) The equation for line L can be written as $y=-\frac{A}{B}x+\frac{C}{B}$, so its slope is $-\frac{A}{B}$. The perpendicular line has slope $\frac{-1}{-A/B}=\frac{B}{A}$ and passes through (a, b), so its equation is $y=\frac{B}{A}(x-a)+b$.

(b) Substituting $\frac{B}{A}(x-a)+b$ for y in the equation for line L gives:

$$Ax+B\left[\frac{B}{A}(x-a)+b\right]=C$$
$$A^2x+B^2(x-a)+ABb=AC$$
$$(A^2+B^2)x=B^2a+AC-ABb$$
$$x=\frac{B^2a+AC-ABb}{A^2+B^2}$$

Substituting the expression for x in the equation for line L gives:

$$A\left(\frac{B^2a+AC-ABb}{A^2+B^2}\right)+By=C$$
$$By=\frac{-A(B^2a+AC-ABb)}{A^2+B^2}+\frac{C(A^2+B^2)}{A^2+B^2}$$
$$By=\frac{-AB^2a-A^2C+A^2Bb+A^2C+B^2C}{A^2+B^2}$$
$$By=\frac{A^2Bb+B^2C-AB^2a}{A^2+B^2}$$
$$y=\frac{A^2b+BC-ABa}{A^2+B^2}$$

The coordinates of Q are

$$\left(\frac{B^2a+AC-ABb}{A^2+B^2},\frac{A^2b+BC-ABa}{A^2+B^2}\right).$$

(c) Distance

$$=\sqrt{(x-a)^2+(y-b)^2}$$
$$=\sqrt{\left(\frac{B^2a+AC-ABb}{A^2+B^2}-a\right)^2+\left(\frac{A^2b+BC-ABa}{A^2+B^2}-b\right)^2}$$
$$=\sqrt{\left(\frac{B^2a+AC-ABb-a(A^2+B^2)}{A^2+B^2}\right)^2+\left(\frac{A^2b+BC-ABa-b(A^2+B^2)}{A^2+B^2}\right)^2}$$
$$=\sqrt{\left(\frac{AC-ABb-A^2a}{A^2+B^2}\right)^2+\left(\frac{BC-ABa-B^2b}{A^2+B^2}\right)^2}$$
$$=\sqrt{\left(\frac{A(C-Bb-Aa)}{A^2+B^2}\right)^2+\left(\frac{B(C-Aa-Bb)}{A^2+B^2}\right)^2}$$
$$=\sqrt{\frac{A^2(C-Aa-Bb)^2}{(A^2+B^2)^2}+\frac{B^2(C-Aa-Bb)^2}{(A^2+B^2)^2}}$$
$$=\sqrt{\frac{(A^2+B^2)(C-Aa-Bb)^2}{(A^2+B^2)^2}}$$
$$=\sqrt{\frac{(C-Aa-Bb)^2}{A^2+B^2}}$$
$$=\frac{|C-Aa-Bb|}{\sqrt{A^2+B^2}}$$
$$=\frac{|Aa+Bb-C|}{\sqrt{A^2+B^2}}$$

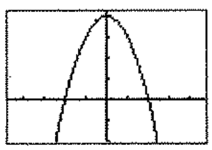
Section 1.2 Functions and Graphs (pp. 12–21)

Exploration 1 Composing Functions

1. $y_3=g\circ f$, $y_4=f\circ g$

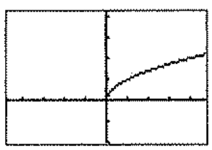
2. Domain of y_3: [−2, 2] Range of y_3: [0, 2]

y_1:



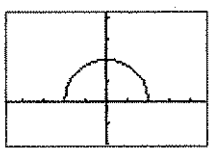
[−4.7, 4.7] by [−2, 4.2]

y_2:



[−4.7, 4.7] by [−2, 4.2]

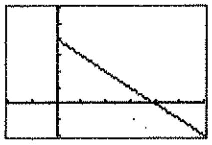
y_3:



[−4.7, 4.7] by [−2, 4.2]

3. Domain of y_4: $[0, \infty)$; Range of y_4: $(-\infty, 4]$

y_4:



$[-2, 6]$ by $[-2, 6]$

4. $y_3 = y_2(y_1(x)) = \sqrt{y_1(x)} = \sqrt{4-x^2}$

$y_4 = y_1(y_2(x)) = 4-(y_2(x))^2 = 4-(\sqrt{x})^2 = 4-x,\ x \ge 0$

Quick Review 1.2

1. $3x-1 \le 5x+3$

$-2x \le 4$

$x \ge -2$

Solution: $[-2, \infty)$

2. $x(x-2) > 0$

Solutions to $x(x-2) = 0$: $x = 0, x = 2$

Test $x = -1$: $-1(-1-2) = 3 > 0$

$x(x-2) > 0$ is true when $x < 0$.

Test $x = 1$: $1(1-2) = -1 < 0$

$x(x-2) > 0$ is false when $0 < x < 2$.

Test $x = 3$: $3(3-2) = 3 > 0$

$x(x-2) > 0$ is true when $x > 2$.

Solution set: $(-\infty, 0) \cup (2, \infty)$

3. $|x-3| \le 4$

$-4 \le x-3 \le 4$

$-1 \le x \le 7$

Solution set: $[-1, 7]$

4. $|x-2| \ge 5$

$x-2 \le -5$ or $x-2 \ge 5$

$x \le -3$ or $x \ge 7$

Solution set: $(-\infty, -3] \cup [7, \infty)$

5. $x^2 < 16$

Solution to $x^2 = 16$: $x = -4, x = 4$

Test $x = -6$: $(-6)^2 = 36 > 16$

$x^2 < 16$ is false when $x < -4$

Test $x = 0$: $0^2 = 0 < 16$

$x^2 < 16$ is true when $-4 < x < 4$

Test $x = 6$: $6^2 = 36 > 16$

$x^2 < 16$ is false when $x > 4$.

Solution set: $(-4, 4)$

6. $9 - x^2 \ge 0$

Solutions to $9 - x^2 = 0$: $x = -3, x = 3$

Test $x = -4$: $9-(-4)^2 = 9-16 = -7 < 0$

$9 - x^2 \ge 0$ is false when $x < -3$.

Test $x = 0$: $9 - 0^2 = 9 > 0$

$9 - x^2 \ge 0$ is true when $-3 < x < 3$.

Test $x = 4$: $9 - 4^2 = 9 - 16 = -7 < 0$

$9 - x^2 \ge 0$ is false when $x > 3$.

Solution set: $[-3, 3]$

7. Translate the graph of f 2 units left and 3 units downward.

8. Translate the graph of f 5 units right and 2 units upward.

9. (a) $f(x) = 4$

$x^2 - 5 = 4$

$x^2 - 9 = 0$

$(x+3)(x-3) = 0$

$x = -3$ or $x = 3$

(b) $f(x) = -6$

$x^2 - 5 = -6$

$x^2 = -1$

No real solution

10. (a) $f(x) = -5$

$\frac{1}{x} = -5$

$x = -\frac{1}{5}$

(b) $f(x) = 0$

$\frac{1}{x} = 0$

No solution

11. (a) $f(x) = 4$

$\sqrt{x+7} = 4$

$x + 7 = 16$

$x = 9$

Check: $\sqrt{9+7} = \sqrt{16} = 4$; it checks.

(b) $f(x) = 1$

$\sqrt{x+7} = 1$

$x + 7 = 1$

$x = -6$

Check: $\sqrt{-6+7} = 1$; it checks.

12. (a) $f(x) = -2$

$\sqrt[3]{x-1} = -2$

$x - 1 = -8$

$x = -7$

(b) $f(x) = 3$

$\sqrt[3]{x-1} = 3$

$x - 1 = 27$

$x = 28$

Section 1.2 Exercises

1. $A(d) = \pi\left(\dfrac{d}{2}\right)^2$

   $A(d) = \pi\left(\dfrac{4\text{ in}}{2}\right)^2 = \pi(2\text{ in})^2 = 4\pi\text{ in}^2$

2. $h(s) = \dfrac{\sqrt{3}}{2}s = 3\dfrac{\sqrt{3}}{2}m = 1.5\sqrt{3}m$

3. $S(e) = 6e^2 = 6(5\text{ ft})^2 = 6(25\text{ ft}^2) = 150\text{ ft}^2$

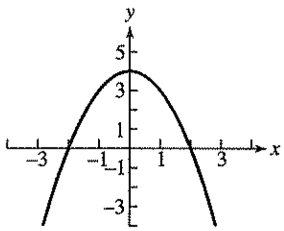
4. $v(r) = \dfrac{4}{3}\pi r^3 = \dfrac{4}{3}\pi(3\text{cm})^3$

   $= \dfrac{4}{3}\pi(27\text{ cm}^3) = 36\pi\text{ cm}^3$

5. **(a)** $(-\infty, \infty)$ or all real numbers

   (b) $(-\infty, 4]$

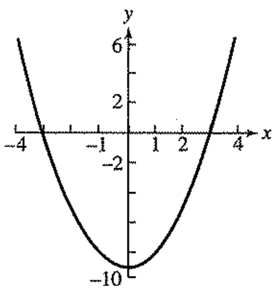
   (c)



6. **(a)** $(-\infty, \infty)$ or all real numbers

   (b) $[-9, \infty)$

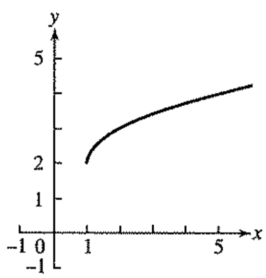
   (c)



7. **(a)** Since we require $x - 1 \geq 0$, the domain is $[1, \infty)$.

   (b) $[2, \infty)$

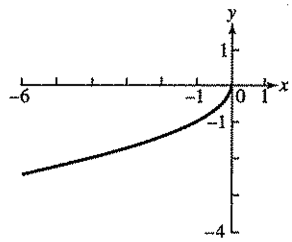
   (c)



8. **(a)** Since we require $-x \geq 0$, the domain is $(-\infty, 0]$.

   (b) $(-\infty, 0]$

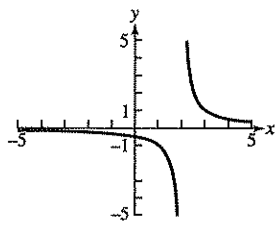
   (c)



9. **(a)** Since we require $x - 2 \neq 0$, the domain is $(-\infty, 2) \cup (2, \infty)$.

   (b) Since $\dfrac{1}{x-2}$ can assume any value except 0, the range is $(-\infty, 0) \cup (0, \infty)$.

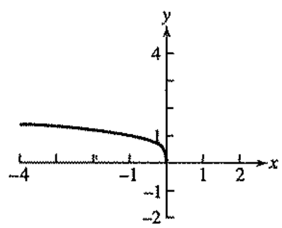
   (c)



10. **(a)** Since we require $-x \geq 0$, the domain is $(-\infty, 0]$.

    (b) $[0, \infty)$

    (c)

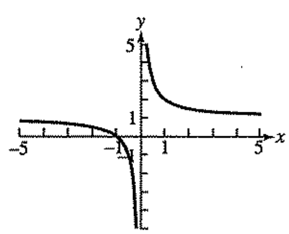


11. **(a)** Since we require $x \neq 0$, the domain is $(-\infty, 0) \cup (0, \infty)$.

    (b) Note that $\dfrac{1}{x}$ can assume any value except 0, so $1 + \dfrac{1}{x}$ can assume any value except 1.

