17. $y = x^2, x \le 0$
$x = -\sqrt{y}$

Interchange x and y.

$y = -\sqrt{x}$
$f^{-1}(x) = -\sqrt{x}$ or $-x^{1/2}$

Verify.

For $x \ge 0$ (the domain of f^{-1}),

$(f \circ f^{-1})(x) = f(-\sqrt{x}) = (-\sqrt{x})^2 = x$

For $x \le 0$, (the domain of f),

$(f^{-1} \circ f)(x) = f^{-1}(x^2) = -\sqrt{x^2} = -|x| = x$

18. $y = x^{2/3}, x \ge 0$
$y^{3/2} = (x^{2/3})^{3/2}, x \ge 0$
$y^{3/2} = x$

Interchange x and y.

$x^{3/2} = y$
$f^{-1}(x) = x^{3/2}$

Verify.

For $x \ge 0$ (the domain of f^{-1}),

$(f \circ f^{-1})(x) = f(x^{3/2}) = (x^{3/2})^{2/3} = x$

for $x \ge 0$, (the domain of f),

$(f^{-1} \circ f)(x) = f^{-1}(x^{2/3}) = (x^{2/3})^{3/2} = |x| = x$

19. $y = -(x-2)^2, x \le 2$
$(x-2)^2 = -y, x \le 2$
$x - 2 = -\sqrt{-y}$
$x = 2 - \sqrt{-y}$

Interchange x and y.

$y = 2 - \sqrt{-x}$
$f^{-1}(x) = 2 - \sqrt{-x}$ or $2 - (-x)^{1/2}$

Verify.

For $x \le 0$ (the domain of f^{-1}),

$$\begin{aligned}(f \circ f^{-1})(x) &= f(2 - \sqrt{-x}) \\ &= -[(2 - \sqrt{-x}) - 2]^2 \\ &= -(-\sqrt{-x})^2 = -|x| = x\end{aligned}$$

For $x \le 2$ (the domain of f),

$$\begin{aligned}(f^{-1} \circ f)(x) &= f^{-1}(-(x-2)^2) \\ &= 2 - \sqrt{(x-2)^2} \\ &= 2 - |x-2| = 2 + (x-2) = x\end{aligned}$$

20. $y = (x^2 + 2x + 1), x \ge -1$
$y = (x+1)^2, x \ge -1$
$\sqrt{y} = x + 1$
$\sqrt{y} - 1 = x$

Interchange x and y.

$\sqrt{x} - 1 = y$
$f^{-1}(x) = \sqrt{x} - 1$ or $x^{1/2} - 1$

Verify.

For $x \ge 0$ (the domain of f^{-1}),

$$\begin{aligned}(f \circ f^{-1})(x) &= f(\sqrt{x} - 1) \\ &= [(\sqrt{x} - 1)^2 + 2(\sqrt{x} - 1) + 1] \\ &= (\sqrt{x})^2 - 2\sqrt{x} + 1 + 2\sqrt{x} - 2 + 1 \\ &= (\sqrt{x})^2 = x\end{aligned}$$

For $x \ge -1$ (the domain of f),

$$\begin{aligned}(f^{-1} \circ f)(x) &= f^{-1}(x^2 + 2x + 1) \\ &= \sqrt{x^2 + 2x + 1} - 1 \\ &= \sqrt{(x+1)^2} - 1 \\ &= |x+1| - 1 \\ &= (x+1) - 1 = x\end{aligned}$$

21. $y = \dfrac{1}{x^2}, x > 0$
$x^2 = \dfrac{1}{y}, x > 0$
$x = \sqrt{\dfrac{1}{y}} = \dfrac{1}{\sqrt{y}}$

Interchange x and y.

$y = \dfrac{1}{\sqrt{x}}$
$f^{-1}(x) = \dfrac{1}{\sqrt{x}}$ or $\dfrac{1}{x^{1/2}}$

Verify.

For $x > 0$ (the domain of f^{-1}),

$(f \circ f^{-1})(x) = f\left(\dfrac{1}{\sqrt{x}}\right) = \dfrac{1}{(1/\sqrt{x})^2} = x$

For $x > 0$ (the domain of f),

$(f^{-1} \circ f)(x) = f^{-1}\left(\dfrac{1}{x^2}\right) = \dfrac{1}{\sqrt{1/x^2}} = \sqrt{x^2} = |x| = x$

22. $y=\frac{1}{x^3}$

$x^3=\frac{1}{y}$

$x=\sqrt[3]{\frac{1}{y}}=\frac{1}{\sqrt[3]{y}}$

Interchange x and y.

$y=\frac{1}{\sqrt[3]{x}}$

$f^{-1}(x)=\frac{1}{\sqrt[3]{x}}$ or $\frac{1}{x^{1/3}}$

Verify.

$(f\circ f^{-1})(x)=f\left(\frac{1}{\sqrt[3]{x}}\right)=\frac{1}{(1/\sqrt[3]{x})^3}=x$

$(f^{-1}\circ f)(x)=f^{-1}\left(\frac{1}{x^3}\right)=\frac{1}{\sqrt[3]{1/x^3}}=x$

23. $y=\frac{2x+1}{x+3}$

$xy+3y=2x+1$

$xy-2x=1-3y$

$(y-2)x=1-3y$

$x=\frac{1-3y}{y-2}$

Interchange x and y.

$y=\frac{1-3x}{x-2}$

$f^{-1}(x)=\frac{1-3x}{x-2}$

Verify.

$$(f\circ f^{-1})(x)=f\left(\frac{1-3x}{x-2}\right)=\frac{2\left(\frac{1-3x}{x-2}\right)+1}{\frac{1-3x}{x-2}+3}=\frac{2(1-3x)+(x-2)}{(1-3x)+3(x-2)}=\frac{-5x}{-5}=x$$

$$(f^{-1}\circ f)(x)=f^{-1}\left(\frac{2x+1}{x+3}\right)=\frac{1-3\left(\frac{2x+1}{x+3}\right)}{\frac{2x+1}{x+3}-2}=\frac{(x+3)-3(2x+1)}{(2x+1)-2(x+3)}=\frac{-5x}{-5}=x$$

24. $y=\frac{x+3}{x-2}$

$xy-2y=x+3$

$xy-x=2y+3$

$x(y-1)=2y+3$

$x=\frac{2y+3}{y-1}$

Interchange x and y.

$y=\frac{2x+3}{x-1}$

$f^{-1}(x)=\frac{2x+3}{x-1}$

Verify.

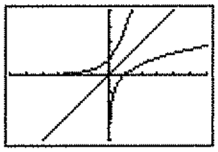
$$(f\circ f^{-1})(x)=f\left(\frac{2x+3}{x-1}\right)=\frac{\frac{2x+3}{x-1}+3}{\frac{2x+3}{x-1}-2}=\frac{(2x+3)+3(x-1)}{(2x+3)-2(x-1)}=\frac{5x}{5}=x$$

$$(f^{-1}\circ f)(x)=f^{-1}\left(\frac{x+3}{x-2}\right)=\frac{2\left(\frac{x+3}{x-2}\right)+3}{\frac{x+3}{x-2}-1}=\frac{2(x+3)+3(x-2)}{(x+3)-(x-2)}=\frac{5x}{5}=x$$

25. Graph of f: $x_1 = t,\ y_1 = e^t$

Graph of $f^{-1}: x_2 = e^t,\ y_2 = t$

Graph of $y = x$: $x_3 = t,\ y_3 = t$

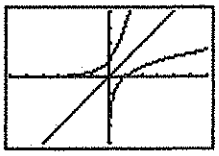


[−6, 6] by [−4, 4]

26. Graph of f: $x_1 = t,\ y_1 = 3^t$

Graph of $f^{-1}: x_2 = 3^t,\ y_2 = t$

Graph of $y = x$: $x_3 = t,\ y_3 = t$

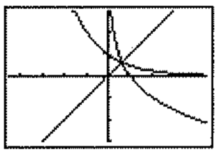


[−6, 6] by [−4, 4]

27. Graph of f: $x_1 = t,\ y_1 = 2^{-t}$

Graph of $f^{-1} = x_2 = 2^{-t},\ y_2 = t$

Graph of $y = x$: $x_3 = t,\ y_3 = t$

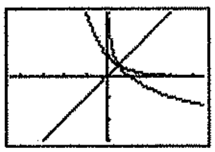


[−4.5, 4.5] by [−3, 3]

28. Graph of f: $x_1 = t,\ y_1 = 3^{-t}$

Graph of $f^{-1}: x_2 = 3^{-t},\ y_2 = t$

Graph of $y = x$: $x_3 = t,\ y_3 = t$

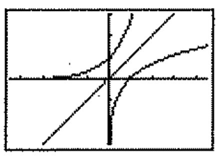


[−4.5, 4.5] by [−3, 3]

29. Graph of f: $x_1 = t,\ y_1 = \ln t$

Graph of $f^{-1}: x_2 = \ln t,\ y_2 = t$

Graph of $y = x$: $x_3 = t,\ y_3 = t$

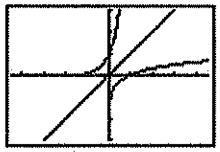


[−4.5, 4.5] by [−3, 3]

30. Graph of f: $x_1 = t,\ y_1 = \log t$

Graph of $f^{-1}: x_2 = \log t,\ y_2 = t$

Graph of $y = x$: $x_3 = t,\ y_3 = t$



[−4.5, 4.5] by [−3, 3]

31. Graph of f: $x_1 = t,\ y_1 = \sin^{-1} t$

Graph of $f^{-1}: x_2 = \sin^{-1} t,\ y_2 = t$

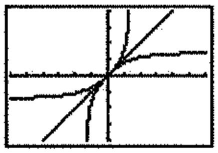
Graph of $y = x$: $x_3 = t,\ y_3 = t$

[−3, 3] by [−2, 2]

32. Graph of f: $x_1 = t,\ y_1 = \tan^{-1} t$

Graph of $f^{-1}: x_2 = \tan^{-1} t,\ y_2 = t$

Graph of $y = x$: $x_3 = t,\ y_3 = t$



[−6, 6] by [−4, 4]

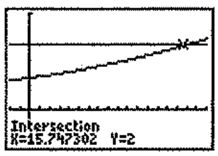
33. $(1.045)^t = 2$

$\ln(1.045)^t = \ln 2$

$t \ln 1.045 = \ln 2$

$t = \dfrac{\ln 2}{\ln 1.045} \approx 15.75$

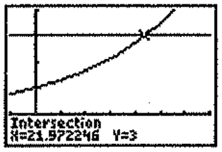
Graphical support:



[−2, 18] by [−1, 3]

