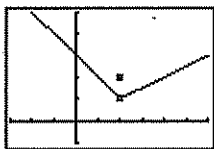


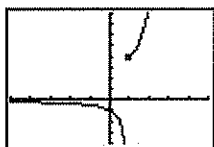
20.



[-3, 6] by [-1, 5]

(a) $x = 2$ (b) Removable, assign the value 1 to $f(2)$.

21.



[-5, 5] by [-4, 8]

(a) $x = 1$

(b) Not removable, it's an infinite discontinuity.

22.



[-4.7, 4.7] by [-3.1, 3.1]

(a) $x = -1$ (b) Removable, assign the value 0 to $f(-1)$.23. (a) All points not in the domain along with $x = 0, 1$ (b) $x = 0$ is a removable discontinuity, assign $f(0) = 0$.
 $x = 1$ is not removable, the one-sided limits are different.24. (a) All points not in the domain along with $x = 1, 2$ (b) $x = 1$ is not removable, the one-sided limits are different.
 $x = 2$ is a removable discontinuity, assign $f(2) = 1$.25. For $x \neq -3$, $f(x) = \frac{x^2 - 9}{x + 3} = \frac{(x + 3)(x - 3)}{x + 3} = x - 3$.The extended function is $y = x - 3$.26. For $x \neq 1$, $f(x) = \frac{x^3 - 1}{x^2 - 1}$
 $= \frac{(x - 1)(x^2 + x + 1)}{(x + 1)(x - 1)}$
 $= \frac{x^2 + x + 1}{x + 1}$.The extended function is $y = \frac{x^2 + x + 1}{x + 1}$.27. Since $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$, the extended function is

$$y = \begin{cases} \frac{\sin x}{x}, & x \neq 0 \\ 1, & x = 0. \end{cases}$$

28. Since $\lim_{x \rightarrow 0} \frac{\sin 4x}{x} = 4 \lim_{x \rightarrow 0} \frac{\sin 4x}{4x} = 4(1) = 4$, the extended function is

$$y = \begin{cases} \frac{\sin 4x}{x}, & x \neq 0 \\ 4, & x = 0. \end{cases}$$

29. For $x \neq 4$ (and $x > 0$),

$$f(x) = \frac{x - 4}{\sqrt{x} - 2} = \frac{(\sqrt{x} + 2)(\sqrt{x} - 2)}{\sqrt{x} - 2} = \sqrt{x} + 2.$$

The extended function is $y = \sqrt{x} + 2$.30. For $x \neq 2$ (and $x \neq -2$),

$$\begin{aligned} f(x) &= \frac{x^3 - 4x^2 - 11x + 30}{x^2 - 4} \\ &= \frac{(x - 2)(x - 5)(x + 3)}{(x - 2)(x + 2)} \\ &= \frac{(x - 5)(x + 3)}{x + 2} \\ &= \frac{x^2 - 2x - 15}{x + 2}. \end{aligned}$$

The extended function is $y = \frac{x^2 - 2x - 15}{x + 2}$.31. The domain of f is all real numbers $x \neq 3$. f is continuous at all those points so f is a continuous function.32. The domain of g is all real numbers $x > 1$. f is continuous at all those points so g is a continuous function.33. f is the composite of two continuous functions $g \circ h$ where

$$g(x) = \sqrt{x} \text{ and } h(x) = \frac{x}{x + 1}.$$

34. f is the composite of two continuous functions $g \circ h$ where

$$g(x) = \sin x \text{ and } h(x) = x^2 + 1.$$

35. f is the composite of three continuous functions $g \circ h \circ k$

$$\text{where } g(x) = \cos x, h(x) = \sqrt[3]{x}, \text{ and } k(x) = 1 - x.$$

36. f is the composite of two continuous functions $g \circ h$ where

$$g(x) = \tan x \text{ and } h(x) = \frac{x^2}{x^2 + 4}.$$

37. One possible answer:

Assume $y = x$, constant functions, and the square root function are continuous.

By the sum theorem, $y = x + 2$ is continuous.

By the composite theorem, $y = \sqrt{x+2}$ is continuous.

By the quotient theorem, $y = \frac{1}{\sqrt{x+2}}$ is continuous.

Domain: $(-2, \infty)$

38. One possible answer:

Assume $y = x$, constant functions, and the cube root function are continuous.

By the difference theorem, $y = 4 - x$ is continuous.

By the composite theorem, $y = \sqrt[3]{4-x}$ is continuous.

By the product theorem, $y = x^2 = x \cdot x$ is continuous.

By the sum theorem, $y = x^2 + \sqrt[3]{4-x}$ is continuous.

Domain: $(-\infty, \infty)$

39. Possible answer:

Assume $y = x$ and $y = |x|$ are continuous.

By the product theorem, $y = x^2 = x \cdot x$ is continuous.

By the constant multiple theorem, $y = 4x$ is continuous.

By the difference theorem, $y = x^2 - 4x$ is continuous.

By the composite theorem, $y = |x^2 - 4x|$ is continuous.

Domain: $(-\infty, \infty)$

40. One possible answer:

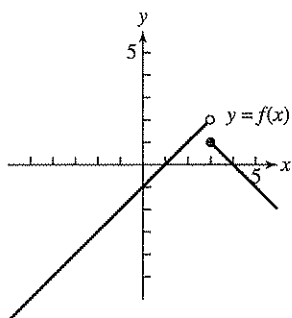
Assume $y = x$ and $y = 1$ are continuous.