    The range is $(-\infty, 1) \cup (1, \infty)$.

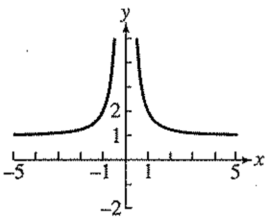
    (c)



12. (a) Since we require $x^2 \neq 0$, the domain is $(-\infty, 0) \cup (0, \infty)$

(b) Since $\frac{1}{x^2} > 0$ for all x, the range is $(1, \infty)$.

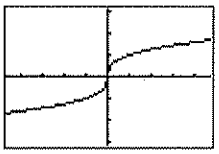
(c)



13. (a) $(-\infty, \infty)$ or all real numbers

(b) $(-\infty, \infty)$ or all real numbers

(c)

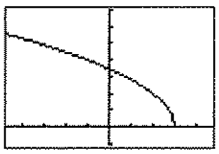


[–4.7, 4.7] by [–3.1, 3.1]

14. (a) Since we require $3 - x \geq 0$, the domain is $(-\infty, 3]$.

(b) $[0, \infty)$

(c)

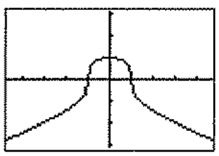


[–4.7, 4.7] by [–1, 7]

15. (a) $(-\infty, \infty)$ or all real numbers

(b) The maximum function value is attained at the point (0, 1), so the range is $(-\infty, 1]$.

(c)

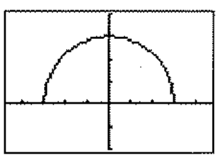


[–4.7, 4.7] by [–3.1, 3.1]

16. (a) Since we require $9 - x^2 \geq 0$ the domain is $[-3, 3]$

(b) $[0, 3]$

(c)

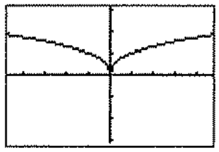


[–4.7, 4.7] by [–2, 4]

17. (a) $(-\infty, \infty)$ or all real numbers

(b) Since $x^{2/5}$ is equivalent to $\sqrt[5]{x^2}$, the range is $[0, \infty)$

(c)

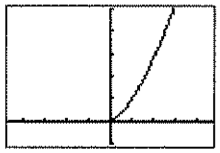


[–4.7, 4.7] by [–3.1, 3.1]

18. (a) This function is equivalent to $y = \sqrt{x^3}$, so its domain is $[0, \infty)$.

(b) $[0, \infty)$

(c)

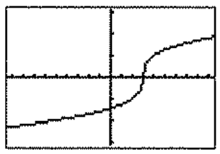


[–4.7, 4.7] by [–1, 5]

19. (a) $(-\infty, \infty)$ or all real numbers

(b) $(-\infty, \infty)$ or all real numbers

(c)

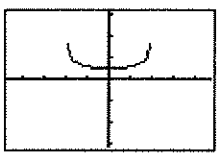


[–9.4, 9.4] by [–3.1, 3.1]

20. (a) Since $(4 - x^2) > 0$, the domain is $(-2, 2)$

(b) $[0.5, \infty)$

(c)



[–4.7, 4.7] by [–3.1, 3.1]

21. Even, since the function is an even power of x.

22. Neither, since the function is a sum of even and odd powers of x.

23. Neither, since the function is a sum of even and odd powers of x $(x^1 + 2x^0)$.

24. Even, since the function is a sum of even powers of x $(x^2 - 3x^0)$.

25. Even, since the function involves only even powers of x.

26. Odd, since the function is a sum of odd powers of x.

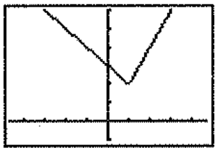
27. Odd, since the function is a quotient of an odd function (x^3) and an even function (x^2-1).

28. Neither, since, (for example), $y(-2) = 4^{1/3}$ and $y(2) = 0$.

29. Neither, since, (for example), $y(-1)$ is defined and $y(1)$ is undefined.

30. Even, since the function involves only even powers of x.

31. **(a)**

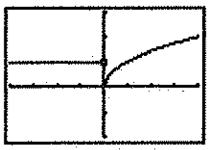


$[-4.7, 4.7]$ by $[-1, 6]$

(b) $(-\infty, \infty)$ or all real numbers

(c) $[2, \infty)$

32. **(a)**

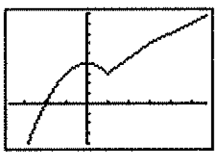


$[-4, 4]$ by $[-2, 3]$

(b) $(-\infty, \infty)$ or all real numbers

(c) $[0, \infty)$

33. **(a)**

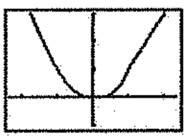


$[-3.7, 5.7]$ by $[-4, 9]$

(b) $(-\infty, \infty)$ or all real numbers

(c) $(-\infty, \infty)$ or all real numbers

34. **(a)**



$[-2.35, 2.35]$ by $[-1, 3]$

(b) $(-\infty, \infty)$ or all real numbers

(c) $[0, \infty)$

35. Because if the vertical line test holds, then for each x-coordinate, there is at most one y-coordinate giving a point on the curve. This y-coordinate would correspond to the value assigned to the x-coordinate. Since there is only one y-coordinate, the assignment would be unique.

36. If the curve is not $y = 0$, there must be a point (x, y) on the curve where $y \neq 0$. That would mean that (x, y) and $(x, -y)$ are two different points on the curve and it is not the graph of a function, since it fails the vertical line test.

37. No

38. Yes

39. Yes

40. No

41. Line through (0, 0) and (1, 1): $y = x$

Line through (1, 1) and (2, 0): $y = -x + 2$

$$f(x) = \begin{cases} x, & 0 \le x \le 1 \\ -x+2, & 1 < x \le 2 \end{cases}$$

42. $$f(x) = \begin{cases} 2, & 0 \le x < 1 \\ 0, & 1 \le x < 2 \\ 2, & 2 \le x < 3 \\ 0, & 3 \le x \le 4 \end{cases}$$

43. Line through (0, 2) and (2, 0): $y = -x + 2$

Line through (2, 1) and (5, 0): $m = \frac{0-1}{5-2} = \frac{-1}{3} = -\frac{1}{3}$,

so $y = -\frac{1}{3}(x-2) + 1 = -\frac{1}{3}x + \frac{5}{3}$

$$f(x) = \begin{cases} -x+2, & 0 < x \le 2 \\ -\frac{1}{3}x + \frac{5}{3}, & 2 < x \le 5 \end{cases}$$

44. Line through (−1, 0) and (0, −3):

$$m = \frac{-3-0}{0-(-1)} = \frac{-3}{1} = -3, \text{ so } y = -3x - 3$$

Line through (0, 3) and (2, −1):

$$m = \frac{-1-3}{2-0} = \frac{-4}{2} = -2, \text{ so } y = -2x + 3$$

$$f(x) = \begin{cases} -3x-3, & -1 < x \le 0 \\ -2x+3, & 0 < x \le 2 \end{cases}$$

45. Line through (–1, 1) and (0, 0): $y = -x$
Line through (0, 1) and (1, 1): $y = 1$
Line through (1, 1) and (3, 0):

$$m = \frac{0-1}{3-1} = \frac{-1}{2} = -\frac{1}{2},$$

$$\text{so } y = -\frac{1}{2}(x-1)+1 = -\frac{1}{2}x+\frac{3}{2}$$

$$f(x) = \begin{cases} -x, & -1 \le x < 0 \\ 1, & 0 < x \le 1 \\ -\frac{1}{2}x+\frac{3}{2}, & 1 < x < 3 \end{cases}$$

46. Line through (–2, –1) and (0, 0): $y = \frac{1}{2}x$

Line through (0, 2) and (1, 0): $y = -2x + 2$
Line through (1, –1) and (3, –1): $y = -1$

$$f(x) = \begin{cases} \frac{1}{2}x, & -2 \le x \le 0 \\ -2x+2, & 0 < x \le 1 \\ -1, & 1 < x \le 3 \end{cases}$$

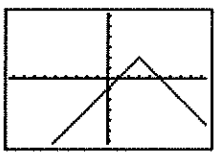
47. Line through $\left(\frac{T}{2}, 0\right)$ and $(T, 1)$:

$$m = \frac{1-0}{T-(T/2)} = \frac{2}{T}, \text{ so } y = \frac{2}{T}\left(x - \frac{T}{2}\right) + 0 = \frac{2}{T}x - 1$$

$$f(x) = \begin{cases} 0, & 0 \le x \le \frac{T}{2} \\ \frac{2}{T}x - 1, & \frac{T}{2} < x \le T \end{cases}$$

48. $$f(x) = \begin{cases} A, & 0 \le x < \frac{T}{2} \\ -A, & \frac{T}{2} \le x < T \\ A, & T \le x < \frac{3T}{2} \\ -A, & \frac{3T}{2} \le x \le 2T \end{cases}$$

49. (a)



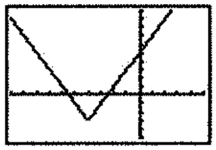
[–9.4, 9.4] by [–6.2, 6.2]

Note that $f(x) = -|x-3| + 2$, so its graph is the graph of the absolute value function reflected across the x-axis and then shifted 3 units right and 2 units upward.

(b) $(-\infty, \infty)$

(c) $(-\infty, 2]$

50. (a) The graph of $f(x)$ is the graph of the absolute value function stretched vertically by a factor of 2 and then shifted 4 units to the left and 3 units downward.



[–10, 5] by [–5, 10]

(b) $(-\infty, \infty)$ or all real numbers

(c) $[-3, \infty)$

51. (a) $f(g(x)) = (x^2 - 3) + 5 = x^2 + 2$

(b) $g(f(x)) = (x+5)^2 - 3$
$= (x^2 + 10x + 25) - 3$
$= x^2 + 10x + 22$

(c) $f(g(0)) = 0^2 + 2 = 2$

(d) $g(f(0)) = 0^2 + 10 \cdot 0 + 22 = 22$

(e) $g(g(-2)) = [(-2^2) - 3]^2 - 3 = 1^2 - 3 = -2$

(f) $f(f(x)) = (x + 5) + 5 = x + 10$

52. (a) $f(g(x)) = (x - 1) + 1 = x$

(b) $g(f(x)) = (x + 1) - 1 = x$

(c) $f(g(x)) = 0$

(d) $g(f(0)) = 0$

(e) $g(g(-2)) = (-2 - 1) - 1 = -3 - 1 = -4$

(f) $f(f(x)) = (x + 1) + 1 = x + 2$

53. (a) Since $(f \circ g)(x) = \sqrt{g(x) - 5} = \sqrt{x^2 - 5}$, $g(x) = x^2$.

(b) Since $(f \circ g)(x) = 1 + \frac{1}{g(x)} = x$, we know that

$$\frac{1}{g(x)} = x - 1, \text{ so } g(x) = \frac{1}{x-1}.$$

(c) Since $(f \circ g)(x) = f\left(\frac{1}{x}\right) = x, f(x) = \frac{1}{x}$.

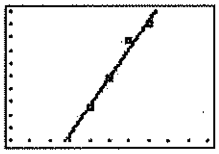
(d) Since $(f \circ g)(x) = f(\sqrt{x}) = |x|, f(x) = x^2$.

The completed table is shown. Note that the absolute value sign in part (d) is optional.