34. $e^{0.05t} = 3$
$\ln e^{0.05t} = \ln 3$
$0.05t = \ln 3$
$t = \dfrac{\ln 3}{0.05} = 20 \ln 3 \approx 21.97$

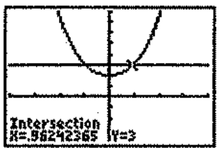
Graphical support:



$[-5, 35]$ by $[-1, 4]$

35. $e^x + e^{-x} = 3$
$e^x - 3 + e^{-x} = 0$
$e^x(e^x - 3 + e^{-x}) = e^x(0)$
$(e^x)^2 - 3e^x + 1 = 0$
$e^x = \dfrac{3 \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)}$
$e^x = \dfrac{3 \pm \sqrt{5}}{2}$
$x = \ln\left(\dfrac{3 \pm \sqrt{5}}{2}\right) \approx -0.96 \text{ or } 0.96$

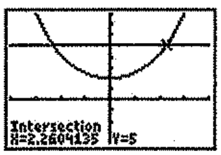
Graphical support:



$[-4, 4]$ by $[-4, 8]$

36. $2^x + 2^{-x} = 5$
$2^x - 5 + 2^{-x} = 0$
$2^x(2^x - 5 + 2^{-x}) = 2^x(0)$
$(2^x)^2 - 5(2^x) + 1 = 0$
$2^x = \dfrac{5 \pm \sqrt{(-5)^2 - 4(1)(1)}}{2(1)}$
$2^x = \dfrac{5 \pm \sqrt{21}}{2}$
$x = \log_2\left(\dfrac{5 \pm \sqrt{21}}{2}\right) \approx -2.26 \text{ or } 2.26$

Graphical support:

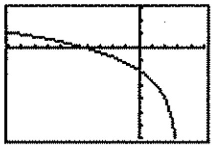


$[-4, 4]$ by $[-4, 8]$

37. $\ln y = 2t + 4$
$e^{\ln y} = e^{2t+4}$
$y = e^{2t+4}$

38. $\ln(y-1) - \ln 2 = x + \ln x$
$\ln(y-1) = x + \ln x + \ln 2$
$e^{\ln(y-1)} = e^{x + \ln x + \ln 2}$
$y - 1 = e^x(x)(2)$
$y = 2xe^x + 1$

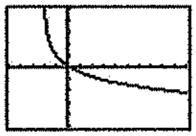
39.



$[-10, 5]$ by $[-7, 3]$

Domain: $(-\infty, 3)$
Range: $(-\infty, \infty)$

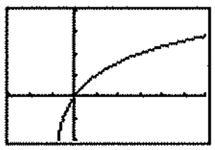
40.



$[-5, 10]$ by $[-5, 5]$

Domain: $(-2, \infty)$
Range: $(-\infty, \infty)$

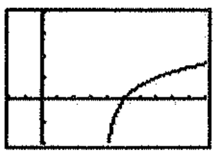
41.



$[-3, 6]$ by $[-2, 4]$

Domain: $(-1, \infty)$
Range: $(-\infty, \infty)$

42.



$[-2, 10]$ by $[-2, 4]$

Domain: $(4, \infty)$
Range: $(-\infty, \infty)$

43. $y=\dfrac{100}{1+2^{-x}}$

$$1+2^{-x}=\frac{100}{y}$$

$$2^{-x}=\frac{100}{y}-1$$

$$\log_2(2^{-x})=\log_2\left(\frac{100}{y}-1\right)$$

$$-x=\log_2\left(\frac{100}{y}-1\right)$$

$$x=-\log_2\left(\frac{100}{y}-1\right)$$

$$=-\log_2\left(\frac{100-y}{y}\right)$$

$$=\log_2\left(\frac{y}{100-y}\right)$$

Interchange x and y.

$$y=\log_2\left(\frac{x}{100-x}\right)$$

$$f^{-1}(x)=\log_2\left(\frac{x}{100-x}\right)$$

Verify.

$$(f\circ f^{-1})(x)=f\left(\log_2\frac{x}{100-x}\right)$$

$$=\frac{100}{1+2^{-\log_2\left(\frac{x}{100-x}\right)}}$$

$$=\frac{100}{1+2^{\log_2\left(\frac{100-x}{x}\right)}}$$

$$=\frac{100}{1+\frac{100-x}{x}}$$

$$=\frac{100x}{x+(100-x)}=\frac{100x}{100}=x$$

$$(f^{-1}\circ f)(x)=f^{-1}\left(\frac{100}{1+2^{-x}}\right)$$

$$=\log_2\left(\frac{\frac{100}{1+2^{-x}}}{100-\frac{100}{1+2^{-x}}}\right)$$

$$=\log_2\left(\frac{100}{100(1+2^{-x})-100}\right)$$

$$=\log_2\left(\frac{1}{2^{-x}}\right)=\log_2(2^x)=x$$

44. $y=\dfrac{50}{1+1.1^{-x}}$

$$1+1.1^{-x}=\frac{50}{y}$$

$$1.1^{-x}=\frac{50}{y}-1$$

$$\log_{1.1}(11^{-x})=\log_{1.1}\left(\frac{50}{y}-1\right)$$

$$-x=\log_{1.1}\left(\frac{50}{y}-1\right)$$

$$x=-\log_{1.1}\left(\frac{50}{y}-1\right)=-\log_{1.1}\left(\frac{50-y}{y}\right)=\log_{1.1}\left(\frac{y}{50-y}\right)$$

Interchange x and y:

$$y=\log_{1.1}\left(\frac{x}{50-x}\right)$$

$$f^{-1}(x)=\log_{1.1}\left(\frac{x}{50-x}\right)$$

Verify.

$$(f\circ f^{-1})(x)=f\left(\log_{1.1}\left(\frac{x}{50-x}\right)\right)$$

$$=\frac{50}{1+1.1^{-\log_{1.1}\left(\frac{x}{50-x}\right)}}$$

$$=\frac{50}{1+1.1^{\log_{1.1}\left(\frac{50-x}{x}\right)}}$$

$$=\frac{50}{1+\frac{50-x}{x}}$$

$$=\frac{50x}{x+(50-x)}=\frac{50x}{50}=x$$

$$(f^{-1}\circ f)(x)=f^{-1}\left(\frac{50}{1+1.1^{-x}}\right)$$

$$=\log_{1.1}\left(\frac{\frac{50}{1+1.1^{-x}}}{50-\frac{50}{1+1.1^{-x}}}\right)$$

$$=\log_{1.1}\left(\frac{50}{50(1+1.1^{-x})-50}\right)$$

$$=\log_{1.1}\left(\frac{1}{1.1^{-x}}\right)=\log_{1.1}(1.1^x)=x$$

45. **(a)** $f(f(x)) = \sqrt{1-(f(x))^2}$
$= \sqrt{1-(1-x^2)}$
$= \sqrt{x^2} = |x| = x$, since $x \geq 0$

(b) $f(f(x)) = f\left(\frac{1}{x}\right) = \frac{1}{1/x} = x$ for all $x \neq 0$

46. **(a)** $\text{Amount} = 8\left(\frac{1}{2}\right)^{t/12}$

(b) $8\left(\frac{1}{2}\right)^{t/12} = 1$
$\left(\frac{1}{2}\right)^{t/12} = \frac{1}{8}$
$\left(\frac{1}{2}\right)^{t/12} = \left(\frac{1}{2}\right)^3$
$\frac{t}{12} = 3$
$t = 36$

There will be 1 gram remaining after 36 hours.

47. $500(1.0475)^t = 1000$
$1.0475^t = 2$
$\ln(1.0475^t) = \ln 2$
$t \ln 1.0475 = \ln 2$
$t = \frac{\ln 2}{\ln 1.0475} \approx 14.936$

It will take about 14.936 years. (If the interest is paid at the end of each year, it will take 15 years.)

48. $375{,}000(1.0225)^t = 1{,}000{,}000$

$1.0225^t = \frac{8}{3}$

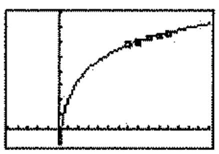
$\ln(1.0225^t) = \ln\left(\frac{8}{3}\right)$

$t \ln 1.0225 = \ln\left(\frac{8}{3}\right)$

$t = \frac{\ln(8/3)}{\ln 1.0225} \approx 44.081$

It will take about 44.081 years.

49. **(a)** $f(x) = 2.0010 + (1.9285)\ ln\ (x)$



[–5, 15] by [–1, 8]

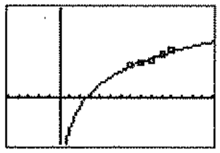
(b) $2.0010 + (1.9285)\ ln\ (12) = 6.79$ trillion
$6.79 - 6.63 = 0.16$ trillion

The estimate exceeds the actual by 0.16 trillion

(c) $2.0010 + (1.9285) ln(x) = 7$
$(1.9285) ln(x) = 7 - 2.0010$
$ln(x) = \frac{7-2.0010}{1.9285}$

$x = e^{(2.592)} = 13.35$, or sometime during 2003.

50. **(a)** $f(x) = -0.6782 + (0.7142)\ ln(x)$



[–5, 15] by [–1, 2]

(b) $-0.6782 + (0.7142)\ ln\ (x)$

$-0.6782 + (0.7142)\ ln\ (12) = 1.10$ trillion

$1.10 - 1.15 = -0.05$ trillion cubic feet.

The estimate is an under estimate of 0.05 trillion cubic feet.

(c) $-0.6782 + (0.7142) ln(x) = 1.5$
$(0.7142) ln(x) = 1.5 + 0.6782$
$ln(x) = \frac{2.1782}{0.7142}$
$ln(x) = 3.050$
$x = e^{3.050}$
$x = 21.1$
or sometime during 2011.

51. **(a)** Suppose that $f(x_1) = f(x_2)$. Then $mx_1 + b = mx_2 + b$ so $mx_1 = mx_2$. Since $m \neq 0$, this gives $x_1 = x_2$.

(b) $y = mx + b$
$y - b = mx$
$\frac{y-b}{m} = x$

Interchange x and y.

$\frac{x-b}{m} = y$
$f^{-1}(x) = \frac{x-b}{m}$

The slopes are reciprocals.

(c) If the original functions both have slope m, each of the inverse functions will have slope $\frac{1}{m}$. The graphs of the inverses will be parallel lines with nonzero slope.

(d) If the original functions have slopes m and $-\frac{1}{m}$, respectively, then the inverse functions will have slopes $\frac{1}{m}$ and $-m$, respectively. Since each of $\frac{1}{m}$ and $-m$ is the negative reciprocal of the other, the graphs of the inverses will be perpendicular lines with nonzero slopes.