Use the product, difference, and quotient theorems. One also needs to verify that the limit of this function as x approaches 1 is 2.

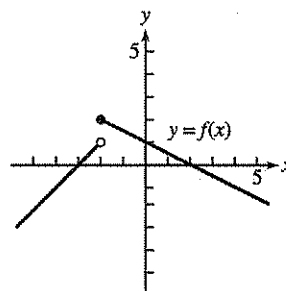
Alternately, observe that the function is equivalent to $y = x + 1$ (for all x), which is continuous by the sum theorem.

Domain: $(-\infty, \infty)$

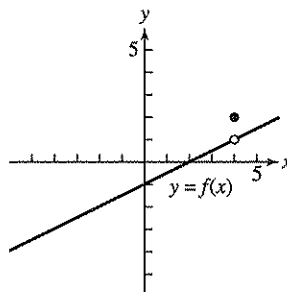
41. One possible answer:



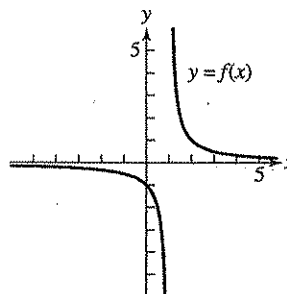
42. One possible answer:



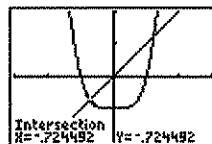
43. One possible answer:



44. One possible answer:



45.

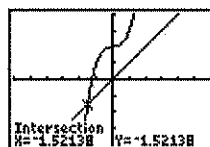


Intersection
 $x = -0.724492$ $y = -0.724492$

$[-3, 3]$ by $[-2, 2]$

Solving $x = x^4 - 1$, we obtain the solutions $x \approx -0.724$ and $x \approx 1.221$.

46.



Intersection
 $x = -1.52138$ $y = -1.52138$

$[-6, 6]$ by $[-4, 4]$

Solving $x = x^3 + 2$, we obtain the solution $x \approx -1.521$.

47. We require that $\lim_{x \rightarrow 3^+} 2ax = \lim_{x \rightarrow 3^-} (x^2 - 1)$:

$$2a(3) = 3^2 - 1$$

$$6a = 8$$

$$a = \frac{4}{3}$$

48. Solve at $x = 2$

$$f(x) = 2x + 3 = 2(2) + 3 = 4 + 3 = 7$$

$$f(x) = ax + 1 \text{ at } x = 2.1$$

$$7 = 2a + 1$$

$$6 = 2a$$

$$a = 3$$

49. Solve at $x = -1$

$$f(x) = 4 - x^2 = 4 - (-1)^2 = 4 + 1 = 5$$

$$f(x) = ax + 1 \text{ at } x = 1$$

$$5 = a(1) + 1$$

$$a = 4$$

50. Solve at $x = 1$

$$f(x) = x^3 = 1^3 = 1$$

$$f(x) = x^2 + x + a$$

$$1 = 1^2 + 1 + a$$

$$a = -1$$

51. Consider $f(x) = x - e^{-x}$. f is continuous, $f(0) = -1$, and

$$f(1) = 1 - \frac{1}{e} > 0.5. \text{ By the Intermediate Value Theorem, for}$$

some c in $(0, 1)$, $f(c) = 0$ and $e^{-c} = c$.

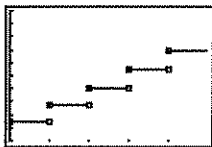
52. (a) Sarah's salary is $\$36,500 = \$36,500(1.035)^0$ for the first year ($0 \leq t < 1$), $\$36,500(1.035)$ for the second year

($1 \leq t < 2$), $\$36,500(1.035)^2$ for the third year

($2 \leq t < 3$), and so on. This corresponds to

$$y = 36,500(1.035)^{\text{int}t}.$$

(b)



$[0, 4.8]$ by $[35000, 45000]$

The function is continuous at all points in the domain $[0, 5)$ except at $t = 1, 2, 3, 4$.

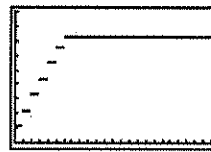
53. (a) We require:

$$f(x) = \begin{cases} 0 & x = 0 \\ 1.10, & 0 < x \leq 1 \\ 2.20, & 1 < x \leq 2 \\ 3.30, & 2 < x \leq 3 \\ 4.40, & 3 < x \leq 4 \\ 5.50, & 4 < x \leq 5 \\ 6.60, & 5 < x \leq 6 \\ 7.25, & 6 < x \leq 24. \end{cases}$$

This may be written more compactly as

$$f(x) = \begin{cases} -1.10 \text{ int}(-x), & 0 \leq x \leq 6 \\ 7.25, & 6 < x \leq 24 \end{cases}$$

(b)



$[0, 24]$ by $[0, 9]$

This is continuous for all values of x in the domain $[0, 24]$ except for $x = 0, 1, 2, 3, 4, 5, 6$.

54. False. Consider $f(x) = 1/x$ which is continuous and has a point of discontinuity at $x = 0$.

55. True. If f has a jump discontinuity at $x = a$, then

$$\lim_{x \rightarrow a^-} f(x) \neq \lim_{x \rightarrow a^+} f(x) \text{ so } f \text{ is not continuous at } x = a.$$

56. B. $f(x) = \frac{1}{\sqrt{0}}$ is not defined.

57. E. $f(x) = \sqrt{x-1} = \sqrt{1-1} = \sqrt{0}$ is the only defined option.

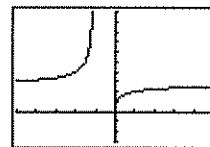
58. A. $f(1) = 1$.