$g(x)$	$f(x)$	$(f \circ g)(x)$
x^2	$\sqrt{x-5}$	$\sqrt{x^2-5}$
$\frac{1}{x-1}$	$1+\frac{1}{x}$	$x, x \ne -1$
$\frac{1}{x}$	$\frac{1}{x}$	$x, x \ne 0$
$\sqrt{x}$	x^2	$\lvert x \rvert, x \ge 0$

54. (a) $y = 2.893x^2 - 24.107x + 590.214$

(b)



[0, 15] by [500, 800]

(c) $y = 2.893(18)^2 - 24.107(18) + 590.214$
$= 937.332 - 433.926 + 590.214$
$= \$1093$ million or $1.093 billion

(d) linear regression: $y = 33.75x + 312.5$

$y = 33.75(18) + 312.5 = \$920$ million in 2008.

55. (a) Because the circumference of the original circle was 8π and a piece of length x was removed.

(b) $r = \dfrac{8\pi - x}{2\pi} = 4 - \dfrac{x}{2\pi}$

(c) $h = \sqrt{16 - r^2}$

$$= \sqrt{16 - \left(4 - \frac{x}{2\pi}\right)^2}$$
$$= \sqrt{16 - \left(16 - \frac{4x}{\pi} + \frac{x^2}{4\pi^2}\right)}$$
$$= \sqrt{\frac{4x}{\pi} - \frac{x^2}{4\pi^2}}$$
$$= \sqrt{\frac{16\pi x}{4\pi^2} - \frac{x^2}{4\pi^2}}$$
$$= \frac{\sqrt{16\pi x - x^2}}{2\pi}$$

(d) $V = \dfrac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\pi\left(\frac{8\pi - x}{2\pi}\right)^2 \cdot \frac{\sqrt{16\pi x - x^2}}{2\pi}$$
$$= \frac{(8\pi - x)^2\sqrt{16\pi x - x^2}}{24\pi^2}$$

56. (a) Note that 2 mi = 10,560 ft, so there are $\sqrt{800^2 + x^2}$ feet of river cable at $180 per foot and $(10{,}560 - x)$ feet of land cable at $100 per foot. The cost is

$C(x) = 180\sqrt{800^2 + x^2} + 100(10{,}560 - x)$

(b) $C(0) = \$1{,}200{,}000$
$C(500) \approx \$1{,}175{,}812$
$C(1000) \approx \$1{,}186{,}512$
$C(1500) = \$1{,}212{,}000$
$C(2000) \approx \$1{,}243{,}732$
$C(2500) \approx \$1{,}278{,}479$
$C(3000) \approx \$1{,}314{,}870$

Values beyond this are all larger. It would appear that the least expensive location is less than 2000 ft from point P.

57. False: $x^4 + x^2 + x \neq (-x)^4 + (-x)^2 + (-x)$.

58. True: $(-x)^3 = -x^3$

59. B: Since $9 - x^2 > 0$, the domain is $(-3, 3)$

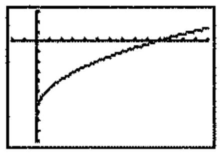
60. A: $y \neq 1$

61. D: $(f \circ g)(2) = f(x+3)(2) = 2(x+3) - 1$
$= 2(2+3) - 1 = 2(5) - 1 = 10 - 1 = 9$

62. C: $A(w) = Lw$
$L = 2w$
$A(w) = 2w^2$

63. (a) Enter $y_1 = f(x) = x - 7$, $y_2 = g(x) = \sqrt{x}$,
$y_3 = (f \circ g)(x) = y_1(y_2(x))$, and
$y_4 = (g \circ f)(x) = y_2(y_1(x))$

$f \circ g$:

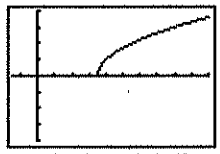


[–10, 70] by [–10, 3]

Domain: $[0, \infty)$

Range: $[-7, \infty)$

$g \circ f$:



[–3, 20] by [–4, 4]

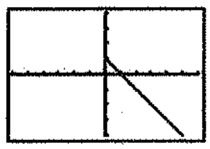
Domain: $[7, \infty)$

Range: $[0, \infty)$

(b) $(f \circ g)(x) = \sqrt{x} - 7$ $\quad$ $(g \circ f)(x) = \sqrt{x - 7}$

64. (a) Enter $y_1 = f(x) = 1 - x^2$, $y_2 = g(x) = \sqrt{x}$,
$y_3 = (f \circ g)(x) = y_1(y_2(x))$, and $y_4 = (g \circ f)(x)$
$= y_2(y_1(x))$

$f \circ g$:

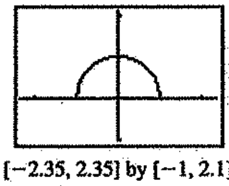


[–6, 6] by [–4, 4]

Domain: $[0, \infty)$

Range: $(-\infty, 1]$

$g \circ f$:



[–2.35, 2.35] by [–1, 2.1]

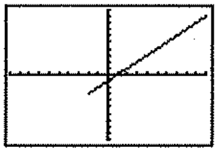
Domain: $[-1, 1]$

Range: $[0, 1]$

(b) $(f \circ g)(x) = 1 - (\sqrt{x})^2 = 1 - x, x \geq 0$
$(g \circ f)(x) = \sqrt{1 - x^2}$

65. (a) Enter $y_1 = f(x) = x^2 - 3, y_2 = g(x) = \sqrt{x+2},$ $y_3 = (f \circ g)(x) = y_1(y_2(x))$, and $y_4 = (g \circ f)(x)$ $= y_2(y_1(x))$.

$f \circ g$:

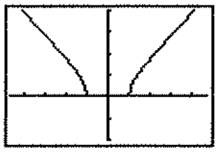


[–10, 10] by [–10, 10]

Domain: $[-2, \infty)$

Range: $[-3, \infty)$

$g \circ f$:



[–4.7, 4.7] by [–2, 4]

Domain: $(-\infty, -1] \cup [1, \infty)$

Range: $[0, \infty)$

(b) $(f \circ g)(x) = (\sqrt{x+2})^2 - 3$

$= (x+2) - 3, x \geq -2$

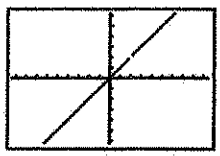
$= x - 1, x \geq -2$

$(g \circ f)(x) = \sqrt{(x^2-3)+2} = \sqrt{x^2-1}$

66. (a) Enter $y_1(x) = f(x) = \dfrac{2x-1}{x+3}, y_2 = \dfrac{3x+1}{2-x},$ $y_3 = (f \circ g)(x) = y_1(y_2(x))$, and $y_4 = (g \circ f)(x)$ $= y_2(y_1(x))$.

Use a "decimal window" such as the one shown.

$f \circ g$:

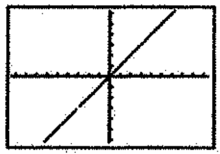


[−9.4, 9.4] by [−6.2, 6.2]

Domain: $(-\infty, 2) \cup (2, \infty)$

Range: $(-\infty, 2) \cup (2, \infty)$

$g \circ f$:



[−9.4, 9.4] by [−6.2, 6.2]

Domain: $(-\infty, -3) \cup (-3, \infty)$

Range: $(-\infty, -3) \cup (-3, \infty)$

(b) $$(f \circ g)(x) = \frac{2\left(\frac{3x+1}{2-x}\right) - 1}{\frac{3x+1}{2-x} + 3}$$

$$= \frac{2(3x+1) - (2-x)}{(3x+1) + 3(2-x)}, \ x \neq 2$$

$$= \frac{7x}{7}, \ x \neq 2$$

$$= x, \ x \neq 2$$

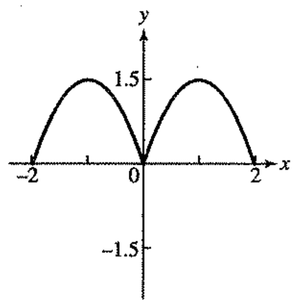
$$(g \circ f)(x) = \frac{3\left(\frac{2x-1}{x+3}\right) + 1}{2 - \frac{2x-1}{x+3}}$$

$$= \frac{3(2x-1) + (x+3)}{2(x+3) - (2x-1)}, x \neq -3$$

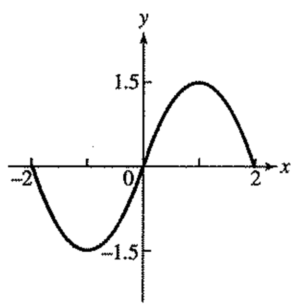
$$= \frac{7x}{7}, \ x \neq -3$$

$$= x, \ x \neq -3$$

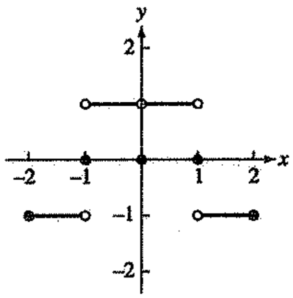
67. (a)



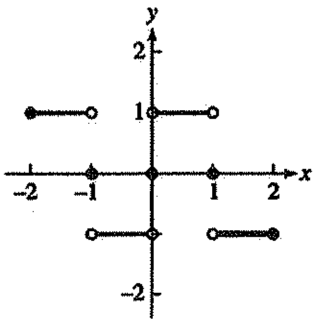
(b)



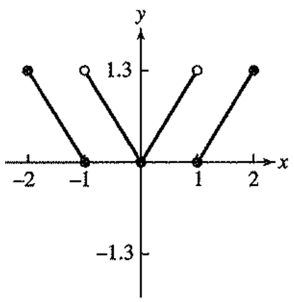
68. (a)



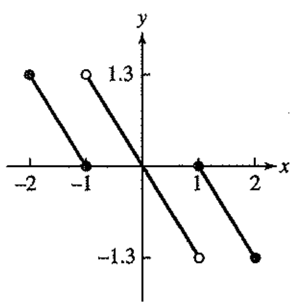
(b)



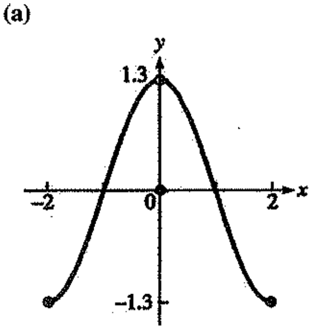
69. (a)



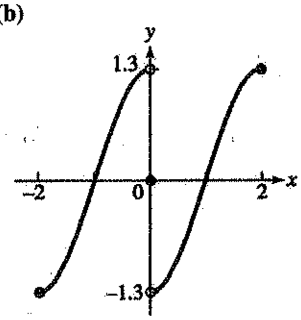
(b)



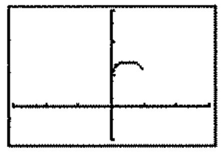
70. (a)



(b)



71. (a)



[–3, 3] by [–1, 3]

(b) Domain of y_1: $[0, \infty)$

Domain of y_2: $(-\infty, 1]$

Domain of y_3: $[0, 1]$

(c) The functions $y_1 - y_2$, $y_2 - y_1$, and $y_1 \cdot y_2$ all have domain [0, 1], the same as the domain of $y_1 + y_2$ found in part (b).

Domain of $\frac{y_1}{y_2}$: $[0,1)$

Domain of $\frac{y_2}{y_1}$: $(0,1]$

(d) The domain of a sum, difference, or product of two functions is the intersection of their domains.
The domain of a quotient of two functions is the intersection of their domains with any zeros of the denominator removed.