52. False. For example, the horizontal line test finds three answers for $x = 0$.

53. False. Must satisfy $(f \circ g)(x) = (g \circ f)(x) = x$, not just $(f \circ g)(x) = x$.

54. C. $\ln(x) > 0$, requiring $x + 2 > 0$, or $x > -2$.

55. A. As $(x + 2) \to 0, f(x) \to -\infty$.

56. E.

$$\begin{aligned} f(x) &= 3x-2 \\ y &= 3x-2 \\ x &= 3y-2 \\ x+2 &= 3y \\ y &= \frac{x+2}{3} \end{aligned}$$

57. B.
$$\begin{aligned} 2-3^{-x} &= -1 \\ -3^{-x} &= -3 \\ (-3)^{-x} &= (-3)^1 \\ -x &= 1 \\ x &= -1 \end{aligned}$$

58. **(a)** y_2 is a vertical shift (upward) of y_1, although it's difficult to see that near the vertical asymptote at $x = 0$. One might use "trace" or "table" to verify this.

(b) Each graph of y_3 is a horizontal line.

(c) The graphs of y_4 and $y = a$ are the same.

(d) $e^{y_2-y_1} = a$, $\ln(e^{y_2-y_1}) = \ln a$,

$y_2 - y_1 = \ln a$, $y_1 = y_2 - \ln a = \ln x - \ln a$

59. If the graph of $f(x)$ passes the horizontal line test, so will the graph of $g(x) = -f(x)$ since it's the same graph reflected about the x-axis.

Alternate answer: If $g(x_1) = g(x_2)$ then $-f(x_1) = -f(x_2)$, $f(x_1) = f(x_2)$, and $x_1 = x_2$ since f is one-to-one.

60. Suppose that $g(x_1) = g(x_2)$. Then $\frac{1}{f(x_1)} = \frac{1}{f(x_2)}$, $f(x_1) = f(x_2)$, and x_1 and x_2 since f is one-to-one.

61. **(a)** The expression $a(b^{c-x}) + d$ is defined for all values of x, so the domain is $(-\infty, \infty)$. Since b^{c-x} attains all positive values, the range is (d, ∞) if $a > 0$ and the range is $(-\infty, d)$ if $a < 0$.

(b) The expression $a \log_b(x - c) + d$ is defined when $x - c > 0$, so the domain is (c, ∞).

Since $a \log_b(x - c) + d$ attains every real value for some value of x, the range is $(-\infty, \infty)$.

62. **(a)** Suppose $f(x_1) = f(x_2)$. Then:

$$\begin{aligned} &\frac{ax_1+b}{cx_1+d} = \frac{ax_2+b}{cx_2+d} \\ &(ax_1+b)(cx_2+d) = (ax_2+b)(cx_1+d) \\ &acx_1x_2 + adx_1 + bcx_2 + bd \\ &\quad = acx_1x_2 + adx_2 + bcx_1 + bd \\ &adx_1 + bcx_2 = adx_2 + bcx_1 \\ &(ad-bc)x_1 = (ad-bc)x_2 \end{aligned}$$

Since $ad - bc \neq 0$, this means that $x_1 = x_2$.

(b)
$$\begin{aligned} y &= \frac{ax+b}{cx+d} \\ cxy + dy &= ax+b \\ (cy-a)x &= -dy+b \\ x &= \frac{-dy+b}{cy-a} \end{aligned}$$

Interchange x and y:

$$\begin{aligned} y &= \frac{-dx+b}{cx-a} \\ f^{-1}(x) &= \frac{-dx+b}{cx-a} \end{aligned}$$

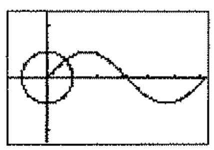
(c) As $x \to \pm\infty$, $f(x) = \frac{ax+b}{cx+d} \to \frac{a}{c}$, so the horizontal asymptote is $y = \frac{a}{c} (c \neq 0)$. Since $f(x)$ is undefined at $x = -\frac{d}{c}$, the vertical asymptote is $x = -\frac{d}{c}$.

(d) As $x \to \pm\infty$, $f^{-1}(x) = \frac{-dx+b}{cx-a} \to -\frac{d}{c}$, so the horizontal asymptote is $y = -\frac{d}{c} (c \neq 0)$. Since $f^{-1}(x)$ is undefined at $x = \frac{a}{c}$, the vertical asymptote is $x = \frac{a}{c}$. The horizontal asymptote of f becomes the vertical asymptote of f^{-1} and vice versa due to the reflection of the graph about the line $y = x$.

Section 1.6 Trigonometric Functions (pp. 46–55)

Exploration 1 Unwrapping Trigonometric Functions

1. (x_1, y_1) is the circle of radius 1 centered at the origin (unit circle). (x_2, y_2) is one period of the graph of the sine function.

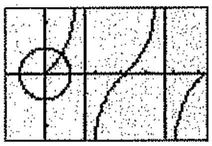


2. The y-values are the same in the interval $0 \le t \le 2\pi$.

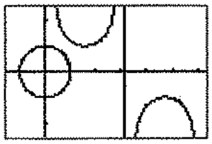
3. The y-values are the same in the interval $0 \le t \le 4\pi$.

4. The x_1-values and the y_2-values are the same in each interval.

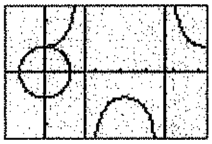
5. $y_2 = \tan t$:



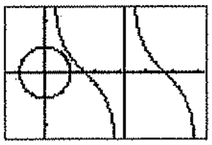
$y_2 = \csc t$:



$y_2 = \sec t$:



$y_2 = \cot t$:



For each value of t, the value of $y_2 = \tan t$ is equal to the ratio $\frac{y_1}{x_1}$.

For each value of t, the value of $y_2 = \csc t$ is equal to the ratio $\frac{1}{y_1}$.

For each value of t, the value of $y_2 = \sec t$ is equal to the ratio $\frac{1}{x_1}$.

For each value of t, the value of $y_2 = \cot t$ is equal to the ratio $\frac{x_1}{y_1}$.

Quick Review 1.6

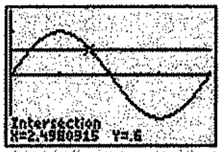
1. $\frac{\pi}{3} \cdot \frac{180^\circ}{\pi} = 60^\circ$

2. $-2.5 \cdot \frac{180^\circ}{\pi} = \left(-\frac{450}{\pi}\right)^\circ \approx -143.24^\circ$

3. $-40^\circ \cdot \frac{\pi}{180^\circ} = -\frac{2\pi}{9}$

4. $45^\circ \cdot \frac{\pi}{180^\circ} = \frac{\pi}{4}$

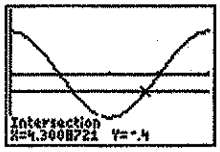
5.



$[0, 2\pi]$ by $[-1.5, 1.5]$

$x \approx 0.6435, x \approx 2.4981$

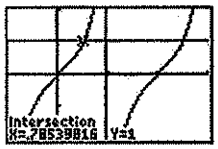
6.



$[0, 2\pi]$ by $[-1.5, 1.5]$

$x \approx 1.9823, x \approx 4.3009$

7.



$\left[-\frac{\pi}{2}, \frac{3\pi}{2}\right]$ by $[-2, 2]$

$x \approx 0.7854 \left(\text{or } \frac{\pi}{4}\right), x \approx 3.9270 \left(\text{or } \frac{5\pi}{4}\right)$

8. $f(-x) = 2(-x)^2 - 3 = 2x^2 - 3 = f(x)$

The graph is symmetric about the y-axis because if a point (a, b) is on the graph, then so is the point $(-a, b)$.

9. $f(-x) = (-x)^3 - 3(-x)$
$= -x^3 + 3x$
$= -(x^3 - 3x) = -f(x)$

The graph is symmetric about the origin because if a point (a, b) is on the graph, then so is the point $(-a, -b)$.

10. $x \ge 0$

Section 1.6 Exercises

1. Arc length $= \left(\frac{5\pi}{8}\right)(2) = \frac{5\pi}{4}$

2. Radius $= \frac{10}{175^\circ\left(\frac{\pi}{180^\circ}\right)} = \frac{72}{7\pi} \approx 3.274$

3. Angle $= \frac{7}{14} = \frac{1}{2}$ radian or about 28.65°

4. Angle $= \frac{3\pi/2}{6} = \frac{\pi}{4}$ radian or 45°

5. Even. $\sec(-\theta) = \frac{1}{\cos(-\theta)} = \frac{1}{\cos(\theta)} = \sec(\theta)$

6. Odd. $\tan(-\theta) = \frac{\sin(-\theta)}{\cos(-\theta)} = \frac{-\sin(\theta)}{\cos(\theta)} = -\tan\theta$

7. Odd. $\csc(\theta) = \frac{1}{\sin(-\theta)} = \frac{1}{-\sin(\theta)} = -\csc\theta$

8. Odd. $\cot(-\theta) = \frac{\cos(-\theta)}{\sin(-\theta)} = \frac{\cos(\theta)}{-\sin(-\theta)} = -\cot(\theta)$

9. Using a triangle with sides: −15, 17 and 8; $\sin\theta = 8/17$, $\tan\theta = -8/15$, $\csc\theta = 17/8$, $\sec\theta = -17/15$, $\cot\theta = -15/8$.

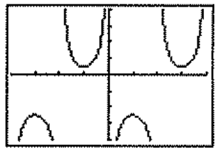
10. Using a triangle with sides: −1, 1 and $\sqrt{2}$;
$\sin\theta = -\sqrt{2}/2,\ \cos\theta = \sqrt{2}/2,\ \csc\theta = -\sqrt{2},$
$\sec\theta = \sqrt{2},\ \cot\theta = -1$

11. (a) Period $= \frac{2\pi}{3}$

(b) Domain: Since $\csc(3x+\pi) = \frac{1}{\sin(3x+\pi)}$, we require $3x + \pi \neq k\pi$, or $x \neq \frac{(k-1)\pi}{3}$. This requirement is equivalent to $x \neq \frac{k\pi}{3}$ for integers k.

(c) Since $|\csc(3x+\pi)| \geq 1$, the range excludes numbers between $-3-2=-5$ and $3-2=1$. The range is $(-\infty, -5] \cup [1, \infty)$.