59. E. $x = 3$ causes a zero to be in the denominator.

60. (a) The function is defined when $1 + \frac{1}{x} > 0$, that is, on

$(-\infty, -1) \cup (0, \infty)$. (It can be argued that the domain should also include certain values in the interval $(-1, 0)$, namely, those rational numbers that have odd denominators when expressed in lowest terms.)

(b)



$[-5, 5]$ by $[-3, 10]$

60. Continued

(c) If we attempt to evaluate $f(x)$ at these values, we

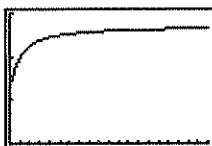
obtain $f(-1) = \left(1 + \frac{1}{-1}\right)^{-1} = 0^{-1} = \frac{1}{0}$ (undefined) and

$f(0) = \left(1 + \frac{1}{0}\right)^0$ (undefined). Since f is undefined at

these values due to division by zero, both values are points of discontinuity.

(d) The discontinuity at $x = 0$ is removable because the right-hand limit is 0. The discontinuity at $x = -1$ is not removable because it is an infinite discontinuity.

(e)



$[0, 20]$ by $[0, 3]$

X	Y
10	2.5937
100	2.7048
1000	2.7183
10000	2.7183
100000	2.7183
1E6	2.7183
1E7	2.7183

The limit is about 2.718, or e .

61. This is because $\lim_{h \rightarrow 0} f(a+h) = \lim_{x \rightarrow a} f(x)$.

62. Suppose not. Then f would be negative somewhere in the interval and positive somewhere else in the interval. So, by the Intermediate Value Theorem, it would have to be zero somewhere in the interval, which contradicts the hypothesis.

63. Since the absolute value function is continuous, this follows from the theorem about continuity of composite functions.

64. For any real number a , the limit of this function as x approaches a cannot exist. This is because as x approaches a , the values of the function will continually oscillate between 0 and 1.

Section 2.4 Rates of Change and Tangent Lines (pp. 87–94)

Quick Review 2.4

1. $\Delta x = 3 - (-5) = 8$
 $\Delta y = 5 - 2 = 3$

2. $\Delta x = a - 1$
 $\Delta y = b - 3$

3. $m = \frac{-1-3}{5-(-2)} = \frac{-4}{7} = -\frac{4}{7}$

4. $m = \frac{3-(-1)}{3-(-3)} = \frac{4}{6} = \frac{2}{3}$

5. $y = \frac{3}{2}[x - (-2)] + 3$
 $y = \frac{3}{2}x + 6$

6. $m = \frac{-1-6}{4-1} = \frac{-7}{3} = -\frac{7}{3}$
 $y = -\frac{7}{3}(x-1) + 6$
 $y = -\frac{7}{3}x + \frac{25}{3}$

7. $y = -\frac{3}{4}(x-1) + 4$
 $y = -\frac{3}{4}x + \frac{19}{4}$

8. $m = -\frac{1}{-3/4} = \frac{4}{3}$
 $y = \frac{4}{3}(x-1) + 4$
 $y = \frac{4}{3}x + \frac{8}{3}$

9. Since $2x + 3y = 5$ is equivalent to $y = -\frac{2}{3}x + \frac{5}{3}$, we use

$$m = -\frac{2}{3}$$

$$y = -\frac{2}{3}[x - (-1)] + 3$$

$$y = -\frac{2}{3}x + \frac{7}{3}$$

10. $\frac{b-3}{4-2} = \frac{5}{3}$
 $b-3 = \frac{10}{3}$
 $b = \frac{19}{3}$

Section 2.4 Exercises

1. (a) $\frac{\Delta f}{\Delta x} = \frac{f(3) - f(2)}{3 - 2} = \frac{28 - 9}{1} = 19$

(b) $\frac{\Delta f}{\Delta x} = \frac{f(1) - f(-1)}{1 - (-1)} = \frac{2 - 0}{2} = 1$

2. (a) $\frac{\Delta f}{\Delta x} = \frac{f(2) - f(0)}{2 - 0} = \frac{3 - 1}{2} = 1$

(b) $\frac{\Delta f}{\Delta x} = \frac{f(12) - f(10)}{12 - 10} = \frac{7 - \sqrt{41}}{2} \approx 0.298$

3. (a) $\frac{\Delta f}{\Delta x} = \frac{f(0) - f(-2)}{0 - (-2)} = \frac{1 - e^{-2}}{2} \approx 0.432$

(b) $\frac{\Delta f}{\Delta x} = \frac{f(3) - f(1)}{3 - 1} = \frac{e^3 - e}{2} \approx 8.684$

$$4. (a) \frac{\Delta f}{\Delta x} = \frac{f(4) - f(1)}{4 - 1} = \frac{\ln 4 - 0}{3} = \frac{\ln 4}{3} \approx 0.462$$

$$(b) \frac{\Delta f}{\Delta x} = \frac{f(103) - f(100)}{103 - 100} = \frac{\ln 103 - \ln 100}{3} = \frac{1}{3} \ln \frac{103}{100} \\ = \frac{1}{3} \ln 1.03 \approx 0.0099$$

$$5. (a) \frac{\Delta f}{\Delta x} = \frac{f(3\pi/4) - f(\pi/4)}{(3\pi/4) - (\pi/4)} = \frac{-1 - 1}{\pi/2} = -\frac{4}{\pi} \approx -1.273$$