72. (a) Yes. Since $(f \cdot g)(-x) = f(-x) \cdot g(-x) = f(x) \cdot g(x) = (f \cdot g)(x)$, the function $(f \cdot g)(x)$ will also be even.

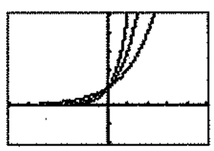
(b) The product will be even, since

$$\begin{aligned}(f \cdot g)(-x) &= f(-x) \cdot g(-x) \\ &= (-f(x)) \cdot (-g(x)) \\ &= f(x) \cdot g(x) \\ &= (f \cdot g)(x).\end{aligned}$$

Section 1.3 Exponential Functions (pp. 22–29)

Exploration 1 Exponential Functions

1.



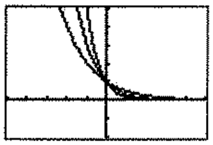
[–5, 5] by [–2, 5]

2. $x > 0$

3. $x < 0$

4. $x = 0$

5.



$[-5, 5]$ by $[-2, 5]$

6. $2^{-x} < 3^{-x} < 5^{-x}$ for $x < 0$; $2^{-x} > 3^{-x} > 5^{-x}$ for $x > 0$; $2^{-x} = 3^{-x} = 5^{-x}$ for $x = 0$.

Quick Review 1.3

1. Using a calculator, $5^{2/3} \approx 2.924$.

2. Using a calculator, $3^{\sqrt{2}} \approx 4.729$.

3. Using a calculator, $3^{-1.5} \approx 0.192$.

4. $x^3 = 17$

$x = \sqrt[3]{17}$
$x \approx 2.5713$

5. $x^5 = 24$

$x = \sqrt[5]{24}$
$x \approx 1.8882$

6. $x^{10} = 1.4567$

$x = \pm\sqrt[10]{1.4567}$
$x \approx \pm 1.0383$

7. $500(1.0475)^5 \approx \$630.58$

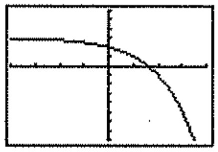
8. $1000(1.063)^3 \approx \$1201.16$

9. $$\frac{(x^{-3}y^2)^2}{(x^4y^3)^3} = \frac{x^{-6}y^4}{x^{12}y^9} = x^{-6-12}y^{4-9} = x^{-18}y^{-5} = \frac{1}{x^{18}y^5}$$

10. $$\left(\frac{a^3b^{-2}}{c^4}\right)^2\left(\frac{a^4c^{-2}}{b^3}\right)^{-1} = \frac{a^6b^{-4}}{c^8}\cdot\frac{b^3}{a^4c^{-2}} = \frac{a^6}{b^4c^8}\cdot\frac{b^3c^2}{a^4} = a^{6-4}b^{-4+3}c^{-8+2} = a^2b^{-1}c^{-6} = \frac{a^2}{bc^6}$$

Section 1.3 Exercises

1.

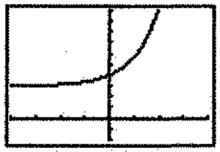


$[-4, 4]$ by $[-8, 6]$

Domain: $(-\infty, \infty)$

Range: $(-\infty, 3)$

2.

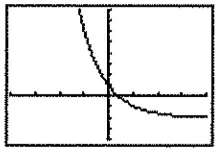


$[-4, 4]$ by $[-2, 10]$

Domain: $(-\infty, \infty)$

Range: $(3, \infty)$

3.



$[-4, 4]$ by $[-4, 8]$

Domain: $(-\infty, \infty)$

Range: $(-2, \infty)$

4.

$[-4, 4]$ by $[-8, 4]$

Domain: $(-\infty, \infty)$

Range: $(-\infty, -1)$

5. $9^{2x} = (3^2)^{2x} = 3^{4x}$

6. $16^{3x} = (2^4)^{3x} = 2^{12x}$

7. $\left(\frac{1}{8}\right)^{2x} = (2^{-3})^{2x} = 2^{-6x}$

8. $\left(\frac{1}{27}\right)^{x} = (3^{-3})^{x} = 3^{-3x}$

9. x-intercept: ≈ -2.322

y-intercept: -4.0

10. x-intercept: ≈ 1.386

y-intercept: -3.0

11. x-intercept: ≈ -0.631

y-intercept: 0.5

12. x-intercept: 2.0

y-intercept: ≈ 1.585

13. The graph of $y = 2^x$ is increasing from left to right and has the negative x-axis as an asymptote. (a)

14. The graph of $y = 3^{-x}$ or, equivalently, $y = \left(\frac{1}{3}\right)^x$, is decreasing from left to right and has the positive x-axis as an asymptote. (d)

15. The graph of $y = -3^{-x}$ is the reflection about the x-axis of the graph in Exercise 2. (e)

16. The graph of $y = -0.5^{-x}$ or, equivalently, $y = -2^{-x}$, is the reflection about the x-axis of the graph in Exercise 1. (c)

17. The graph of $y = 2^{-x} - 2$ is decreasing from left to right and has the line $y = -2$ as an asymptote. (b)

18. The graph of $y = 1.5^x - 2$ is increasing from left to right and has the line $y = -2$ as an asymptote. (f)

19. (a) $\frac{1,935}{1,853} = 1.0443$

$\frac{1,998}{1,935} = 1.0326$

$\frac{2,095}{1,998} = 1.0485$

$\frac{2,167}{2,095} = 1.0344$

$\frac{2,241}{2,167} = 1.0341$

(b) One possibility is $1,853(1.04)^n$

(c) $1,853(1.04)^{12} = 2967$ thousand or 2,967,000

20. (a) $\frac{7,000}{6,901} = 1.0143$

$\frac{7,078}{7,000} = 1.0111$

$\frac{7,193}{7,078} = 1.0162$

$\frac{7,288}{7,193} = 1.0132$

$\frac{7,386}{7,288} = 1.0134$

(b) One possibility is $6,901\ (1.013)^n$

(c) $6,901\ (1.013)^{10} = 7852$ thousand or 7,852,000

21. Let t be the number of years. Solving $500,000\ (1.0375)^t = 1,000,000$ graphically, we find that $t \approx 18.828$. The population will reach 1 million in about 19 years.

22. (a) The population is given by $P(t) = 6250\ (1.0275)^t$, where t is the number of years after 1890.

Population in 1915: $P(25) \approx 12,315$

Population in 1940: $P(50) \approx 24,265$

(b) Solving $P(t) = 50,000$ graphically, we find that $t \approx 76.651$. The population reached 50,000 about 77 years after 1890, in 1967.

23. (a) $A(t) = 6.6\left(\frac{1}{2}\right)^{t/14}$

(b) Solving $A(t) = 1$ graphically, we find that $t \approx 38.1145$. There will be 1 gram remaining after about 38.1145 days.

24. Let t be the number of years. Solving $2300\ (1.06)^t = 4150$ graphically, we find that $t \approx 10.129$. It will take about 10.129 years. (If the interest is not credited to the account until the end of each year, it will take 11 years.)

25. Let A be the amount of the initial investment, and let t be the number of years. We wish to solve $A\ (1.0625)^t = 2A$, which is equivalent to $1.0625^t = 2$. Solving graphically, we find that $t \approx 11.433$. It will take about 11.433 years. (If the interest is credited at the end of each year, it will take 12 years.)

26. Let A be the amount of the initial investment, and let t be the number of years. We wish to solve

$A\left(1 + \frac{0.0625}{12}\right)^{12t} = 2A$, which is equivalent to

$\left(1 + \frac{0.0625}{12}\right)^{12t} = 2$. Solving graphically, we find that

$t \approx 11.119$. It will take about 11.119 years. (If the interest is credited at the end of each month, it will take 11 years 2 months.)

27. Let A be the amount of the initial investment, and let t be the number of years. We wish to solve $A\ e^{0.0625t} = 2A$, which is equivalent to $e^{0.0625t} = 2$. Solving graphically, we find that $t \approx 11.090$. It will take about 11.090 years.

28. Let A be the amount of the initial investment, and let t be the number of years. We wish to solve $A\ (1.0575)^t = 3A$, which is equivalent to $1.0575^t = 3$. Solving graphically, we find that $t \approx 19.650$. It will take about 19.650 years. (If the interest is credited at the end of each year, it will take 20 years.)

29. Let A be the amount of the initial investment, and let t be the number of years. We wish to solve $A\left(1+\frac{0.0575}{365}\right)^{365t}=3A$, which is equivalent to $\left(1+\frac{0.0575}{365}\right)^{365t}=3$. Solving graphically, we find that $t\approx 19.108$. It will take about 19.108 years.

30. Let A be the amount of the initial investment, and let t be the number of years. We wish to solve $A\,e^{0.0575t}=3A$, which is equivalent to $e^{0.0575t}=3$. Solving graphically, we find that $t\approx 19.106$. It will take about 19.106 years.

31. After t hours, the population is $P(t)=2^{t/0.5}$ or, equivalently, $P(t)=2^{2t}$. After 24 hours, the population is $P(24)=2^{48}\approx 2.815\times 10^{14}$ bacteria.

32. **(a)** Each year, the number of cases is 100% − 20% = 80% of the previous year's number of cases. After t years, the number of cases will be $C(t)=10{,}000\ (0.8)^t$. Solving $C(t)=1000$ graphically, we find that $t\approx 10.319$. It will take about 10.319 years.

(b) Solving $C(t)=1$ graphically, we find that $t\approx 41.275$. It will take about 41.275 years.

33.

x	y	Δy
1	−1	
		2
2	1	
		2
3	3	
		2
4	5	

34.

x	y	Δy
1	1	
		−3
2	−2	
		−3
3	−5	
		−3
4	−8	

35.

x	y	Δy
1	1	
		3
2	4	
		5
3	9	
		7
4	16	

36.

x	y	ratio
1	8.155	
		2.718
2	22.167	
		2.718
3	60.257	
		2.718
4	163.794	

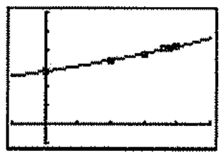
37. Since $\Delta x=1$, the corresponding value of Δy is equal to the slope of the line. If the changes in x are constant for a linear function, then the corresponding changes in y are constant as well.

38. **(a)** When $t=0$, $B=100\,e^0=100$. There were 100 bacteria present initially.

(b) When $t=6$, $B=100\,e^{0.639(6)}\approx 6394.351$. After 6 hours, there are about 6394 bacteria.

(c) Solving $100e^{0.639t}=200$ graphically, we find that $t\approx 1.000$. The population will be 200 after about 1 hour. Since the population doubles (from 100 to 200) in about 1 hour, the doubling time is about 1 hour.

39. **(a)** $y=14153.84\,(1.01963)^x$



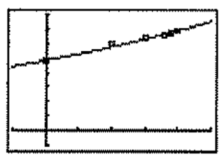
[−5, 25] by [−5000, 30000]

(b) Estimate: $14153.84\,(1.01963)^{23}=22{,}133$ thousand or 22,133,000.

22,133,000 − 22,119,000 = 14,000. The estimate exceeds the actual by 14,000.

(c) $\frac{14{,}229}{(22{,}133)(23)}=0.02$ or 2%

40. **(a)** $y=24121.49\,(1.0178)^x$



[−5, 25] by [−5000, 40000]

(b) Estimate: $24121.49\,(1.0178)^{23}=36{,}194$ thousand or 36,194,000.

36,194,000 − 35,484,000 = 710,000.

The estimate exceeds the actual by 710,000.