(d)



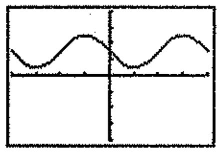
$\left[-\frac{2\pi}{3}, \frac{2\pi}{3}\right]$ by [−8, 8]

12. (a) Period $= \frac{2\pi}{4} = \frac{\pi}{2}$

(b) Domain: $(-\infty, \infty)$

(c) Since $|\sin(4x+\pi)| \leq 1$, the range extends from $-2+3=1$ to $2+3=5$. The range is [1, 5].

(d)



$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ by [−8, 8]

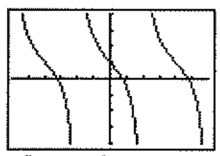
13. (a) Period $= \frac{\pi}{3}$

(b) Domain: We require $3x + \pi \neq \frac{k\pi}{2}$ for odd integers k.

Therefore, $x \neq \frac{(k-2)\pi}{6}$ for odd integers k. This requirement is equivalent to $x \neq \frac{k\pi}{6}$ for odd integers k.

(c) Since the tangent function attains all real values, the range is $(-\infty, \infty)$.

(d)



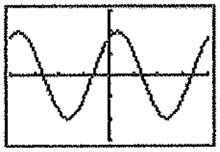
$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ by [−8, 8]

14. (a) Period $= \frac{2\pi}{2} = \pi$

(b) Domain: $(-\infty, \infty)$

(c) Range: Since $\left|\sin\left(2x+\frac{\pi}{3}\right)\right| \le 1$, the range is $[-2, 2]$.

(d)



$[-\pi, \pi]$ by $[-3, 3]$

15. (a) The period of $y = \sec x$ is 2π, so the window should have length 4π.

One possible answer: $[0, 4\pi]$ by $[-3, 3]$

(b) The period of $y = \csc x$ is 2π, so the window should have length 4π.

One possible answer: $[0, 4\pi]$ by $[-3, 3]$

(c) The period of $y = \cot x$ is π, so the window should have length 2π.

One possible answer: $[0, 2\pi]$ by $[-3, 3]$

16. (a) The period of $y = \sin x$ is 2π, so the window should have length 4π.

One possible answer: $[0, 4\pi]$ by $[-2, 2]$

(b) The period of $y = \cos x$ is 2π, so the window should have length 4π.

One possible answer: $[0, 4\pi]$ by $[-2, 2]$

(c) The period of $y = \tan x$ is π, so the window should have length 2π.

One possible answer: $[0, 2\pi]$ by $[-3, 3]$

17. (a) Period $= \frac{2\pi}{2} = \pi$

(b) Amplitude $= 1.5$

(c) $[-2\pi, 2\pi]$ by $[-2, 2]$

18. (a) Period $= \frac{2\pi}{3}$

(b) Amplitude $= 2$

(c) $\left[-\frac{2\pi}{3}, \frac{2\pi}{3}\right]$ by $[-4, 4]$

19. (a) Period $= \frac{2\pi}{2} = \pi$

(b) Amplitude $= 3$

(c) $[-2\pi, 2\pi]$ by $[-4, 4]$

20. (a) Period $= \frac{2\pi}{1/2} = 4\pi$

(b) Amplitude $= 5$

(c) $[-4\pi, 4\pi]$ by $[-10, 10]$

21. (a) Period $= \frac{2\pi}{\pi/3} = 6$

(b) Amplitude $= 4$

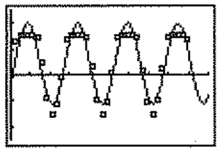
(c) $[-3, 3]$ by $[-5, 5]$

22. (a) Period $= \frac{2\pi}{\pi} = 2$

(b) Amplitude $= 1$

(c) $[-4, 4]$ by $[-2, 2]$

23. (a) Using a graphing calculator with the sinusoidal regression feature, the equation is $y = 1.543 \sin(2468.635x - 0.494) + 0.438$.



$[0, 0.01]$by $[-2.5, 2.5]$

(b) The frequency is 2468.635 radians per second, which is equivalent to $\frac{2468.635}{2\pi} \approx 392.9$ cycles per second (Hz).

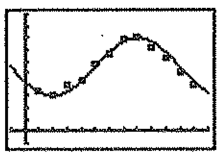
The note is a " G ".

24. (a) $b = \frac{2\pi}{12} = \frac{\pi}{6}$

(b) It's half of the difference, so $a = \frac{80-30}{2} = 25$.

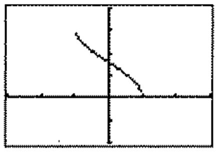
(c) $k = \frac{80+30}{2} = 55$

(d) The function should have its minimum at $t = 2$ (when the temperature is 30°F) and its maximum at $t = 8$ (when the temperature is 80°F). The value of h is $\frac{2+8}{2} = 5$ .Equation: $y = 25\sin\left[\frac{\pi}{6}(x-5)\right] + 55$



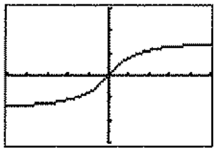
$[-1, 13]$ by $[-10, 100]$

25. The portion of the curve $y = \cos x$ between $0 \le x \le \pi$ passes the horizontal line test, so it is one-to-one.



[–3, 3] by [–2, 4]

26. The portion of the curve $y = \tan x$ between $-\pi/2 < x < \pi/2$ passes the horizontal line test, so it is one-to-one. [In parametric mode, use $T_{min} = -\pi/2 + \varepsilon$ and $T_{max} = \pi/2 - \varepsilon$, where ε is a very small positive number, say 0.00001.]



[–5, 5] by [–3, 3]

27. Since $\frac{\pi}{6}$ is in the range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ of $y = \sin^{-1} x$ and

$\sin\frac{\pi}{6} = 0.5$, $\sin^{-1}(0.5) = \frac{\pi}{6}$ radian or $\frac{\pi}{6} \cdot \frac{180°}{\pi} = 30°$.

28. Since $-\frac{\pi}{4}$ is the range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ of $y = \sin^{-1} x$ and

$\sin\left(-\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$, $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$ radian

or $-\frac{\pi}{4} \cdot \frac{180°}{\pi} = -45°$.

29. Using a calculator, $\tan^{-1}(-5) \approx -1.3734$ radians or $-78.6901°$.

30. Using a calculator, $\cos^{-1}(0.7) \approx 0.7954$ radian or $45.5730°$.

31. The angle $\tan^{-1}(2.5) \approx 1.190$ is the solution to this equation in the interval $-\frac{\pi}{2} < x < \frac{\pi}{2}$. Another solution in $0 \le x \le 2\pi$ is $\tan^{-1}(2.5) + \pi \approx 4.332$. The solutions are $x \approx 1.190$ and $x \approx 4.332$.

32. The angle $\cos^{-1}(-0.7) \approx 2.346$ is the solution to this equation in the interval $0 \le x \le \pi$. Since the cosine function is even, the value $-\cos^{-1}(-0.7) \approx -2.346$ is also a solution, so any value of the form $\pm\cos^{-1}(-0.7) + 2k\pi$ is a solution, where k is an integer. In $2\pi \le x < 4\pi$ the solutions are $x = \cos^{-1}(-0.7) + 2\pi \approx 8.629$ and $x = -\cos^{-1}(-0.7) + 4\pi \approx 10.220$.

33. This equation is equivalent to $\sin x = \frac{1}{2}$, so the solutions in the interval $0 \le x < 2\pi$ are $x = \frac{\pi}{6}$ and $x = \frac{5\pi}{6}$.

34. This equation is equivalent to $\cos x = -\frac{1}{3}$, so the solution in the interval $0 \le x \le \pi$ is

$y = \cos^{-1}\left(-\frac{1}{3}\right) \approx 1.911$. Since the cosine function is even, the solutions in the interval $-\pi \le x < \pi$ are $x \approx -1.911$ and $x \approx 1.911$.

35. The solutions in the interval $0 \le x < 2\pi$ are $x = \frac{7\pi}{6}$ and $x = \frac{11\pi}{6}$. Since $y = \sin x$ has period 2π, the solutions are all of the form $x = \frac{7\pi}{6} + 2k\pi$ or $x = \frac{11\pi}{6} + 2k\pi$, where k is any integer.

36. The equation is equivalent to $\tan x = \frac{1}{-1} = -1$, so the solution in the interval $-\frac{\pi}{2} < x < \frac{\pi}{2}$ is

$x = \tan^{-1}(-1) = -\frac{\pi}{4}$. Since the period of $y = \tan x$ is π, all solutions are of the form $x = -\frac{\pi}{4} + k\pi$, where k is any integer. This is equivalent to $x = \frac{3\pi}{4} + k\pi$, where k is any integer.

37. Note that $\sqrt{8^2 + 15^2} = 17$.

Since $\sin\theta = \frac{8}{17}$ and $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$,

$\cos\theta = \sqrt{1 - \sin^2\theta} = \sqrt{1 - \left(\frac{8}{17}\right)^2} = \frac{15}{17}$.

Therefore: $\sin\theta = \frac{8}{17}$, $\cos\theta = \frac{15}{17}$, $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{8}{15}$,

$\cot\theta = \frac{1}{\tan\theta} = \frac{15}{8}$, $\sec\theta = \frac{1}{\cos\theta} = \frac{17}{15}$,

$\csc\theta = \frac{1}{\sin\theta} = \frac{17}{8}$

38. Note that $\sqrt{5^2+12^2}=13$.

Since $\tan\theta=-\frac{5}{12}=\frac{-5/13}{12/13}=\frac{\sin\theta}{\cos\theta}$ and $-\frac{\pi}{2}\le\theta\le\frac{\pi}{2}$, we have $\sin\theta=-\frac{5}{13}$ and $\cos\theta=\frac{12}{13}$.