$$(b) \frac{\Delta f}{\Delta x} = \frac{f(\pi/2) - f(\pi/6)}{(\pi/2) - (\pi/6)} = \frac{0 - \sqrt{3}}{\pi/3} = -\frac{3\sqrt{3}}{\pi} \approx -1.654$$

$$6. (a) \frac{\Delta f}{\Delta x} = \frac{f(\pi) - f(0)}{\pi - 0} = \frac{1 - 3}{\pi} = -\frac{2}{\pi} \approx -0.637$$

$$(b) \frac{\Delta f}{\Delta x} = \frac{f(\pi) - f(-\pi)}{\pi - (-\pi)} = \frac{1 - 1}{2\pi} = 0$$

7. We use $Q_1 = (10, 225)$, $Q_2 = (14, 375)$, $Q_3 = (16.5, 475)$, $Q_4 = (18, 550)$, and $P = (20, 650)$.

$$(a) \text{ Slope of } PQ_1: \frac{650 - 225}{20 - 10} \approx 43$$

$$\text{Slope of } PQ_2: \frac{650 - 375}{20 - 14} \approx 46$$

$$\text{Slope of } PQ_3: \frac{650 - 475}{20 - 16.5} = 50$$

$$\text{Slope of } PQ_4: \frac{650 - 550}{20 - 18} = 50$$

Secant Slope

$$PQ_1 \quad 43$$

$$PQ_2 \quad 46$$

$$PQ_3 \quad 50$$

$$PQ_4 \quad 50$$

The appropriate units are meters per second.

(b) Approximately 50 m/sec

8. We use $Q_1 = (5, 20)$, $Q_2 = (7, 38)$, $Q_3 = (8.5, 56)$, $Q_4 = (9.5, 72)$, and $P = (10, 80)$.

$$(a) \text{ Slope of } PQ_1: \frac{80 - 20}{10 - 5} = 12$$

$$\text{Slope of } PQ_2: \frac{80 - 38}{10 - 7} = 14$$

$$\text{Slope of } PQ_3: \frac{80 - 56}{10 - 8.5} = 16$$

$$\text{Slope of } PQ_4: \frac{80 - 72}{10 - 9.5} = 16$$

Secant Slope

$$PQ_1 \quad 12$$

$$PQ_2 \quad 14$$

$$PQ_3 \quad 16$$

$$PQ_4 \quad 16$$

The appropriate units are meters per second.

(b) Approximately 16 m/sec

$$9. (a) \lim_{h \rightarrow 0} \frac{y(-2+h) - y(-2)}{h} = \lim_{h \rightarrow 0} \frac{(-2+h)^2 - (-2)^2}{h} \\ = \lim_{h \rightarrow 0} \frac{4 - 4h + h^2 - 4}{h} \\ = \lim_{h \rightarrow 0} \frac{-4h + h^2}{h} \\ = \lim_{h \rightarrow 0} (-4 + h) \\ = -4$$

(b) The tangent line has slope -4 and passes through

$$(-2, y(-2)) = (-2, 4)$$

$$y = -4[x - (-2)] + 4$$

$$y = -4x - 4$$

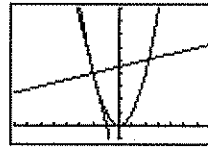
(c) The normal line has slope $-\frac{1}{-4} = \frac{1}{4}$ and passes through

$$(-2, y(-2)) = (-2, 4).$$

$$y = \frac{1}{4}[x - (-2)] + 4$$

$$y = \frac{1}{4}x + \frac{9}{2}$$

(d)



$[-8, 7]$ by $[-1, 9]$

$$10. (a) \lim_{h \rightarrow 0} \frac{y(1+h) - y(1)}{h} \\ = \lim_{h \rightarrow 0} \frac{[(1+h)^2 - 4(1+h)] - [1^2 - 4(1)]}{h} \\ = \lim_{h \rightarrow 0} \frac{1 + 2h + h^2 - 4 - 4h + 3}{h} \\ = \lim_{h \rightarrow 0} \frac{h^2 - 2h}{h} \\ = \lim_{h \rightarrow 0} (h - 2) \\ = -2$$

(b) The tangent line has slope -2 and passes through

$$(1, y(1)) = (1, -3).$$

$$y = -2(x - 1) - 3$$

$$y = -2x - 1$$

10. Continued

(c) The normal line has slope $-\frac{1}{-2} = \frac{1}{2}$ and passes through

$$(1, y(1)) = (1, -3).$$

$$y = \frac{1}{2}(x-1) - 3$$

$$y = \frac{1}{2}x - \frac{7}{2}$$

(d)

$[-6, 6]$ by $[-6, 2]$

$$\begin{aligned} 11. (a) \lim_{h \rightarrow 0} \frac{y(2+h) - y(2)}{h} &= \lim_{h \rightarrow 0} \frac{\frac{1}{(2+h)-1} - \frac{1}{2-1}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{1}{h+1} - 1}{h} \\ &= \lim_{h \rightarrow 0} \frac{1 - (h+1)}{h(h+1)} \\ &= \lim_{h \rightarrow 0} \left(-\frac{1}{h+1} \right) \\ &= -1 \end{aligned}$$