(c) $\frac{23{,}668}{(36{,}194)(23)}=0.018$ or 1.8%

41. False. $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$. It is positive.

42. True. $4^3 = (22)^3 = 2^6$

43. *D.* $y = pe^{rt}$

$$\frac{400}{200} = \frac{200e^{0.045t}}{200}$$
$$\ln\ (2 = e^{0.045t})$$
$$\ln 2 = 0.045t$$
$$t = \frac{\ln 2}{0.045} = 15.4 \text{ years}$$

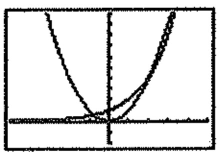
44. A.

45. B.

46. *C*, $f(x) = 4 - e^x$

$$0 = 4 - e^x$$
$$+e^x \quad +e^x$$
$$ln(e^x = 4)$$
$$x = ln 4$$
$$x = 1.386$$

47. **(a)**



[−5, 5] by [−2, 10]

In this window, it appears they cross twice, although a third crossing off-screen appears likely.

(b)

x	change in $Y1$	change in $Y2$
1		
	3	2
2		
	5	4
3		
	7	8
4		

It happens by the time $x = 4$.

(c) Solving graphically, $x \approx -0.7667$, $x = 2$, $x = 4$.

(d) The solution set is approximately $(-0.7667, 2) \cup (4, \infty)$.

48. Since $f(1) = 4.5$ we have $ka = 4.5$, and since $f(-1) = 0.5$ we have $ka^{-1} = 0.5$.

Dividing, we have

$$\frac{ka}{ka^{-1}} = \frac{4.5}{0.5}$$
$$a^2 = 9$$
$$a = \pm 3$$

Since $f(x) = k \cdot a^x$ is an exponential function, we require $a > 0$, so $a = 3$. Then $ka = 4.5$ gives $3k = 4.5$, so $k = 1.5$. The values are $a = 3$ and $k = 1.5$.

49. Since $f(1) = 1.5$ we have $ka = 1.5$, and since $f(-1) = 6$ we have $ka^{-1} = 6$. Dividing, we have

$$\frac{ka}{ka^{-1}} = \frac{1.5}{6}$$
$$a^2 = 0.25$$
$$a = \pm 0.5$$

Since $f(x) = k \cdot a^x$ is an exponential function, we require $a > 0$, so $a = 0.5$. Then $ka = 1.5$ gives $0.5k = 1.5$, so $k = 3$. The values are $a = 0.5$ and $k = 3$.

Quick Quiz (Sections 1.1–1.3)

1. C, $m = -2$

$$y = -1$$
$$-1 = -2(3) + 5$$
$$-1 = -6 + 5$$
$$-1 = -1$$

2. D, $g(2) = 2(2) - 1 = 4 - 1 = 3$

$$f(3) = (3)^2 + 1 = 9 + 1 = 10$$

3. E.

4. **(a)** $(-\infty, \infty)$

(b) $(-2, \infty)$

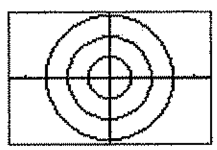
(c) $0 = e^{-x} - 2$

$$ln 2 = -x$$
$$x \approx -0.693$$

Section 1.4 Parametric Equations (pp.30–36)

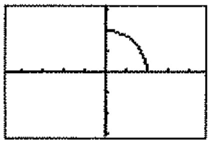
Exploration 1 Parametrizing Circles

1. Each is a circle with radius $|a|$. As $|a|$ increases, the radius of the circle increases.



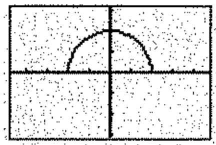
[−4.7, 4.7] by [−3.1, 3.1]

2. $0 \le t \le \frac{\pi}{2}$:



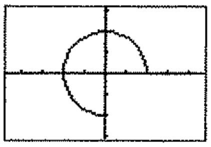
$[-4.7, 4.7]$ by $[-3.1, 3.1]$

$0 \le t \le \pi$:



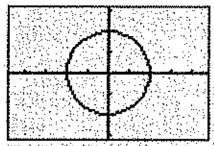
$[-4.7, 4.7]$ by $[-3.1, 3.1]$

$0 \le t \le \frac{3\pi}{2}$:



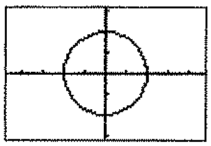
$[-4.7, 4.7]$ by $[-3.1, 3.1]$

$2\pi \le t \le 4\pi$:



$[-4.7, 4.7]$ by $[-3.1, 3.1]$

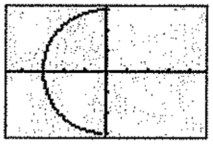
$0 \le t \le 4\pi$



$[-4.7, 4.7]$ by $[-3.1, 3.1]$

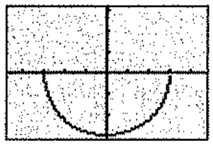
Let d be the length of the parametric interval. If $d < 2\pi$, you get $\frac{d}{2\pi}$ of a complete circle. If $d = 2\pi$, you get the complete circle. If $d > 2\pi$, you get the complete circle but portions of the circle will be traced out more than once. For example, if $d = 4\pi$ the entire circle is traced twice.

3.



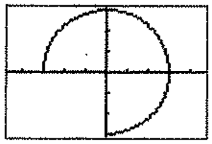
$\frac{\pi}{2} \le t \le \frac{3\pi}{2}$

initial point: (0, 3)
terminal point: (0, –3)



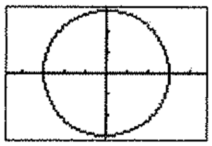
$\pi \le t \le 2\pi$

initial point: (–3, 0)
terminal point: (3, 0)



$\frac{3\pi}{2} \le t \le 3\pi$

initial point: (0, –3)
terminal point: (–3, 0)



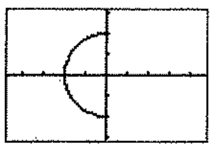
$\pi \le t \le 5\pi$

initial point: (–3, 0)
terminal point: (–3, 0)

4. For $0 \le t \le 2\pi$ the complete circle is traced once clockwise beginning and ending at (2, 0).

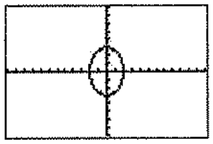
For $\pi \le t \le 3\pi$ the complete circle is traced once clockwise beginning and ending at (–2, 0).

For $\frac{\pi}{2} \le t \le \frac{3\pi}{2}$ the half circle below is traced clockwise starting at (0, –2) and ending at (0, 2).



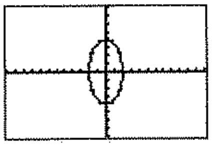
Exploration 2 Parametrizing Ellipses

1. $a = 2, b = 3$:



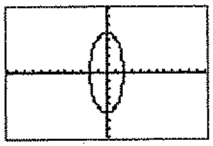
$[-12, 12]$ by $[-8, 8]$

$a = 2, b = 4$:



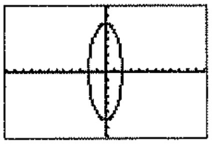
$[-12, 12]$ by $[-8, 8]$

$a = 2, b = 5$:



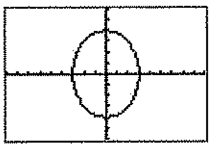
$[-12, 12]$ by $[-8, 8]$

$a = 2, b = 6$:



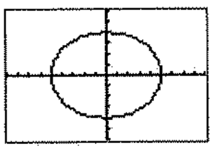
$[-12, 12]$ by $[-8, 8]$

2. $a = 3, b = 4$:



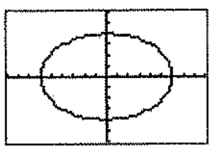
$[-9, 9]$ by $[-6, 6]$

$a = 5, b = 4$:



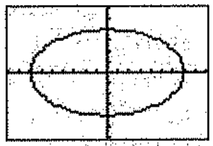
$[-9, 9]$ by $[-6, 6]$

$a = 6, b = 4$:



$[-9, 9]$ by $[-6, 6]$

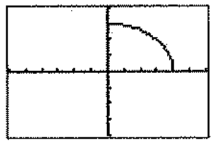
$a = 7, b = 4$:



$[-9, 9]$ by $[-6, 6]$

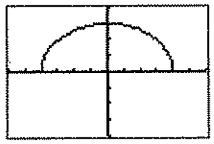
3. If $|a| > |b|$, then the major axis is on the x-axis and the minor on the y-axis. If $|a| < |b|$, then the major axis is on the y-axis and the minor on the x-axis.

4. $0 \le t \le \frac{\pi}{2}$:



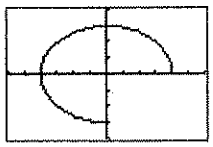
$[-6, 6]$ by $[-4, 4]$

$0 \le t \le \pi$:



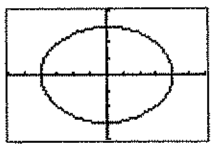
$[-6, 6]$ by $[-4, 4]$

$0 \le t \le \frac{3\pi}{2}$:



$[-6, 6]$ by $[-4, 4]$

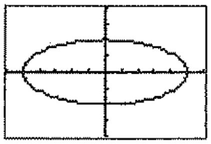
$0 \le t \le 4\pi$:



$[-6, 6]$ by $[-4, 4]$

Let d be the length of the parametric interval. If $d < 2\pi$, you get $\frac{d}{2\pi}$ of a complete ellipse. If $d = 2\pi$, you get the complete ellipse. If $d > 2\pi$, you get the complete ellipse but portions of the ellipse will be traced out more than once. For example, if $d = 4\pi$ the entire ellipse is traced twice.

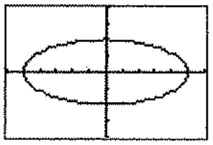
5. $0 \le t \le 2\pi$:



$[-6, 6]$ by $[-4, 4]$

initial point: $(5, 0)$
terminal point: $(5, 0)$

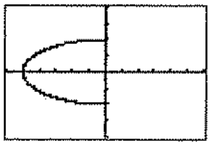
$\pi \le t \le 3\pi$:



$[-6, 6]$ by $[-4, 4]$

initial point: $(-5, 0)$
terminal point: $(-5, 0)$

$\frac{\pi}{2} \le t \le \frac{3\pi}{2}$:



$[-6, 6]$ by $[-4, 4]$

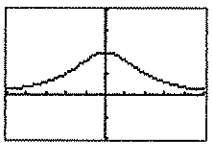
initial point: $(0, -2)$
terminal point: $(0, 2)$

Each curve is traced clockwise from the initial point to the terminal point.

Exploration 3 Graphing the Witch of Agnesi

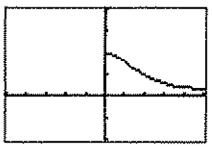
1. We used the parameter interval $[0, \pi]$ because our graphing calculator ignored the fact that the curve is not defined when $t = 0$ or π. The curve is traced from right to left across the screen. x ranges from $-\infty$ to ∞.

2. $-\frac{\pi}{2} \le t \le \frac{\pi}{2}$:



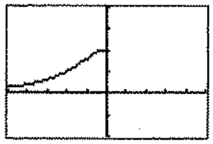
$[-5, 5]$ by $[-2, 4]$

$0 < t \le \frac{\pi}{2}$:



$[-5, 5]$ by $[-2, 4]$

$\frac{\pi}{2} \le t < \pi$:



$[-5, 5]$ by $[-2, 4]$

For $-\frac{\pi}{2} \le t \le \frac{\pi}{2}$, the entire graph described in part 1 is drawn. The left branch is drawn from right to left across the screen starting at the point $(0, 2)$. Then the right branch is drawn from right to left across the screen stopping at the point $(0, 2)$. If you leave out $-\frac{\pi}{2}$ and $\frac{\pi}{2}$, then the point $(0, 2)$ is not drawn.