In summary:

$$\sin\theta=-\frac{5}{13},\ \cos\theta=\frac{12}{13},\ \tan\theta=-\frac{5}{12}$$

$$\cot\theta=\frac{1}{\tan\theta}=-\frac{12}{5},\ \sec\theta=\frac{1}{\cos\theta}=\frac{13}{12},\ \csc\theta=\frac{1}{\sin\theta}=-\frac{13}{5}$$

39. Note that $r=\sqrt{(-3)^2+4^2}=5$. Then:

$$\sin\theta=\frac{y}{r}=\frac{4}{5},\ \cos\theta=\frac{x}{r}=-\frac{3}{5},\ \tan\theta=\frac{y}{x}=-\frac{4}{3},$$

$$\cot\theta=\frac{x}{y}=-\frac{3}{4},\ \sec\theta=\frac{r}{x}=-\frac{5}{3},\ \csc\theta=\frac{r}{y}=\frac{5}{4}$$

40. Note that $r=\sqrt{(-2)^2+2^2}=2\sqrt{2}$. Then:

$$\sin\theta=\frac{y}{r}=\frac{2}{2\sqrt{2}}=\frac{1}{\sqrt{2}}\cos\theta=\frac{x}{r}=\frac{-2}{2\sqrt{2}}=-\frac{1}{\sqrt{2}}$$

$$\tan\theta=\frac{y}{x}=\frac{2}{-2}=-1,\ \cot\theta=\frac{x}{y}=\frac{-2}{2}=-1,$$

$$\sec\theta=\frac{r}{x}=\frac{2\sqrt{2}}{-2}=-\sqrt{2},\csc\theta=\frac{r}{y}=\frac{2\sqrt{2}}{2}=\sqrt{2}$$

41. Let $\theta=\cos^{-1}\left(\frac{7}{11}\right)$. Then $0\le\theta\le\pi$ and $\cos\theta=\frac{7}{11}$, so

$$\sin\left(\cos^{-1}\left(\frac{7}{11}\right)\right)=\sin\theta=\sqrt{1-\cos^2\theta}$$

$$=\sqrt{1-\left(\frac{7}{11}\right)^2}=\frac{\sqrt{72}}{11}=\frac{6\sqrt{2}}{11}\approx 0.771$$

42. Let $\theta=\sin^{-1}\left(\frac{9}{13}\right)$. Then $-\frac{\pi}{2}\le\theta\le\frac{\pi}{2}$ and $\sin\theta=\frac{9}{13}$, so

$\cos\theta=\sqrt{1-\sin^2\theta}=\sqrt{1-\left(\frac{9}{13}\right)^2}=\frac{\sqrt{88}}{13}$. Therefore,

$$\tan\left(\sin^{-1}\left(\frac{9}{13}\right)\right)=\tan\theta=\frac{\sin\theta}{\cos\theta}=$$

$$\frac{9/13}{\sqrt{88}/13}=\frac{9}{\sqrt{88}}\approx 0.959.$$

43. **(a)** Amplitude = 37

(b) Period $=\frac{2\pi}{(2\pi/365)}=365$

(c) Horizontal shift = 101

(d) Vertical shift = 25

(e) $f(x)=37\sin\left[\frac{2\pi}{365}(x-101)\right]+25$

44. **(a)** Highest: $25+37=62^\circ$F
Lowest: $25-37=-12^\circ$F

(b) Average $=\frac{62+(-12)}{2}=25^\circ$F

This average is the same as the vertical shift because the average of the highest and lowest values of $y=\sin x$ is 0.

45. **(a)** $\cot(-x)=\frac{\cos(-x)}{\sin(-x)}=\frac{\cos(x)}{-\sin(x)}=-\cot(x)$

(b) Assume that f is even and g is odd.

Then $\frac{f(-x)}{g(-x)}=\frac{f(x)}{-g(x)}=-\frac{f(x)}{g(x)}$ so $\frac{f}{g}$ is odd. The situation is similar for $\frac{g}{f}$.

46. **(a)** $\csc(-x)=\frac{1}{\sin(-x)}=\frac{1}{-\sin(x)}=-\csc(x)$

(b) Assume that f is odd.

Then $\frac{1}{f(-x)}=\frac{1}{-f(x)}=-\frac{1}{f(x)}$ so $\frac{1}{f}$ is odd.

47. Assume that f is even and g is odd.
Then $f(-x)g(-x)=(f(x))(-g(x))=-f(x)g(x)$ so $f(g)$ is odd.

48. If (a, b) is the point on the unit circle corresponding to the angle θ, then $(-a, -b)$ is the point on the unit circle corresponding to the angle $(\theta+\pi)$ since it is exactly half way around the circle. This means that both $\tan(\theta)$ and $\tan(\theta+\pi)$ have the same value, $\frac{b}{a}$.

49. **(a)** Using a graphing calculator with the sinusoidal regression feature, the equation is $y = 3.0014 \sin(0.9996x + 2.0012) + 2.9999$.

(b) $y = 3\sin(x + 2) + 3$

50. False. It is 4π because $2\pi/B = 1/2$ implies the period B is 4π.

51. False. The amplitude is 1/2.

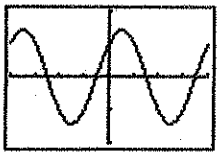
52. D.

53. B. The curve oscillates between −3 and 1.

54. E.

55. A.

56. (a)



$[-2\pi, 2\pi]$ by $[-2, 2]$

The graph is a sine/cosine type graph, but it is shifted and has an amplitude greater than 1.

(b) Amplitude ≈ 1.414 (that is, $\sqrt{2}$)
Period $= 2\pi$
Horizontal shift ≈ -0.785 $\left(\text{that is}, -\frac{\pi}{4}\right)$ or 5.498 $\left(\text{that is}, -\frac{7\pi}{4}\right)$
Vertical shift: 0

(c) $\sin\left(x+\frac{\pi}{4}\right) = (\sin x)\left(\cos\frac{\pi}{4}\right) + (\cos x)\left(\sin\frac{\pi}{4}\right)$

$= (\sin x)\left(\frac{1}{\sqrt{2}}\right) + (\cos x)\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{1}{\sqrt{2}}(\sin x + \cos x)$

Therefore, $\sin x + \cos x = \sqrt{2}\sin\left(x+\frac{\pi}{4}\right)$.

57. (a) $\sqrt{2}\sin\left(ax+\frac{\pi}{4}\right)$

(b) See part (a).

(c) It works.

(d) $\sin\left(ax+\frac{\pi}{4}\right) = (\sin ax)\left(\cos\frac{\pi}{4}\right) + (\cos ax)\left(\sin\frac{\pi}{4}\right)$

$= (\sin ax)\left(\frac{1}{\sqrt{2}}\right) + (\cos ax)\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{1}{\sqrt{2}}(\sin ax + \cos ax)$

So, $\sin(ax) + \cos(ax) = \sqrt{2}\sin\left(ax+\frac{\pi}{4}\right)$.

58. (a) One possible answer:

$y = \sqrt{a^2+b^2}\sin\left(x+\tan^{-1}\left(\frac{b}{a}\right)\right)$

(b) See part (a).

(c) It works.

(d) $\sin\left(x+\tan^{-1}\left(\frac{b}{a}\right)\right)$

$= \sin(x)\cos\left(\tan^{-1}\left(\frac{b}{a}\right)\right) + \cos(x)\sin\left(\tan^{-1}\left(\frac{b}{a}\right)\right)$

$= \sin(x)\left(\frac{a}{\sqrt{a^2+b^2}}\right) + \cos(x)\left(\frac{b}{\sqrt{a^2+b^2}}\right)$

$= \frac{1}{\sqrt{a^2+b^2}}\cdot(a\sin x + b\cos x)$

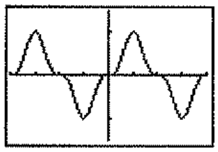
and multiplying through by the square root gives the desired result. Note that the substitutions

$\cos\left(\tan^{-1}\frac{b}{a}\right) = \frac{a}{\sqrt{a^2+b^2}}$ and

$\sin\left(\tan^{-1}\frac{b}{a}\right) = \frac{b}{\sqrt{a^2+b^2}}$ depend on the requirement

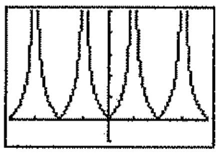
that a is positive. If a is negative, the formula does not work.

59. Since $\sin x$ has period 2π, $\sin^3(x+2\pi) = \sin^3(x)$. This function has period 2π. A graph shows that no smaller number works for the period.



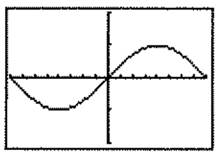
$[-2\pi, 2\pi]$ by $[-1.5, 1.5]$

60. Since $\tan x$ has period π, $|\tan(x+\pi)| = |\tan x|$. This function has period π. A graph shows that no smaller number works for the period.



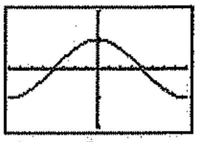
$[-2\pi, 2\pi]$ by $[-1, 5]$

61. The period is $\frac{2\pi}{60} = \frac{\pi}{30}$. One possible graph:



$\left[-\frac{\pi}{60}, \frac{\pi}{60}\right]$ by $[-2, 2]$

62. The period is $\frac{2\pi}{60\pi} = \frac{1}{30}$. One possible graph:



$\left[-\frac{1}{60}, \frac{1}{60}\right]$ by $[-2, 2]$

Quick Quiz (Sections 1.4–1.6)

1. C.

2. D.

3. E.

4. (a) $f(x) = 5x - 3$

$$\begin{aligned} y &= 5x - 3 \\ x &= 5y - 3 \\ x + 3 &= 5y \\ \frac{x+3}{5} &= y \\ g(x) &= \frac{x+3}{5} \end{aligned}$$

(b) $$\begin{aligned} f \circ g(x) &= f\left(\frac{x+3}{5}\right) \\ &= 5\left(\frac{x+3}{5}\right) - 3 \\ &= x + 3 - 3 = x \end{aligned}$$

(c) $$\begin{aligned} g \circ f(x) &= g(5x - 3) \\ &= \frac{(5x-3)+3}{5} = \frac{5x}{5} = x \end{aligned}$$

Chapter 1 Review Exercises (pp. 56–57)

1. $y = 3(x-1) + (-6)$
$y = 3x - 9$

2. $y = -\frac{1}{2}(x+1) + 2$
$y = -\frac{1}{2}x + \frac{3}{2}$

3. $x = 0$

4. $m = \frac{-2-6}{1-(-3)} = \frac{-8}{4} = -2$
$y = -2(x+3) + 6$
$y = -2x$

5. $y = 2$

6. $m = \frac{5-3}{-2-3} = \frac{2}{-5} = -\frac{2}{5}$
$y = -\frac{2}{5}(x-3) + 3$
$y = -\frac{2}{5} + \frac{21}{5}$

7. $y = -3x + 3$

8. Since $2x - y = -2$ is equivalent to $y = 2x + 2$, the slope of the given line (and hence the slope of the desired line) is 2.
$y = 2(x-3) + 1$
$y = 2x - 5$

9. Since $4x + 3y = 12$ is equivalent to $y = -\frac{4}{3}x + 4$, the slope of the given line (and hence the slope of the desired line) is $-\frac{4}{3}$.
$y = -\frac{4}{3}(x-4) - 12$
$y = -\frac{4}{3}x - \frac{20}{3}$

10. Since $3x - 5y = 1$ is equivalent to $y = \frac{3}{5}x - \frac{1}{5}$, the slope of the given line is $\frac{3}{5}$ and the slope of the perpendicular line is $-\frac{5}{3}$.
$y = -\frac{5}{3}(x+2) - 3$
$y = -\frac{5}{3}x - \frac{19}{3}$

11. Since $\frac{1}{2}x + \frac{1}{3}y = 1$ is equivalent to $y = -\frac{3}{2}x + 3$, the slope of the given line is $-\frac{3}{2}$ and the slope of the perpendicular line is $\frac{2}{3}$.
$y = \frac{2}{3}(x+1) + 2$
$y = \frac{2}{3}x + \frac{8}{3}$

12. The line passes through $(0, -5)$ and $(3, 0)$.
$m = \frac{0-(-5)}{3-0} = \frac{5}{3}$
$y = \frac{5}{3}x - 5$

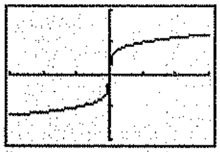
13. $m = \frac{2-4}{2-(-2)} = \frac{-2}{4} = -\frac{1}{2}$
$f(x) = -\frac{1}{2}(x+2) + 4$
$f(x) = -\frac{1}{2}x + 3$

Check: $f(4) = -\frac{1}{2}(4) + 3 = 1$, as expected.