(b) The tangent line has slope -1 and passes through

$$(2, y(2)) = (2, 1).$$

$$y = -(x-2) + 1$$

$$y = -x + 3$$

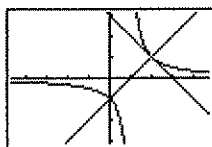
(c) The normal line has slope $-\frac{1}{-1} = 1$ and passes through

$$(2, y(2)) = (2, 1).$$

$$y = 1(x-2) + 1$$

$$y = x - 1$$

(d)



$[-4.7, 4.7]$ by $[-3.1, 3.1]$

$$\begin{aligned} 12. (a) \lim_{h \rightarrow 0} \frac{y(0+h) - y(0)}{h} &= \lim_{h \rightarrow 0} \frac{(h^2 - 3h - 1) - (-1)}{h} \\ &= \lim_{h \rightarrow 0} \frac{h^2 - 3h}{h} \\ &= \lim_{h \rightarrow 0} (h - 3) \\ &= -3 \end{aligned}$$

(b) The tangent line has slope -3 and passes through

$$(0, y(0)) = (0, -1).$$

$$y = -3(x-0) - 1$$

$$y = -3x - 1$$

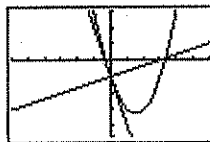
(c) The normal line has slope $-\frac{1}{-3} = \frac{1}{3}$ and passes through

$$(0, y(0)) = (0, -1).$$

$$y = \frac{1}{3}(x-0) - 1$$

$$y = \frac{1}{3}x - 1$$

(d)



$[-6, 6]$ by $[-5, 3]$

13. (a) Near $x = 2$, $f(x) = |x| = x$.

$$\lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h} = \lim_{h \rightarrow 0} \frac{(2+h) - 2}{h} = \lim_{h \rightarrow 0} 1 = 1$$

(b) Near $x = -3$, $f(x) = |x| = -x$.

$$\lim_{h \rightarrow 0} \frac{f(-3+h) - f(-3)}{h} = \lim_{h \rightarrow 0} \frac{(3-h) - 3}{h} = \lim_{h \rightarrow 0} -1 = -1$$

14. Near $x = 1$, $f(x) = |x-2| = -(x-2) = 2-x$.

$$\begin{aligned} \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h} &= \lim_{h \rightarrow 0} \frac{[2 - (1+h)] - (2-1)}{h} = \lim_{h \rightarrow 0} \frac{1-h-1}{h} \\ &= \lim_{h \rightarrow 0} -1 = -1 \end{aligned}$$

15. First, note that $f(0) = 2$.

$$\begin{aligned}\lim_{h \rightarrow 0^-} \frac{f(0+h) - f(0)}{h} &= \lim_{h \rightarrow 0^-} \frac{(2-2h-h^2) - 2}{h} \\ &= \lim_{h \rightarrow 0^-} \frac{-2h-h^2}{h} \\ &= \lim_{h \rightarrow 0^-} (-2-h) \\ &= -2\end{aligned}$$

$$\begin{aligned}\lim_{h \rightarrow 0^+} \frac{f(0+h) - f(0)}{h} &= \lim_{h \rightarrow 0^+} \frac{(2h+2) - 2}{h} \\ &= \lim_{h \rightarrow 0^+} 2 \\ &= 2\end{aligned}$$

No, the slope from the left is -2 and the slope from the right is 2 . The two-sided limit of the difference quotient does not exist.

16. First, note that $f(0) = 0$.

$$\begin{aligned}\lim_{h \rightarrow 0^-} \frac{f(0+h) - f(0)}{h} &= \lim_{h \rightarrow 0^-} \frac{-h-0}{h} = -1 \\ \lim_{h \rightarrow 0^+} \frac{f(0+h) - f(0)}{h} &= \lim_{h \rightarrow 0^+} \frac{(h^2-h) - 0}{h} \\ &= \lim_{h \rightarrow 0^+} (h-1) = -1\end{aligned}$$

Yes. The slope is -1 .

17. First, note that $f(2) = \frac{1}{2}$

$$\begin{aligned}\lim_{h \rightarrow 0^-} \frac{f(2+h) - f(2)}{h} &= \lim_{h \rightarrow 0^-} \frac{\frac{1}{2+h} - \frac{1}{2}}{h} \\ &= \lim_{h \rightarrow 0^-} \frac{2 - (2+h)}{2h(2+h)} \\ &= \lim_{h \rightarrow 0^-} \frac{-h}{2h(2+h)} \\ &= \lim_{h \rightarrow 0^-} -\frac{1}{2(2+h)} \\ &= -\frac{1}{4}\end{aligned}$$

$$\begin{aligned}\lim_{h \rightarrow 0^+} \frac{f(2+h) - f(2)}{h} &= \lim_{h \rightarrow 0^+} \frac{\frac{4-(2+h)}{4} - \frac{1}{2}}{h} \\ &= \lim_{h \rightarrow 0^+} \frac{[4-(2+h)] - 2}{4h} \\ &= \lim_{h \rightarrow 0^+} \frac{-h}{4h} \\ &= -\frac{1}{4}\end{aligned}$$

Yes. The slope is $-\frac{1}{4}$.