For $0 < t \le \frac{\pi}{2}$, the right branch is drawn from right to left across the screen stopping at the point $(0, 2)$. If you leave out $\frac{\pi}{2}$, then the point $(0, 2)$ is not drawn.

For $\frac{\pi}{2} \le t < \pi$, the left branch is drawn from right to left across the screen starting at the point $(0, 2)$. If you leave out $\frac{\pi}{2}$, then the point $(0, 2)$ is not drawn.

3. If you replace $x = 2 \cot t$ by $x = -2 \cot t$, the same graph is drawn except it is traced from left to right across the screen. If you replace $x = 2 \cot t$ by $x = 2 \cot(\pi - t)$, the same graph is drawn except it is traced from left to right across the screen.

Quick Review 1.4

1. $m = \frac{3-8}{4-1} = \frac{-5}{3} = -\frac{5}{3}$

$y = -\frac{5}{3}(x-1) + 8$

$y = -\frac{5}{3}x + \frac{29}{3}$

2. $y = -4$

3. $x = 2$

4. When $y = 0$, we have $\frac{x^2}{9} = 1$, so the x-intercepts are -3 and 3. When $x = 0$, we have $\frac{y^2}{16} = 1$, so the y-intercepts are -4 and 4.

5. When $y = 0$, we have $\frac{x^2}{16} = 1$, so the x-intercepts are -4 and 4. When $x = 0$, we have $-\frac{y^2}{9} = 1$, which has no real solution, so there are no y-intercepts.

6. When $y = 0$, we have $0 = x + 1$, so the x-intercept is -1. When $x = 0$, we have $2y^2 = 1$, so the y-intercepts are $-\frac{1}{\sqrt{2}}$ and $\frac{1}{\sqrt{2}}$.

7. (a) $2(1)^2(1) + 1^2 \stackrel{?}{=} 3$

$3 = 3$ Yes

(b) $2(-1)^2(-1) + (-1)^2 \stackrel{?}{=} 3$

$-2 + 1 \stackrel{?}{=} 3$

$-1 \neq 3$ No

(c) $2\left(\frac{1}{2}\right)^2(-2) + (-2)^2 \stackrel{?}{=} 3$

$-1 + 4 \stackrel{?}{=} 3$

$3 = 3$ Yes

8. (a) $9(1)^2 - 18(1) + 4(3)^2 \stackrel{?}{=} 27$

$9 - 18 + 36 \stackrel{?}{=} 27$

$27 = 27$ Yes

(b) $9(1)^2 - 18(1) + 4(-3)^2 \stackrel{?}{=} 27$

$9 - 18 + 36 \stackrel{?}{=} 27$

$27 = 27$ Yes

(c) $9(-1)^2 - 18(-1) + 4(3)^2 \stackrel{?}{=} 27$

$9 + 18 + 36 \stackrel{?}{=} 27$

$63 \neq 27$ No

9. (a) $2x + 3t = -5$

$3t = -2x - 5$

$t = \frac{-2x - 5}{3}$

(b) $3y - 2t = -1$

$-2t = -3y - 1$

$2t = 3y + 1$

$t = \frac{3y + 1}{2}$

10. (a) The equation is true for $a \geq 0$.

(b) The equation is equivalent to "$\sqrt{a^2} = a$ or $\sqrt{a^2} = -a$." Since $\sqrt{a^2} = a$ is true for $a \geq 0$ and $\sqrt{a^2} = -a$ is true for $a \leq 0$, at least one of the two equations is true for all real values of a. Therefore, the given equation $\sqrt{a^2} = \pm a$ is true for all real values of a.

(c) The equation is true for all real values of a.

Section 1.4 Exercises

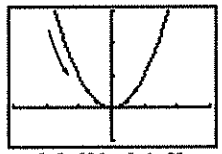
1. Graph (c). Window: $[-4, 4]$ by $[-3, 3]$, $0 \leq t \leq 2\pi$

2. Graph (a). Window: $[-2, 2]$ by $[-2, 2]$, $0 \leq t \leq 2\pi$

3. Graph (d). Window: $[-10, 10]$ by $[-10, 10]$, $0 \leq t \leq 2\pi$

4. Graph (b). Window: $[-15, 15]$ by $[-15, 15]$, $0 \leq t \leq 2\pi$

5. (a)



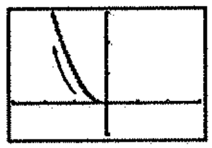
$[-3, 3]$ by $[-1, 3]$

No initial or terminal point.

(b) $y = 9t^2 = (3t)^2 = x^2$

The parametrized curve traces all of the parabola defined by $y = x^2$.

6. (a)



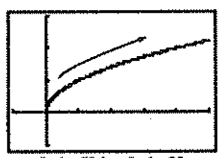
$[-3, 3]$ by $[-1, 3]$

Initial point: (0, 0)
Terminal point: None

(b) $y = t = (-\sqrt{t})^2 = x^2$

The parametrized curve traces the left half of the parabola defined by $y = x^2$ (or all of the curve defined by $x = -\sqrt{y}$).

7. (a)



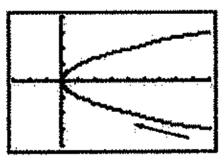
$[-1, 5]$ by $[-1, 3]$

Initial point: (0, 0)
Terminal point: None

(b) $y = \sqrt{t} = \sqrt{x}$

The parametrized curve traces all of the curve defined by $y = \sqrt{x}$ (or the upper half of the parabola defined by $x = y^2$).

8. (a)



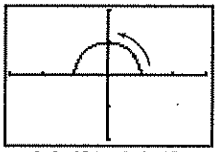
$[-3, 9]$ by $[-4, 4]$

No initial or terminal point.

(b) $x = \sec^2 t - 1 = \tan^2 t = y^2$

The parametrized curve traces all of the parabola defined by $x = y^2$.

9. (a)



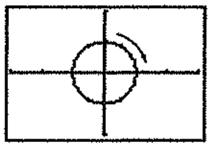
[–3, 3] by [–2, 2]

Initial point: (1, 0)
Terminal point: (–1, 0)

(b) $x^2 + y^2 = \cos^2 t + \sin^2 t = 1$

The parametrized curve traces the upper half of the circle defined by $x^2 + y^2 = 1$ (or all of the semicircle defined by $y = \sqrt{1 - x^2}$).

10. (a)



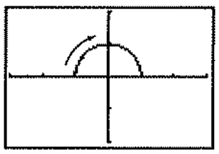
[–3, 3] by [–2, 2]

Initial and terminal point: (0, 1)

(b) $x^2 + y^2 = \sin^2(2\pi t) + \cos^2(2\pi t) = 1$

The parametrized curve traces all of the circle defined by $x^2 + y^2 = 1$.

11. (a)



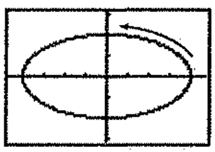
[–3, 3] by [–2, 2]

Initial point: (–1, 0)
Terminal point: (0, 1)

(b) $x^2 + y^2 = \cos^2(\pi - t) + \sin^2(\pi - t) = 1$

The parametrized curve traces the upper half of the circle defined by $x^2 + y^2 = 1$ (or all of the semicircle defined by $y = \sqrt{1 - x^2}$).

12. (a)



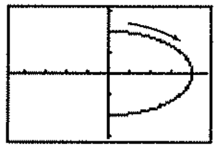
[–4.7, 4.7] by [–3.1, 3.1]

Initial and terminal point: (4, 0)

(b) $\left(\frac{x}{4}\right)^2 + \left(\frac{y}{2}\right)^2 = \cos^2 t + \sin^2 t = 1$

The parametrized curve traces all of the ellipse defined by $\left(\frac{x}{4}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$.

13. (a)



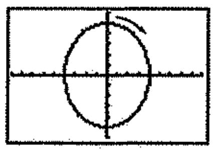
[–4.7, 4.7] by [–3.1, 3.1]

Initial point: (0, 2)
Terminal point: (0, –2)

(b) $\left(\frac{x}{4}\right)^2 + \left(\frac{y}{2}\right)^2 = \sin^2 t + \cos^2 t = 1$

The parametrized curve traces the right half of the ellipse defined by $\left(\frac{x}{4}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$ (or all of the curve defined by $x = 2\sqrt{4 - y^2}$).

14. (a)



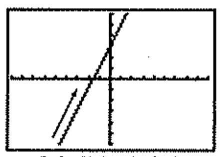
[–9, 9] by [–6, 6]

Initial and terminal point: (0, 5)

(b) $\left(\frac{x}{4}\right)^2 + \left(\frac{y}{5}\right)^2 = \sin^2 t + \cos^2 t = 1$

The parametrized curve traces all of the ellipse defined by $\left(\frac{x}{4}\right)^2 + \left(\frac{y}{5}\right)^2 = 1$.

15. (a)



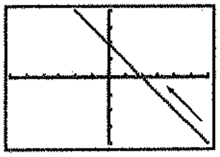
[–9, 9] by [–6, 6]

No initial or terminal point.

(b) $y = 4t - 7 = 2(2t - 5) + 3 = 2x + 3$

The parametrized curve traces all of the line defined by $y = 2x + 3$.

16. (a)



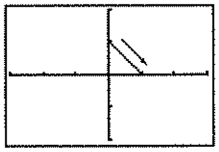
[−6, 6] by [−4, 4]

No initial or terminal point.

(b) $y = 1 + t = 2 - (1 - t) = 2 - x = -x + 2$

The parametrized curve traces all of the line defined by $y = -x + 2$.

17. (a)



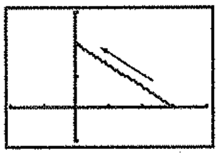
[−3, 3] by [−2, 2]

Initial point: (0, 1)
Terminal point: (1, 0)

(b) $y = 1 - t = 1 - x = -x + 1$

The Cartesian equation is $y = -x + 1$. The portion traced by the parametrized curve is the segment from (0, 1) to (1, 0).

18. (a)



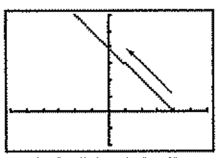
[−2, 4] by [−1, 3]

Initial point: (3, 0)
Terminal point: (0, 2)

(b) $y = 2t = (2t - 2) + 2 = -\frac{2}{3}(3 - 3t) + 2 = -\frac{2}{3}x + 2$

The Cartesian equation is $y = -\frac{2}{3}x + 2$. The portion traced by the curve is the segment from (3, 0) to (0, 2).

19. (a)



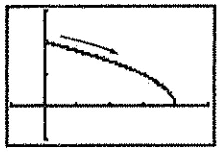
[−6, 6] by [−2, 6]

Initial point: (4, 0)
Terminal point: None

(b) $y = \sqrt{t} = 4 - (4 - \sqrt{t}) = 4 - x = -x + 4$

The parametrized curve traces the portion of the line defined by $y = -x + 4$ to the left of (4, 0), that is, for $x \le 4$.