14. The line passes through (4, –2) and (–3, 0).

$$m=\frac{0-(-2)}{-3-4}=\frac{2}{-7}=-\frac{2}{7}$$
$$y=-\frac{2}{7}(x-4)-2$$
$$y=-\frac{2}{7}x-\frac{6}{7}$$

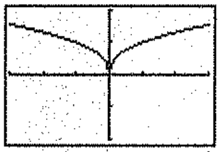
15.



[−3, 3] by [−2, 2]

Symmetric about the origin.

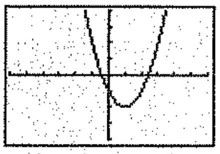
16.



[−3, 3] by [−2, 2]

Symmetric about the *y*-axis.

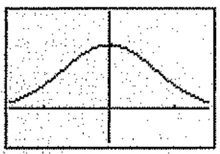
17.



[−6, 6] by [−4, 4]

Neither

18.



[−1.5, 1.5] by [−0.5, 1.5]

Symmetric about the *y*-axis.

19. $y(-x) = (-x)^2 + 1 = x^2 + 1 = y(x)$

Even

20. $y(-x) = (-x)^5 - (-x)^3 - (-x) = -x^5 + x^3 + x = -y(x)$

Odd

21. $y(-x) = 1 - \cos(-x) = 1 - \cos x = y(x)$

Even

22. $y(-x) = \sec(-x)\tan(-x)$
$$=\frac{\sin(-x)}{\cos^2(-x)}=\frac{-\sin x}{\cos^2 x}$$
$$=-\sec x\tan x=-y(x)$$

Odd

23. $y(-x)=\dfrac{(-x)^4+1}{(-x)^3-2(-x)}=\dfrac{x^4+1}{-x^3+2x}=-\dfrac{x^4+1}{x^3-2x}=-y(x)$

Odd

24. $y(-x) = 1 - \sin(-x) = 1 + \sin x$

Neither even nor odd

25. $y(-x) = -x + \cos(-x) = -x + \cos x$

Neither even nor odd

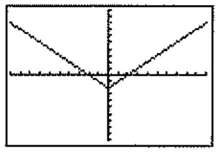
26. $y(-x)=\sqrt{(-x)^4-1}=\sqrt{x^4-1}$

Even

27. **(a)** The function is defined for all values of *x*, so the domain is $(-\infty, \infty)$.

(b) Since $|x|$ attains all nonnegative values, the range is $[-2, \infty)$.

(c)

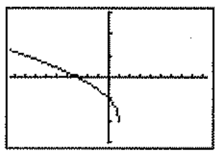


[–10, 10] by [–10, 10]

28. **(a)** Since the square root requires $1 - x \geq 0$, the domain is $(-\infty, 1]$.

(b) Since $\sqrt{1-x}$ attains all nonnegative values, the range is $[-2, \infty)$.

(c)

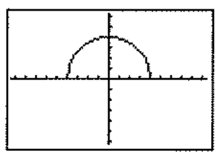


[–9.4, 9.4] by [–3, 3]

29. **(a)** Since the square root requires $16 - x^2 \geq 0$, the domain is $[-4, 4]$.

(b) For values of *x* in the domain, $0 \leq 16 - x^2 \leq 16$, so $0\leq\sqrt{16-x^2}\leq 4$. The range is [0, 4].

(c)

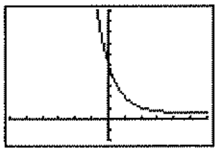


[–9.4, 9.4] by [–6.2, 6.2]

30. (a) The function is defined for all values of x, so the domain is $(-\infty, \infty)$.

(b) Since 3^{2-x} attains all positive values, the range is $(1, \infty)$.

(c)

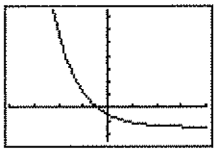


[–6, 6] by [–4, 20]

31. (a) The function is defined for all values of x, so the domain is $(-\infty, \infty)$.

(b) Since $2e^{-x}$ attains all positive values, the range is $(-3, \infty)$.

(c)

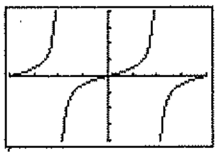


[–4, 4] by [–5, 15]

32. (a) The function is equivalent to $y = \tan 2x$, so we require $2x \neq \frac{k\pi}{2}$ for odd integers k. The domain is given by $x \neq \frac{k\pi}{4}$ for odd integers k.

(b) Since the tangent function attains all values, the range is $(-\infty, \infty)$.

(c)

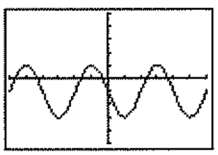


$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ by [–8, 8]

33. (a) The function is defined for all values of x, so the domain is $(-\infty, \infty)$.

(b) The sine function attains values from –1 to 1, so $-2 \leq 2\sin(3x + \pi) \leq 2$, and hence $-3 \leq 2\sin(3x + \pi) - 1 \leq 1$. The range is [–3, 1].

(c)

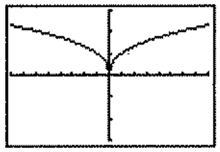


$[-\pi, \pi]$ by [–5, 5]

34. (a) The function is defined for all values of x, so the domain is $(-\infty, \infty)$.

(b) The function is equivalent to $y = \sqrt[5]{x^2}$, which attains all nonnegative values. The range is $[0, \infty)$.

(c)

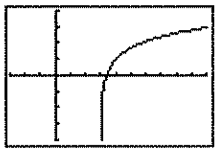


[–8, 8] by [–3, 3]

35. (a) The logarithm requires $x - 3 > 0$, so the domain is $(3, \infty)$.

(b) The logarithm attains all real values, so the range is $(-\infty, \infty)$.

(c)

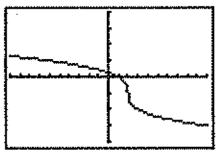


[–3, 10] by [–4, 4]

36. (a) The function is defined for all values of x, so the domain is $(-\infty, \infty)$.

(b) The cube root attains all real values, so the range is $(-\infty,\infty)$.

(c)

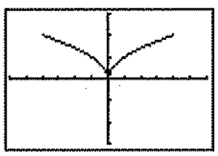


[–10, 10] by [–4, 4]

37. (a) The function is defined for $-4 \leq x \leq 4$, so the domain is [–4, 4].

(b) The function is equivalent to $y = \sqrt{|x|}, -4 \leq x \leq 4$, which attains values from 0 to 2 for x in the domain. The range is [0, 2].

(c)

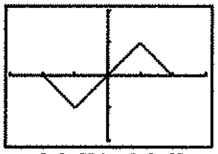


[–6, 6] by [–3, 3]

38. (a) The function is defined for $-2 \le x \le 2$, so the domain is $[-2, 2]$.

(b) See the graph in part (c). The range is $[-1, 1]$.

(c)



$[-3, 3]$ by $[-2, 2]$

39. First piece: Line through (0, 1) and (1, 0)

$$m = \frac{0-1}{1-0} = \frac{-1}{1} = -1$$
$$y = -x + 1 \text{ or } 1 - x$$

Second piece:

Line through (1, 1) and (2, 0)

$$m = \frac{0-1}{2-1} = \frac{-1}{1} = -1$$
$$y = -(x-1) + 1$$
$$y = -x + 2 \text{ or } 2 - x$$
$$f(x) = \begin{cases} 1-x, & 0 \le x < 1 \\ 2-x, & 1 \le x \le 2 \end{cases}$$

40. First piece: Line through (0, 0) and (2, 5)

$$m = \frac{5-0}{2-0} = \frac{5}{2}$$
$$y = \frac{5}{2}x$$

Second piece: Line through (2, 5) and (4, 0)

$$m = \frac{0-5}{4-2} = \frac{-5}{2} = -\frac{5}{2}$$
$$y = -\frac{5}{2}(x-2) + 5$$
$$y = -\frac{5}{2}x + 10 \text{ or } 10 - \frac{5x}{2}$$
$$f(x) = \begin{cases} \frac{5x}{2}, & 0 \le x < 2 \\ 10 - \frac{5x}{2}, & 2 \le x \le 4 \end{cases}$$

(Note: $x = 2$ can be included on either piece.)