18. No. The function is discontinuous at $x = \frac{3\pi}{4}$ because

$$\begin{aligned}\lim_{x \rightarrow (3\pi/4)^-} f(x) &= \lim_{x \rightarrow (3\pi/4)^-} \sin x = \sin \frac{3\pi}{4} = \frac{\sqrt{2}}{2} \text{ but} \\ f\left(\frac{3\pi}{4}\right) &= \cos \frac{3\pi}{4} = -\frac{\sqrt{2}}{2}.\end{aligned}$$

$$\begin{aligned}19. (a) \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} &= \lim_{h \rightarrow 0} \frac{[(a+h)^2 + 2] - (a^2 + 2)}{h} \\ &= \lim_{h \rightarrow 0} \frac{a^2 + 2ah + h^2 + 2 - a^2 - 2}{h} \\ &= \lim_{h \rightarrow 0} \frac{2ah + h^2}{h} \\ &= \lim_{h \rightarrow 0} (2a + h) \\ &= 2a\end{aligned}$$

(b) The slope of the tangent steadily increases as a increases.

$$\begin{aligned}20. (a) \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} &= \lim_{h \rightarrow 0} \frac{\frac{2}{a+h} - \frac{2}{a}}{h} \\ &= \lim_{h \rightarrow 0} \frac{2a - 2(a+h)}{ah(a+h)} \\ &= \lim_{h \rightarrow 0} \frac{-2}{a(a+h)} \\ &= -\frac{2}{a^2}\end{aligned}$$

(b) The slope of the tangent is always negative. The tangents are very steep near $x = 0$ and nearly horizontal as a moves away from the origin.

$$\begin{aligned}21. (a) \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} &= \lim_{h \rightarrow 0} \frac{\frac{1}{a+h-1} - \frac{1}{a-1}}{h} \\ &= \lim_{h \rightarrow 0} \frac{(a-1) - (a+h-1)}{h(a-1)(a+h-1)} \\ &= \lim_{h \rightarrow 0} -\frac{1}{(a-1)(a+h-1)} \\ &= -\frac{1}{(a-1)^2}\end{aligned}$$

(b) The slope of the tangent is always negative. The tangents are very steep near $x = 1$ and nearly horizontal as a moves away from the origin.

$$\begin{aligned}22. (a) \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} &= \lim_{h \rightarrow 0} \frac{[9 - (a+h)^2] - (9 - a^2)}{h} \\ &= \lim_{h \rightarrow 0} \frac{9 - a^2 - 2ah - h^2 - 9 + a^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{-2ah - h^2}{h} \\ &= \lim_{h \rightarrow 0} (-2a - h) \\ &= -2a\end{aligned}$$

(b) The slope of the tangent steadily decreases as a increases.

23. Let
- $f(t) = 100 - 4.9t^2$
- .

$$\begin{aligned}\lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h} &= \lim_{h \rightarrow 0} \frac{[100 - 4.9(2+h)^2] - [100 - 4.9(2)^2]}{h} \\ &= \lim_{h \rightarrow 0} \frac{100 - 19.6 - 19.6h - 4.9h^2 - 100 + 19.6}{h} \\ &= \lim_{h \rightarrow 0} (-19.6 - 4.9h) \\ &= -19.6\end{aligned}$$

The object is falling at a speed of 19.6 m/sec.

24. Let
- $f(t) = 3t^2$
- .

$$\begin{aligned}\lim_{h \rightarrow 0} \frac{f(10+h) - f(10)}{h} &= \lim_{h \rightarrow 0} \frac{3(10+h)^2 - 300}{h} \\ &= \lim_{h \rightarrow 0} \frac{300 + 60h + 3h^2 - 300}{h} \\ &= \lim_{h \rightarrow 0} (60 + 3h) \\ &= 60\end{aligned}$$

The rocket's speed is 60 ft/sec.

25. Let
- $f(r) = \pi r^2$
- , the area of a circle of radius
- r
- .

$$\begin{aligned}\lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h} &= \lim_{h \rightarrow 0} \frac{\pi(3+h)^2 - \pi(3)^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{9\pi + 6\pi h + \pi h^2 - 9\pi}{h} \\ &= \lim_{h \rightarrow 0} (6\pi + \pi h) \\ &= 6\pi\end{aligned}$$

The area is changing at a rate of 6π in²/in., that is, 6π square inches of area per inch of radius.

26. Let
- $f(r) = \frac{4}{3}\pi r^3$
- .

$$\begin{aligned}\lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h} &= \lim_{h \rightarrow 0} \frac{\frac{4}{3}\pi(2+h)^3 - \frac{4}{3}\pi(2)^3}{h} \\ &= \frac{4}{3}\pi \lim_{h \rightarrow 0} \frac{(2+h)^3 - 2^3}{h} \\ &= \frac{4}{3}\pi \lim_{h \rightarrow 0} \frac{8 + 12h + 6h^2 + h^3 - 8}{h} \\ &= \frac{4}{3}\pi \lim_{h \rightarrow 0} (12 + 6h + h^2) \\ &= \frac{4}{3}\pi \cdot 12 \\ &= 16\pi\end{aligned}$$

The volume is changing at a rate of 16π in³/in., that is, 16π cubic inches of volume per inch of radius.

$$\begin{aligned}27. \lim_{h \rightarrow 0} \frac{s(1+h) - s(1)}{h} &= \lim_{h \rightarrow 0} \frac{1.86(1+h)^2 - 1.86(1)^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{1.86 + 3.72h + 1.86h^2 - 1.86}{h} \\ &= \lim_{h \rightarrow 0} (3.72 + 1.86h) \\ &= 3.72\end{aligned}$$

The speed of the rock is 3.72 m/sec.

$$\begin{aligned}28. \lim_{h \rightarrow 0} \frac{s(2+h) - s(2)}{h} &= \lim_{h \rightarrow 0} \frac{11.44(2+h)^2 - 11.44(2)^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{45.76 + 45.76h + 11.44h^2 - 45.76}{h} \\ &= \lim_{h \rightarrow 0} (45.76 + 11.44h) \\ &= 45.76\end{aligned}$$

The speed of the rock is 45.76m/sec.