20. (a)



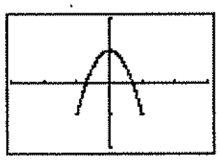
[−1, 5] by [−1, 3]

Initial point: (0, 2)
Terminal point: (4, 0)

(b) $y = \sqrt{4 - t^2} = \sqrt{4 - x}$

The parametrized curve traces the right portion of the curve defined by $y = \sqrt{4 - x}$, that is, for $x \ge 0$.

21. (a)



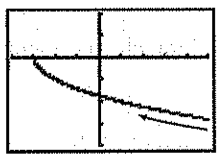
[−3, 3] by [−2, 2]

No initial or terminal point, since the t-interval has no beginning or end. The curve is traced and retraced in both directions.

(b)
$$\begin{aligned} y &= \cos 2t \\ &= \cos^2 t - \sin^2 t \\ &= 1 - 2\sin^2 t \\ &= 1 - 2x^2 \\ &= -2x^2 + 1 \end{aligned}$$

The parametrized curve traces the portion of the parabola defined by $y = -2x^2 + 1$ corresponding to $-1 \le x \le 1$.

22. (a)



[−4, 5] by [−4, 2]

Initial point: None
Terminal point: (−3, 0)

(b) $x = t^2 - 3 = y^2 - 3$

The parametrized curve traces the lower half of the parabola defined by $x = y^2 - 3$ (or all of the curve defined by $y = -\sqrt{x + 3}$).

23. Using $(-1, -3)$ we create the parametric equations $x = -1 + at$ and $y = -3 + bt$, representing a line which goes through $(-1, -3)$ at $t = 0$. We determine a and b so that the line goes through $(4, 1)$ when $t = 1$.

Since $4 = -1 + a$, $a = 5$.

Since $1 = -3 + b$, $b = 4$.

Therefore, one possible parametrization is $x = -1 + 5t$, $y = -3 + 4t$, $0 \le t \le 1$.

24. Using $(-1, 3)$ we create the parametric equations $x = -1 + at$ and $y = 3 + bt$, representing a line which goes through $(-1, 3)$ at $t = 0$. We determine a and b so that the line goes through $(3, -2)$ at $t = 1$.

Since $3 = -1 + a$, $a = 4$.

Since $-2 = 3 + b$, $b = -5$.

Therefore, one possible parametrization is $x = -1 + 4t$, $y = 3 - 5t$, $0 \le t \le 1$.

25. The lower half of the parabola is given by $x = y^2 + 1$ for $y \le 0$. Substituting t for y, we obtain one possible parametrization: $x = t^2 + 1$, $y = t$, $t \le 0$.

26. The vertex of the parabola is at $(-1, -1)$, so the left half of the parabola is given by $y = x^2 + 2x$ for $x \le -1$. Substituting t for x, we obtain one possible parametrization: $x = t$, $y = t^2 + 2t$, $t \le -1$.

27. For simplicity, we assume that x and y are linear functions of t and that point (x, y) starts at $(2, 3)$ for $t = 0$ and passes through $(-1, -1)$ at $t = 1$. Then $x = f(t)$, where $f(0) = 2$ and $f(1) = -1$.

Since slope $= \dfrac{\Delta x}{\Delta t} = \dfrac{-1-2}{1-0} = -3$,

$x = f(t) = -3t + 2 = 2 - 3t$. Also, $y = g(t)$, where $g(0) = 3$ and $g(1) = -1$.

Since slope $= \dfrac{\Delta y}{\Delta t} = \dfrac{-1-3}{1-0} = -4$,

$y = g(t) = -4t + 3 = 3 - 4t$.

One possible parametrization is:

$x = 2 - 3t$, $y = 3 - 4t$, $t \ge 0$.

28. For simplicity, we assume that x and y are linear functions of t and that the point (x, y) starts at $(-1, 2)$ for $t = 0$ and passes through $(0, 0)$ at $t = 1$. Then $x = f(t)$, where $f(0) = -1$ and $f(1) = 0$.

Since slope $= \dfrac{\Delta x}{\Delta t} = \dfrac{0-(-1)}{1-0} = 1$,

$x = f(t) = 1t + (-1) = -1 + t$.

Also, $y = g(t)$, where $g(0) = 2$ and $g(1) = 0$.

Since slope $= \dfrac{\Delta y}{\Delta t} = \dfrac{0-2}{1-0} = -2$,

$y = g(t) = -2t + 2 = 2 - 2t$.

One possible parametrization is:

$x = -1 + t$, $y = 2 - 2t$, $t \ge 0$.

29. The graph is in Quadrant I when $0 < y < 2$, which corresponds to $1 < t < 3$. To confirm, note that $x(1) = 2$ and $x(3) = 0$.

30. The graph is in Quadrant II when $2 < y \le 4$, which corresponds to $3 < t \le 5$. To confirm, note that $x(3) = 0$ and $x(5) = -2$.

31. The graph is in Quadrant III when $-6 \le y < -4$, which corresponds to $-5 \le t < -3$. To confirm, note that $x(-5) = -2$ and $x(-3) = 0$.

32. The graph is in Quadrant IV when $-4 < y < 0$, which corresponds to $-3 < t < 1$. To confirm, note that $x(-3) = 0$ and $x(1) = 2$.

33. The graph of $y = x^2 + 2x + 2$ lies in Quadrant I for all $x > 0$. Substituting t for x, we obtain one possible parametrization: $x = t$, $y = t^2 + 2t + 2$, $t > 0$.

34. The graph of $y = \sqrt{x+3}$ lies in Quadrant I for all $x \ge 0$. Substituting t for x, we obtain one possible parametrization: $x = t$, $y = \sqrt{t+3}$, $t > 0$.

35. Possible answers:

(a) $x = a \cos t$, $y = -a \sin t$, $0 \le t \le 2\pi$

(b) $x = a \cos t$, $y = a \sin t$, $0 \le t \le 2\pi$

(c) $x = a \cos t$, $y = -a \sin t$, $0 \le t \le 4\pi$

(d) $x = a \cos t$, $y = a \sin t$, $0 \le t \le 4\pi$

36. Possible answers:

(a) $x = -a \cos t$, $y = b \sin t$, $0 \le t \le 2\pi$

(b) $x = -a \cos t$, $y = -b \sin t$, $0 \le t \le 2\pi$

(c) $x = -a \cos t$, $y = b \sin t$, $0 \le t \le 4\pi$

(d) $x = -a \cos t$, $y = -b \sin t$, $0 \le t \le 4\pi$

37. False. It is an ellipse.

38. True. Circle starting at $(2, 0)$ and ending at $(2, 0)$.

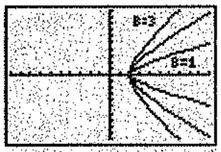
39. D.

40. C.

41. A.

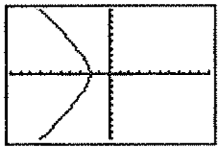
42. E.

43. (a) The resulting graph appears to be the right half of a hyperbola in the first and fourth quadrants. The parameter a determines the x-intercept. The parameter b determines the shape of the hyperbola. If b is smaller, the graph has less steep slopes and appears "sharper". If b is larger, the slopes are steeper and the graph appears more "blunt". The graphs for $a = 2$ and $b = 1$, 2, and 3 are shown.



$[-10, 10]$ by $[-10, 10]$

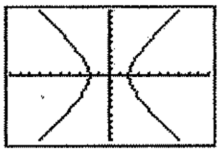
(b)



$[-10, 10]$ by $[-10, 10]$

This appears to be the left half of the same hyberbola.

(c)



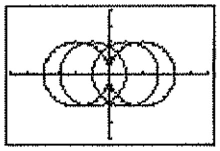
$[-10, 10]$ by $[-10, 10]$

One must be careful because both sec t and tan t are discontinuous at these points. This might cause the grapher to include extraneous lines (the asymptotes of the hyperbola) in its graph. The extraneous lines can be avoided by using the grapher's dot mode instead of connected mode.

(d) Note that $\sec^2 t - \tan^2 t = 1$ by a standard trigonometric identity. Substituting $\frac{x}{a}$ for sec t and $\frac{y}{b}$ for tan t gives $\left(\frac{x}{a}\right)^2 - \left(\frac{y}{b}\right)^2 = 1$.

(e) This changes the orientation of the hyperbola. In this case, b determines the y-intercept of the hyperbola, and a determines the shape. The parameter interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ gives the upper half of the hyperbola. The parameter interval $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ gives the lower half. The same values of t cause discontinuities and may add extraneous lines to the graph. Substituting $\frac{y}{b}$ for sec t and $\frac{x}{a}$ for tan t in the identity $\sec^2 t - \tan^2 t = 1$ gives $\left(\frac{y}{b}\right)^2 - \left(\frac{x}{a}\right)^2 = 1$.

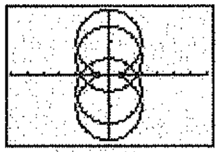
44. (a)



$[-6, 6]$ by $[-4, 4]$

The graph is a circle of radius 2 centered at $(h, 0)$. As h changes, the graph shifts horizontally.

(b)



$[-6, 6]$ by $[-4, 4]$

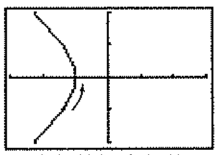
The graph is a circle of radius 2 centered at $(0, k)$. As k changes, the graph shifts vertically.

(c) Since the circle is to be centered at $(2, -3)$, we use $h = 2$ and $k = -3$. Since a radius of 5 is desired, we need to change the coefficients of cos t and sin t to 5.

$x = 5 \cos t + 2,\ y = 5 \sin t - 3,\ 0 \le t \le 2\pi$

(d) $x = 5 \cos t - 3,\ y = 2 \sin t + 4,\ 0 \le t \le 2\pi$

45. (a)



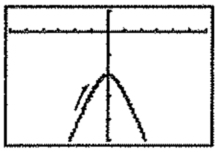
$[-3, 3]$ by $[-2, 2]$

No initial or terminal point. Note that it may be necessary to use a t-interval such as $[-1.57, 1.57]$ or use dot mode in order to avoid "asymptotes" showing on the calculator screen.

(b) $x^2 - y^2 = \sec^2 t - \tan^2 t = 1$

The parametrized curve traces the left branch of the hyperbola defined by $x^2 - y^2 = 1$ (or all of the curve defined by $x = -\sqrt{y^2 + 1}$).

46. (a)



[−6, 6] by [−5, 1]

No initial or terminal point. Note that it may be necessary to use a t-interval such as [−1.57, 1.57] or use dot mode in order to avoid "asymptotes" showing on the calculator screen.

(b) $\left(\frac{y}{2}\right)^2 - x^2 = \sec^2 t - \tan^2 t = 1$

The parametrized curve traces the lower branch of the hyperbola defined by $\left(\frac{y}{2}\right)^2 - x^2 = 1$ (or all of the curve defined by $y = -2\sqrt{x^2+1}$).

47. Note that $m\angle OAQ = t$, since alternate interior angles formed by a transversal of parallel lines are congruent.

Therefore, $\tan t = \frac{OQ}{AQ} = \frac{2}{x}$, so $x = \frac{2}{\tan t} = 2\cot t$.

Now, by equation (iii), we know that

$$\begin{aligned} AB &= \frac{(AQ)^2}{AO} \\ &= \left(\frac{AQ}{AO}\right)(AQ) \\ &= (\cos t)(x) \\ &= (\cos t)(2\cot t) \\ &= \frac{2\cos^2 t}{\sin t}. \end{aligned}$$

Then equation (ii) gives

$$y = 2 - AB\sin t = 2 - \frac{2\cos^2 t}{\sin t}\cdot \sin t = 2 - 2\cos^2 t$$
$$= 2\sin^2 t.$$

The parametric equations are:

$x = 2\cot t,\ y = 2\sin^2 t,\ 0 < t < \pi$

Note: Equation (iii) may not be immediately obvious, but it may be justified as follows. Sketch segment QB. Then $\angle OBQ$ is a right angle, so $\Delta ABQ \sim \Delta AQO$, which gives $\frac{AB}{AQ} = \frac{AQ}{AO}$.