41. (a) $(f \circ g)(-1) = f(g(-1)) = f\left(\frac{1}{\sqrt{-1+2}}\right) = f(1) = \frac{1}{1} = 1$

(b) $(g \circ f)(2) = g(f(2)) = g\left(\frac{1}{2}\right) = \frac{1}{\sqrt{1/2+2}}$

$$= \frac{1}{\sqrt{2.5}} \text{ or } \sqrt{\frac{2}{5}}$$

(c) $(f \circ f)(x) = f(f(x)) = f\left(\frac{1}{x}\right) = \frac{1}{1/x} = x,\ x \ne 0$

(d) $(g \circ g)(x) = g(g(x)) = g\left(\frac{1}{\sqrt{x+2}}\right) = \frac{1}{\sqrt{1/\sqrt{x+2}+2}}$

$$= \frac{\sqrt[4]{x+2}}{\sqrt{1+2\sqrt{x+2}}}$$

42. (a) $(f \circ g)(-1) = f(g(-1))$
$$= f(\sqrt[3]{-1+1})$$
$$= f(0) = 2 - 0 = 2$$

(b) $(g \circ f)(2) = g(f(2)) = g(2-2) = g(0) = \sqrt[3]{0+1} = 1$

(c) $(f \circ f)(x) = f(f(x)) = f(2-x) = 2 - (2-x) = x$

(d) $(g \circ g)(x) = g(g(x)) = g(\sqrt[3]{x+1}) = \sqrt[3]{\sqrt[3]{x+1}+1}$

43. (a) $(f \circ g)(x) = f(g(x))$
$$= f(\sqrt{x+2})$$
$$= 2 - (\sqrt{x+2})^2$$
$$= -x, x \ge -2$$
$(g \circ f)(x) = g(f(x))$
$$= g(2-x^2)$$
$$= \sqrt{(2-x^2)+2} = \sqrt{4-x^2}$$

(b) Domain of $f \circ g$: $[-2, \infty)$

Domain of $g \circ f$: $[-2, 2]$

(c) Range of $f \circ g$: $(-\infty, 2]$

Range of $g \circ f$: $[0, 2]$

44. (a) $(f \circ g)(x) = f(g(x))$
$$= f(\sqrt{1-x})$$
$$= \sqrt{\sqrt{1-x}}$$
$$= \sqrt[4]{1-x}$$
$(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = \sqrt{1-\sqrt{x}}$

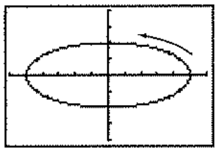
(b) Domain of $f \circ g$: $(-\infty, 1]$

Domain of $g \circ f$: $[0, 1]$

(c) Range of $f \circ g$: $[0, \infty)$

Range of $g \circ f$: $[0, 1]$

45. (a)



[–6, 6] by [–4, 4]

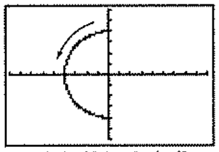
Initial point: (5, 0)
Terminal point: (5, 0)

The ellipse is traced exactly once in a counterclockwise direction starting and ending at the point (5, 0).

(b) Substituting $\cos t = \frac{x}{5}$ and $\sin t = \frac{y}{2}$ in the identity $\cos^2 t + \sin^2 t = 1$ gives the Cartesian equation $\left(\frac{x}{5}\right)^2 + \left(\frac{y}{2}\right)^2 = 1.$

The entire ellipse is traced by the curve.

46. (a)



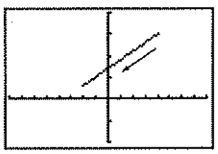
[–9, 9] by [–6, 6]

Initial point: (0, 4)

Terminal point: None (since the endpoint $\frac{3\pi}{2}$ is not included in the t-interval)
The semicircle is traced in a counterclockwise direction starting at (0, 4) and extending to, but not including, (0, –4).

(b) Substituting $\cos t = \frac{x}{4}$ and $\sin t = \frac{y}{4}$ in the identity $\cos^2 t + \sin^2 t = 1$ gives the Cartesian equation $\left(\frac{x}{4}\right)^2 + \left(\frac{y}{4}\right)^2 = 1$, or $x^2 + y^2 = 16$. The left half of the circle is traced by the parametrized curve.

47. (a)



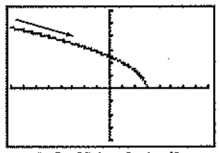
[–8, 8] by [–10, 20]

Initial point: (4, 15)
Terminal point: (–2, 3)

The line segment is traced from right to left starting at (4, 15) and ending at (–2, 3).

(b) Substituting $t = 2 - x$ into $y = 11 - 2t$ gives the Cartesian equation $y = 11 - 2(2 - x)$, or $y = 2x + 7$. The part of the line from (4, 15) to (–2, 3) is traced by the parametrized curve.

48. (a)



[–8, 8] by [–4, 6]

Initial point: None
Terminal point: (3, 0)

The curve is traced from left to right ending at the point (3, 0).

(b) Substituting $t = x - 1$ into $y = \sqrt{4 - 2t}$ gives the Cartesian equation $y = \sqrt{4 - 2(x - 1)}$, or $y = \sqrt{6 - 2x}$. The entire curve is traced by the parametrized curve.

49. (a) For simplicity, we assume that x and y are linear functions of t, and that the point (x, y) starts at (–2, 5) for $t = 0$ and ends at (4, 3) for $t = 1$. Then $x = f(t)$, where $f(0) = -2$ and $f(1) = 4$.

Since slope $= \frac{\Delta x}{\Delta t} = \frac{4 - (-2)}{1 - 0} = 6$, $x = f(t) = 6t - 2 = -2 + 6t$.

Also, $y = g(t)$, where $g(0) = 5$ and $g(1) = 3$. Since slope $= \frac{\Delta y}{\Delta t} = \frac{3 - 5}{1 - 0} = -2$, $y = g(t) = -2t + 5 = 5 - 2t$.

One possible parametrization is: $x = -2 + 6t$, $y = 5 - 2t$, $0 \le t \le 1$

50. For simplicity, we assume that x and y are linear functions of t and that the point (x, y) passes through (–3, –2) for $t = 0$ and (4, –1) for $t = 1$. Then $x = f(t)$, where $f(0) = -3$ and $f(1) = 4$. Since

$$\text{slope} = \frac{\Delta x}{\Delta t} = \frac{4 - (-3)}{1 - 0} = 7,$$

$x = f(t) = 7t - 3 = -3 + 7t$.

Also, $y = g(t)$, where $g(0) = -2$ and $g(1) = -1$.

Since

$$\text{slope} = \frac{\Delta y}{\Delta t} = \frac{-1 - (-2)}{1 - 0} = 1$$

$y = g(t) = t - 2 = -2 + t$.

One possible parametrization is:

$x = -3 + 7t$, $y = -2 + t$, $-\infty < t < \infty$.

51. For simplicity, we assume that x and y are linear functions of t and that the point (x, y) starts at (2, 5) for $t = 0$ and passes through (−1, 0) for $t = 1$. Then $x = f(t)$, where $f(0) = 2$ and $f(1) = -1$. Since

$$\text{slope} = \frac{\Delta x}{\Delta t} = \frac{-1-2}{1-0} = -3, x = f(t) = -3t + 2 = 2 - 3t.$$

Also, $y = g(t)$, where $g(0) = 5$ and $g(1) = 0$. Since

$$\text{slope} = \frac{\Delta y}{\Delta t} = \frac{0-5}{1-0} = -5, y = g(t) = -5t + 5 = 5 - 5t.$$

One possible parametrization is:

$x = 2 - 3t,\ y = 5 - 5t,\ t \ge 0.$

52. One possible parametrization is:

$x = t,\ y = t(t-4),\ t \le 2.$

53. (a) $y = 2 - 3x$

$$3x = 2 - y$$
$$x = \frac{2-y}{3}$$

Interchange x and y.

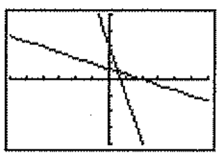
$$y = \frac{2-x}{3}$$
$$f^{-1}(x) = \frac{2-x}{3}$$

Verify.

$$\begin{aligned}(f \circ f^{-1})(x) &= f(f^{-1}(x)) \\ &= f\left(\frac{2-x}{3}\right) \\ &= 2 - 3\left(\frac{2-x}{3}\right) \\ &= 2-(2-x) = x\end{aligned}$$

$$\begin{aligned}(f^{-1} \circ f)(x) &= f^{-1}(f(x)) \\ &= f^{-1}(2-3x) \\ &= \frac{2-(2-3x)}{3} \\ &= \frac{3x}{3} = x\end{aligned}$$

(b)



[−6, 6] by [−4, 4]

54. (a) $y = (x+2)^2, x \ge -2$

$$\sqrt{y} = x + 2$$
$$x = \sqrt{y} - 2$$

Interchange x and y.

$$y = \sqrt{x} - 2$$
$$f^{-1}(x) = \sqrt{x} - 2$$

Verify.

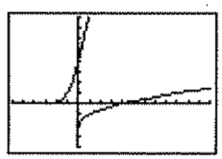
For $x \ge 0$ (the domain of f^{-1})

$$\begin{aligned}(f \circ f^{-1})(x) &= f(f^{-1}(x)) \\ &= f(\sqrt{x} - 2) \\ &= [(\sqrt{x}-2)+2]^2 \\ &= (\sqrt{x})^2 = x\end{aligned}$$

For $x \ge -2$ (the domain of f),

$$\begin{aligned}(f^{-1} \circ f)(x) &= f^{-1}(f(x)) \\ &= f^{-1}((x+2)^2) \\ &= \sqrt{(x+2)^2} - 2 \\ &= |x+2| - 2 \\ &= (x+2) - 2 = x\end{aligned}$$

(b)



[−6, 12] by [−4, 8]

55. Using a calculator, $\sin^{-1}(0.6) \approx 0.6435$ radians or $36.8699°$.

56. Using a calculator, $\tan^{-1}(-2.3) \approx -1.1607$ radians or $-66.5014°$.

57. Since $\cos\theta = \frac{3}{7}$ and $0 \le \theta \le \pi$,

$$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\left(\frac{3}{7}\right)^2} = \sqrt{\frac{40}{49}} = \frac{\sqrt{40}}{7}.$$

Therefore,

$$\sin\theta = \frac{\sqrt{40}}{7}, \cos\theta = \frac{3}{7}, \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sqrt{40}}{3},$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{3}{\sqrt{40}}, \sec\theta = \frac{1}{\cos\theta} = \frac{7}{3},$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{7}{\sqrt{40}}$$

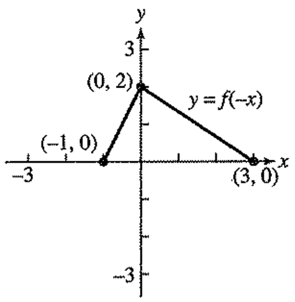
58. (a) Note that $\sin^{-1}(-0.2) \approx -0.2014$. In $[0, 2\pi)$, the solutions are $x = \pi - \sin^{-1}(-0.2) \approx 3.3430$ and $x = \sin^{-1}(-0.2) + 2\pi \approx 6.0818$.

(b) Since the period of $\sin x$ is 2π, the solutions are $x \approx 3.3430 + 2k\pi$ and $x \approx 6.0818 + 2k\pi$, k any integer.

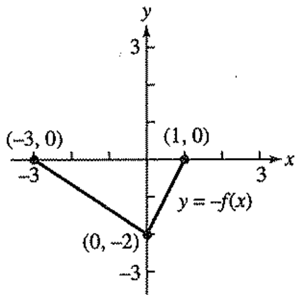
59. $e^{-0.2x} = 4$

$$\ln e^{-0.2x} = \ln 4$$
$$-0.2x = \ln 4$$
$$x = \frac{\ln 4}{-0.2} = -5\ln 4$$

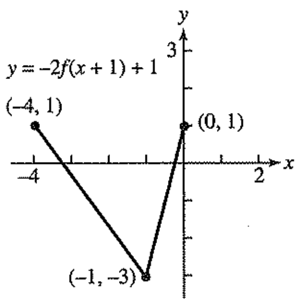
60. (a) The given graph is reflected about the y-axis.