29. First, find the slope of the tangent at
- $x = a$
- .

$$\begin{aligned}\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} &= \lim_{h \rightarrow 0} \frac{[(a+h)^2 + 4(a+h) - 1] - (a^2 + 4a - 1)}{h} \\ &= \lim_{h \rightarrow 0} \frac{a^2 + 2ah + h^2 + 4a + 4h - 1 - a^2 - 4a + 1}{h} \\ &= \lim_{h \rightarrow 0} \frac{2ah + h^2 + 4h}{h} \\ &= \lim_{h \rightarrow 0} (2a + h + 4) \\ &= 2a + 4\end{aligned}$$

The tangent at $x = a$ is horizontal when $2a + 4 = 0$, or $a = -2$. The tangent line is horizontal at $(-2, f(-2)) = (-2, -5)$.

30. First, find the slope of the tangent at
- $x = a$
- .

$$\begin{aligned}\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} &= \lim_{h \rightarrow 0} \frac{[3 - 4(a+h) - (a+h)^2] - (3 - 4a - a^2)}{h} \\ &= \lim_{h \rightarrow 0} \frac{3 - 4a - 4h - a^2 - 2ah - h^2 - 3 + 4a + a^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{-4h - 2ah - h^2}{h} \\ &= \lim_{h \rightarrow 0} (-4 - 2a - h) \\ &= -4 - 2a\end{aligned}$$

The tangent at $x = a$ is horizontal when $-4 - 2a = 0$, or $a = -2$. The tangent line is horizontal at $(-2, f(-2)) = (-2, 7)$.

31. (a) From Exercise 21, the slope of the curve at $x = a$, is

$$-\frac{1}{(a-1)^2}. \text{ The tangent has slope } -1 \text{ when}$$

$$-\frac{1}{(a-1)^2} = -1, \text{ which gives } (a-1)^2 = 1, \text{ so } a = 0 \text{ or}$$

$$a = 2. \text{ Note that } y(0) = \frac{1}{0-1} = -1 \text{ and } y(2) = \frac{1}{2-1} = 1,$$

so we need to find the equations of lines of slope -1 passing through $(0, -1)$ and $(2, 1)$, respectively.

$$\text{At } x = 0: y = -1(x-0) - 1 \\ y = -x - 1$$

$$\text{At } x = 2: y = -1(x-2) + 1 \\ y = -x + 3$$

(b) The normal has slope 1 when the tangent has slope

$$\frac{-1}{1} = -1, \text{ so we again need to find lines through}$$

$(0, -1)$ and $(2, 1)$, this time using slope 1.

$$\text{At } x = 0: y = 1(x-0) - 1 \\ y = x - 1$$

$$\text{At } x = 2: y = 1(x-2) + 1 \\ y = x - 1$$

There is only one such line. It is normal to the curve at two points and its equation is $y = x - 1$.

32. Consider a line that passes through $(1, 12)$ and a point

$(a, 9 - a^2)$ on the curve. Using the result of Exercise 22, this

line will be tangent to the curve at a if its slope is $-2a$.

$$\frac{(9 - a^2) - 12}{a - 1} = -2a \\ 9 - a^2 - 12 = -2a(a - 1) \\ -a^2 - 3 = -2a^2 + 2a \\ a^2 - 2a - 3 = 0 \\ (a + 1)(a - 3) = 0 \\ a = -1 \text{ or } a = 3$$

At $a = -1$ (or $x = -1$), the slope is $-2(-1) = 2$.

$$y = 2(x - 1) + 12 \\ y = 2x + 10$$

At $a = 3$ (or $x = 3$), the slope is $-2(3) = -6$.

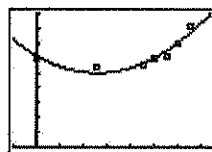
$$y = -6(x - 1) + 12 \\ y = -6x + 18$$

$$33. \text{ (a) } \frac{(272.1 - 299.3) \text{ billion}}{(1995 - 1990) \text{ years}} = \frac{-27.2 \text{ billion}}{5 \text{ years}} = -\$5.4 \frac{\text{billion}}{\text{year}}$$

$$\text{(b) } \frac{(305.5 - 294.5) \text{ billion}}{(2001 - 2000) \text{ years}} = \$11.0 \frac{\text{billion}}{\text{year}}$$

$$\text{(c) } \frac{(404.9 - 348.6) \text{ billion}}{(2003 - 2002) \text{ years}} = \$56.3 \frac{\text{billion}}{\text{year}}$$

$$\text{(d) } y = 2.177x^2 - 22.315x + 306.443$$



$[-2, 15]$ by $[0, 450]$

(e)

$$\begin{aligned} 1990 \text{ to } 1995: & \frac{(2.177(5)^2 - 22.315(5) + 306.443) - (2.177(0)^2 - 22.315(0) + 306.443)}{5 - 0} \\ & = \frac{54.425 - 111.575 + 306.443 - 306.443}{5} \\ & = \frac{-57.15}{5} = -\$11.4 \text{ billion} \end{aligned}$$