48. (a) If $x_2 = x_1$ then the line is a vertical line and the first parametric equation gives $x = x_1$, while the second will give all real values for y since it cannot be the case that $y_2 = y_1$ as well.

Otherwise, solving the first equation for t gives $t = (x - x_1) / (x_2 - x_1)$.

Substituting that into the second equation gives $y = y_1 + [(y_2 - y_1)/(x_2 - x_1)](x - x_1)$ which is the point-slope form of the equation for the line through (x_1, y_1) and (x_2, y_2).
Note that the first equation will cause x to take on all real values, because $(x_2 - x_1)$ is not zero. Therefore, all of the points on the line will be traced out.

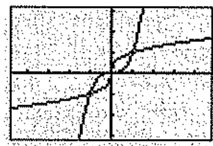
(b) Use the equations for x and y given in part (a), with $0 \le t \le 1$.

Section 1.5 Functions and Logarithms (pp. 37–45)

Exploration 1 Testing for Inverses Graphically

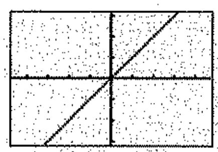
1. It appears that $(f \circ g)(x) = (g \circ f)(x) = x$, suggesting that f and g may be inverses of each other.

(a) f and g:



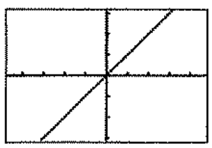
[−4.7, 4.7] by [−3.1, 3.1]

(b) $f \circ g$:



[−4.7, 4.7] by [−3.1, 3.1]

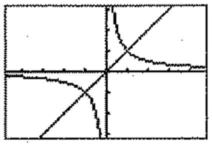
(c) $g \circ f$:



[−4.7, 4.7] by [−3.1, 3.1]

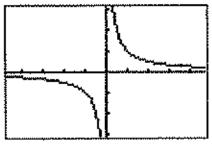
2. It appears that $f \circ g = g \circ f = g$, suggesting that f may be the identity function.

(a) f and g:



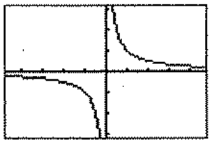
[−4.7, 4.7] by [−3.1, 3.1]

(b) $f \circ g$:



[−4.7, 4.7] by [−3.1, 3.1]

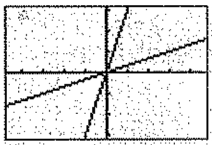
(c) $g \circ f$:



[−4.7, 4.7] by [−3.1, 3.1]

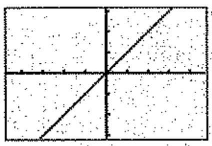
3. It appears that $(f \circ g)(x) = (g \circ f)(x) = x$, suggesting that f and g may be inverses of each other.

(a) f and g:



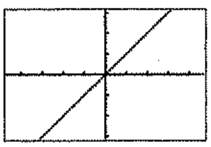
[−4.7, 4.7] by [−3.1, 3.1]

(b) $f \circ g$:



[−4.7, 4.7] by [−3.1, 3.1]

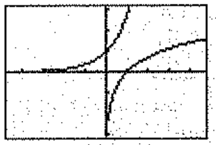
(c) $g \circ f$:



[−4.7, 4.7] by [−3.1, 3.1]

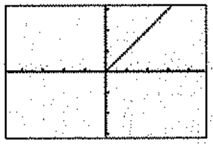
4. It appears that $(f \circ g)(x) = (g \circ f)(x) = x$, suggesting that f and g may be inverses of each other. (Notice that the domain of $f \circ g$ is $(0, \infty)$ and the domain of $g \circ f$ is $(-\infty, \infty)$.

(a) f and g:



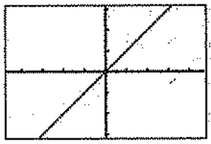
[−4.7, 4.7] by [−3.1, 3.1]

(b) $f \circ g$:



[−4.7, 4.7] by [−3.1, 3.1]

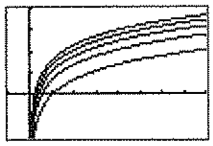
(c) $g \circ f$:



[−4.7, 4.7] by [−3.1, 3.1]

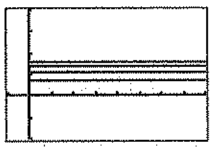
Exploration 2 Supporting the Product Rule

1. They appear to be vertical translations of each other.



[−1, 8] by [−2, 4]

2. This graph suggests that each difference $(y_3 = y_1 - y_2)$ is a constant.



[−1, 8] by [−2, 4]

3. $y_3 = y_1 - y_2 = \ln(ax) - \ln x = \ln a + \ln x - \ln x = \ln a$ Thus, the difference $y_3 = y_1 - y_2$ is the constant In a.

Quick Review 1.5

1. $(f \circ g)(1) = f(g(1)) = f(2) = 1$

2. $(g \circ f)(-7) = g(f(-7)) = g(-2) = 5$

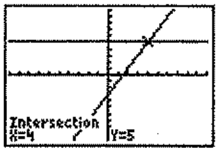
3. $(f \circ g)(x) = f(g(x)) = f(x^2+1) = \sqrt[3]{(x^2+1)-1}$
$= \sqrt[3]{x^2} = x^{2/3}$

4. $(g \circ f)(x) = g(f(x)) = g(\sqrt[3]{x-1})$
$= (\sqrt[3]{x-1})^2 + 1$
$= (x-1)^{2/3} + 1$

5. Substituting t for x, one possible answer is:
$x = t,\ y = \frac{1}{t-1},\ t \geq 2.$

6. Substituting t for x, one possible answer is:
$x = t,\ y = t,\ t < -3.$

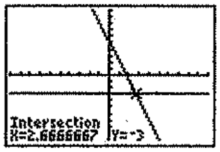
7.



[−10, 10] by [−10, 10]

(4, 5)

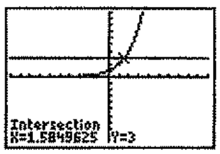
8.



[−10, 10] by [−10, 10]

$\left(\frac{8}{3}, -3\right) \approx (2.67, -3)$

9. (a)

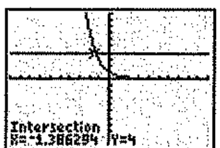


[−10, 10] by [−10, 10]

(1.58, 3)

(b) No points of intersection, since $2^x > 0$ for all values of x.

10. (a)



[−10, 10] by [−10, 10]

(−1.39, 4)

(b) No points of intersection, since $e^{-x} > 0$ for all values of x.

Section 1.5 Exercises

1. No, since (for example) the horizontal line $y = 2$ intersects the graph twice.

2. Yes, since each horizontal line intersects the graph only once.

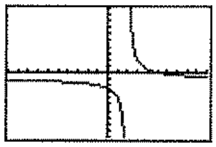
3. Yes, since each horizontal line intersects the graph at most once.

4. No, since (for example) the horizontal line $y = 0.5$ intersects the graph twice.

5. Yes, since each horizontal line intersects the graph only once.

6. No, since (for example) the horizontal line $y = 2$ intersects the graph at more than one point.

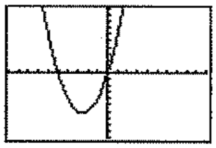
7.



[−10, 10] by [−10, 10]

Yes, the function is one-to-one since each horizontal line intersects the graph at most once, so it has an inverse function.

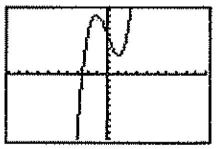
8.



[−10, 10] by [−10, 10]

No, the function is not one-to-one since (for example) the horizontal line $y = 0$ intersects the graph twice, so it does not have an inverse function.

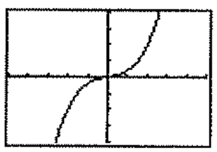
9.



[−10, 10] by [−10, 10]

No, the function is not one-to-one since (for example) the horizontal line $y = 5$ intersects the graph more than once, so it does not have an inverse function.

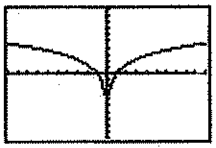
10.



[−5, 5] by [−20, 20]

Yes, the function is one-to-one since each horizontal line intersects the graph only once, so it has an inverse function.

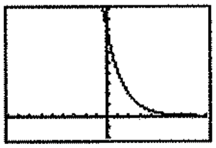
11.



[−10, 10] by [−10, 10]

No, the function is not one-to-one since each horizontal line intersects the graph twice, so it does not have an inverse function.

12.



[−9, 9] by [−2, 10]

Yes, the function is one-to-one since each horizontal line intersects the graph at most once, so it has an inverse function.

13. $y = 2x+3$

$y-3 = 2x$

$\frac{y-3}{2} = x$

Interchange x and y.

$\frac{x-3}{2} = y$

$f^{-1}(x) = \frac{x-3}{2}.$

Verify.

$$(f \circ f^{-1})(x) = f\left(\frac{x-3}{2}\right) = 2\left(\frac{x-3}{2}\right)+3 = (x-3)+3 = x$$

$$(f^{-1} \circ f)(x) = f^{-1}(2x+3) = \frac{(2x+3)-3}{2} = \frac{2x}{2} = x$$

14. $y = 5-4x$

$4x = 5-y$

$x = \frac{5-y}{4}$

Interchange x and y.

$y = \frac{5-x}{4}$

$f^{-1}(x) = \frac{5-x}{4}$

Verify.

$$(f \circ f^{-1})(x) = f\left(\frac{5-x}{4}\right) = 5-4\left(\frac{5-x}{4}\right) = 5-(5-x) = x$$

$$(f^{-1} \circ f)(x) = f^{-1}(5-4x) = \frac{5-(5-4x)}{4} = \frac{4x}{4} = x$$

15. $y = x^3 - 1$

$y+1 = x^3$

$(y+1)^{1/3} = x$

Interchange x and y.

$(x+1)^{1/3} = y$

$f^{-1}(x) = (x+1)^{1/3}$ or $\sqrt[3]{x+1}$

Verify.

$$(f \circ f^{-1})(x) = f(\sqrt[3]{x+1}) = (\sqrt[3]{x+1})^3 - 1 = (x+1) - 1 = x$$

$$(f^{-1} \circ f)(x) = f^{-1}(x^3 - 1) = \sqrt[3]{(x^3-1)+1} = \sqrt[3]{x^3} = x$$

16. $y = x^2 + 1, x \geq 0$

$y - 1 = x^2, x \geq 0$

$\sqrt{y-1} = x$

Interchange x and y.

$\sqrt{x-1} = y$

$f^{-1}(x) = \sqrt{x-1}$ or $(x-1)^{1/2}$

Verify. For $x \geq 1$ (the domain of f^{-1}),

$$(f \circ f^{-1})(x) = f(\sqrt{x-1}) = (\sqrt{x-1})^2 + 1 = (x-1)+1 = x$$

For $x > 0$, (the domain of f),

$$(f^{-1} \circ f)(x) = f^{-1}(x^2+1) = \sqrt{(x^2+1)-1} = \sqrt{x^2} = |x| = x$$