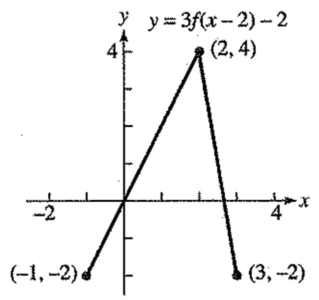
(b) The given graph is reflected about the x-axis.



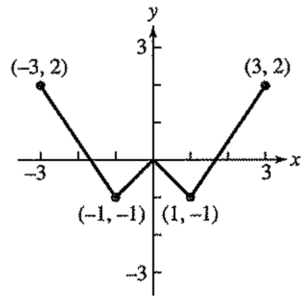
(c) The given graph is shifted left 1 unit, stretched vertically by a factor of 2, reflected about the x-axis, and then shifted upward 1 unit.



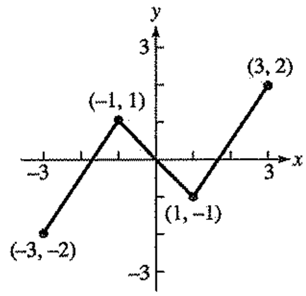
(d) The given graph is shifted right 2 units, stretched vertically by a factor of 3, and then shifted downward 2 units.



61. (a)



(b)



62. (a) $V = 100{,}000 - 10{,}000x,\ 0 \le x \le 10$

(b)

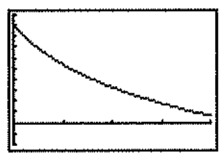
$$\begin{aligned} V &= 55{,}000 \\ 100{,}000 - 10{,}000x &= 55{,}000 \\ -10{,}000x &= -45{,}000 \\ x &= 4.5 \end{aligned}$$

The value is \$55,000 after 4.5 years.

63. (a) $f(0) = 90$ units

(b) $f(2) = 90 - 52 \ln 3 \approx 32.8722$ units

(c)



[0, 4] by [−20, 100]

64. $1500(1.08)^t = 5000$

$$\begin{aligned} 1.08^t &= \frac{5000}{1500} = \frac{10}{3} \\ \ln(1.08)^t &= \ln\frac{10}{3} \\ t \ln 1.08 &= \ln\frac{10}{3} \\ t &= \frac{\ln(10/3)}{\ln 1.08} \\ t &\approx 15.6439 \end{aligned}$$

It will take about 15.6439 years. (If the bank only pays interest at the end of the year, it will take 16 years.)

65. (a) $N(t) = 4 \cdot 2^t$

(b) 4 days: $4 \cdot 2^4 = 64$ guppies

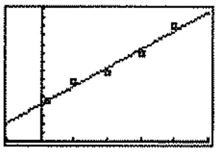
1 week: $4 \cdot 2^7 = 512$ guppies

(c)
$$\begin{aligned} N(t) &= 2000 \\ 4 \cdot 2^t &= 2000 \\ 2^t &= 500 \\ \ln 2^t &= \ln 500 \\ t\ln 2 &= \ln 500 \end{aligned}$$

$t = \dfrac{\ln 500}{\ln 2} \approx 8.9658$ There will be 2000 guppies after 8.9658 days, or after nearly 9 days.

(d) Because it suggests the number of guppies will continue to double indefinitely and become arbitrarily large, which is impossible due to the finite size of the tank and the oxygen supply in the water.

66. (a) $y = 41.770x + 414.342$



[–5, 25] by [0, 1500]

(b) $y = 41.770(22) + 414.342 = 1333$
$1333 - 1432 = -99$

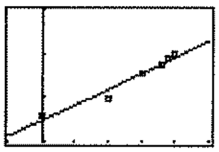
The estimate is 99 less than the actual number.

(c) $y = mx + b$

$m = 41.770$

The slope represents the approximate annual increase in the number of doctorates earned by Hispanic Americans per year.

67. (a) $y = (17467.361)\,(1.00398)^x =$



[–5, 25] by [17000, 20000]

(b) $(17467.361)\ (1.00398)^{23} = 19{,}138$ thousand or 19,138,000

$19{,}138{,}000 - 19{,}190{,}000 = -52{,}000$

The prediction is less than the actual by 52,000.

(c) $\dfrac{17{,}558}{(19{,}138)(23)} = 0.0398$ or 4%

68. (a) $m = 1$

(b) $y = -x - 1$

(c) $y = x + 3$

(d) 2

69. (a) $(2, \infty)$ $x - 2 > 0$

(b) $(-\infty, \infty)$ all real numbers

(c)
$$\begin{aligned} f(x) &= 1 - \ln(x-2) \\ 0 &= 1 - \ln(x-2) \\ 1 &= \ln(x-2) \\ e^1 &= x - 2 \\ x &= e^1 + 2 \approx 4.718 \end{aligned}$$

(d)
$$\begin{aligned} y &= 1 - \ln(x-2) \\ x &= 1 - \ln(y-2) \\ x - 1 &= -\ln(y-2) \\ e^{1-x} &= y - 2 \\ y &= e^{1-x} + 2 \end{aligned}$$

(e)
$$\begin{aligned} (f \circ f^{-1})(x) &= f(f^{-1}(x)) = f(2 + e^{1-x}) \\ &= 1 - \ln(2 + e^{1-x} - 2) = 1 - \ln(e^{1-x}) \\ &= 1 - (1 - x) = x \\ (f^{-1} \circ f)(x) &= f^{-1}(f(x)) = f^{-1}(1 - \ln(x-2)) \\ &= 2 + e^{1-(1-\ln(x-2))} = 2 + e^{\ln(x-2)} \\ &= 2 + (x-2) = x \end{aligned}$$

70. (a) $(-\infty, \infty)$ all real numbers

(b) $[-2, 4]$ $1 - 3\cos(2x)$ oscillates between -2 and 4

(c) π

(d) Even. $\cos(-\theta) = \cos(\theta)$

(e) $x \approx 2.526$

Chapter 2
Limits and Continuity

Section 2.1 Rates of Change and Limits (pp. 59–69)

Quick Review 2.1

1. $f(2) = 2(2^3) - 5(2)^2 + 4 = 0$

2. $f(2) = \dfrac{4(2)^2 - 5}{2^3 + 4} = \dfrac{11}{12}$

3. $f(2) = \sin\left(\pi \cdot \dfrac{2}{2}\right) = \sin \pi = 0$

4. $f(2) = \dfrac{1}{2^2 - 1} = \dfrac{1}{3}$

5. $|x| < 4$
$-4 < x < 4$

6. $|x| < c^2$
$-c^2 < x < c^2$

7. $|x-2| < 3$
$-3 < x-2 < 3$
$-1 < x < 5$

8. $|x-c| < d^2$
$-d^2 < x-c < d^2$
$-d^2+c < x < d^2+c$

9. $\frac{x^2-3x-18}{x+3} = \frac{(x+3)(x-6)}{x+3} = x-6,\ x \neq -3$

10. $\frac{2x^2-x}{2x^2+x-1} = \frac{x(2x-1)}{(2x-1)(x+1)} = \frac{x}{x+1},\ x \neq \frac{1}{2}$

Section 2.1 Exercises

1. $\frac{\Delta y}{\Delta t} = \frac{16(3)^2-16(0)^2}{3-0} = 48\ \text{ft/sec}$

2. $\frac{\Delta y}{\Delta t} = \frac{16(4)^2-16(0)^2}{4-0} = 64\ \text{ft/sec}$

3. $\frac{\Delta y}{\Delta t} = \frac{16(3+h)^2-16(3)^2}{h}$, say $h = 0.01$
$= \frac{16(3+0.01)^2-16(9)}{0.01} = \frac{16(9.0601)-16(9)}{0.01}$
$= \frac{144.9616-144}{0.01} = \frac{0.9616}{0.01} = 96.16\ \text{ft/sec}$

Confirm Algebraically

$$\frac{\Delta y}{\Delta t} = \frac{16(3+h)^2-16(3)^2}{h}$$
$$= \frac{16(9+6h+h^2)-144}{h} = \frac{96h+16h^2}{h} = (96+16h)\ \text{ft/sec}$$

if $h = 0$, then $\frac{\Delta y}{\Delta t} = 96 \frac{\text{ft}}{\text{sec}}$

4. $\frac{\Delta y}{\Delta t} = \frac{16(4+h)^2-16(4)^2}{h}$
say $h = 0.01$
$\frac{16(4+0.01)^2-16(4)^2}{0.01}$
$= \frac{16(16.0801)-16(16)}{0.01}$
$= \frac{257.2816-256}{0.01}$
$= \frac{1.2816}{0.01} = 128.16 \frac{\text{ft}}{\text{sec}}$

Confirm Algebraically

$$\frac{\Delta y}{\Delta t} = \frac{16(4+h)^2-16(4)^2}{h}$$
$$= \frac{16(16+8h+h^2)-256}{h}$$
$$= \frac{128h+16h^2}{h}$$
$$= (128+16h)\frac{\text{ft}}{\text{sec}}$$

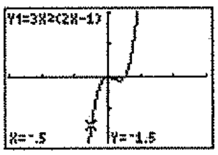
if $h = 0$, then $\frac{\Delta y}{\Delta t} = 128 \frac{\text{ft}}{\text{sec}}$

5. $\lim_{x \to c} (2x^3-3x^2+x-1)$
$= 2c^3-3c^2+c-1$

6. $\lim_{x \to c} \frac{x^4-x^3+1}{x^2+9}$
$= \frac{c^4-c^3+1}{c^2+9}$

7. $\lim_{x \to -1/2} 3x^2(2x-1) = 3\left(-\frac{1}{2}\right)^2\left[2\left(-\frac{1}{2}\right)-1\right] = 3\left(\frac{1}{4}\right)(-2)$
$= -\frac{3}{2}$

Graphical support:



[−3, 3] by [−2, 2]

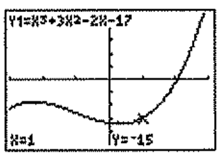
8. $\lim_{x \to -4} (x+3)^{1998} = (-4+3)^{1998} = (-1)^{1998} = 1$

Graphical support:

[−4.001, −3.999] by [0, 5]

9. $\lim_{x \to 1} (x^3+3x^2-2x-17) = (1)^3+3(1)^2-2(1)-17$
$= 1+3-2-17 = -15$

Graphical support:



[−3, 3] by [−25, 25]