$$\begin{aligned} 2000 \text{ to } 2001: & \frac{(2.177(11)^2 - 22.315(11) + 306.443) - (2.177(10)^2 - 22.315(10) + 306.443)}{11 - 10} \\ & = \frac{263.417 - 2700.115 - 217.7 + 2231.5}{1} \\ & = \$23.4 \frac{\text{billion}}{\text{year}} \end{aligned}$$

$$\begin{aligned} 2002 \text{ to } 2003: & \frac{(2.177(13)^2 - 22.315(13) + 306.443) - (2.177(12)^2 - 22.315(12) + 306.443)}{13 - 12} \\ & = \frac{367.913 - 290.095 - 313.488 + 267.756}{1} \\ & = \$32.1 \frac{\text{billion}}{\text{year}} \end{aligned}$$

$$\begin{aligned} \text{(f) } & \frac{2.177(13.1)^2 - 22.315(13.1) + 306.443 - (2.177(13.)^2 - 22.315(13) + 306.443)}{13.1 - 13} \\ & = \frac{373.595 - 292.327 - 367.913 + 290.095}{0.1} = \$34.3 \frac{\text{billion}}{\text{year}} \end{aligned}$$

(g) One possible reason is that the war in Iraq and increased spending to prevent terrorist attacks in the U.S. caused an unusual increase in defense spending.

34. (a)



[-2, 15] by [0, 50]

(b)

$$PQ_1 = \frac{36.6 - 22.6}{2000 - 2003} = -4.7 \quad PQ_2 = \frac{26.4 - 22.6}{2003 - 2001} = -1.9$$

$$PQ_3 = \frac{22.6 - 22.0}{2003 - 2002} = 0.6$$

35. True. The normal line is perpendicular to the tangent line at the point.

36. False. There's no tangent at $x = 0$ because f is undefined at $x = 0$.

37. D. $\frac{-3-5}{-1-2} = \frac{8}{3}$

38. E. $\frac{f(3)-f(1)}{3-1} = \frac{3^2+3-1^2-1}{2} = \frac{10}{2} = 5$

39. C. $\lim_{h \rightarrow 0} \frac{f(a+h)-f(a)}{h}$
 $= \left(\frac{2}{x+h} - \frac{2}{x} \right) \frac{1}{h}$
 $= \frac{2x-2x-2h}{x^2+hx} \left(\frac{1}{h} \right)$
 $= -\frac{2}{x^2}$

$$y = -\frac{2}{(1)^2} = -2$$

$$y = m(x - x_1) + y_1$$

$$y = -2(x - 1) + 2$$

$$y = -2x + 4$$

40. A. From 39, $m_2 = -\frac{1}{m_1}$

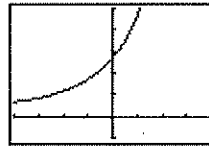
$$m_2 = \frac{1}{2}$$

$$y = \frac{1}{2}(x-1) + 2$$

$$y = \frac{1}{2}x + \frac{3}{2}$$

41. (a) $\frac{f(1+h)-f(1)}{h} = \frac{e^{1+h}-e}{h}$

(b)



[-4, 4] by [-1, 5]

X	Y1
-.01	2.7047
-.5E-4	2.7176
-.1E-4	2.7181
0	ERRRR
.1E-4	2.7184
.5E-4	2.719
.01	2.7319

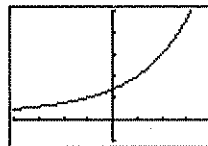
Limit ≈ 2.718

(c) They're about the same.

(d) Yes, it has a tangent whose slope is about e .

42. (a) $\frac{f(1+h)-f(1)}{h} = \frac{2^{1+h}-2}{h}$

(b)



[-4, 4] by [-1, 5]

X	Y1
-.01	1.3815
-.5E-4	1.3861
-.1E-4	1.3862
0	ERRRR
.1E-4	1.3863
.5E-4	1.3865
.01	1.3911

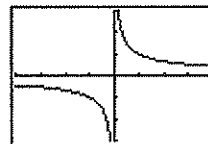
Limit ≈ 1.386

(c) They're about the same.

(d) Yes, it has a tangent whose slope is about $\ln 4$.

43. Let $f(x) = x^{2/5}$. The graph of $y = \frac{f(0+h)-f(0)}{h} = \frac{f(h)}{h}$

is shown.

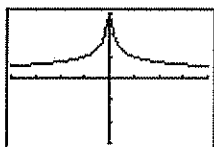


[-4, 4] by [-3, 3]

The left- and right-hand limits are $-\infty$ and ∞ , respectively.

Since they are not the same, the curve does not have a vertical tangent at $x = 0$. No.

44. Let $f(x) = x^{3/5}$. The graph of $y = \frac{f(0+h) - f(0)}{h} = \frac{f(h)}{h}$ is shown.

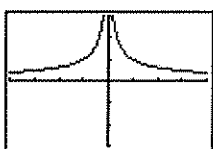


$[-4, 4]$ by $[-3, 3]$

Yes, the curve has a vertical tangent at $x = 0$ because

$$\lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h} = \infty.$$

45. Let $f(x) = x^{1/3}$. The graph of $y = \frac{f(0+h) - f(0)}{h} = \frac{f(h)}{h}$ is shown.

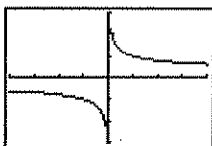


$[-4, 4]$ by $[-3, 3]$

Yes, the curve has a vertical tangent at $x = 0$ because

$$\lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h} = \infty.$$

46. Let $f(x) = x^{2/3}$. The graph of $y = \frac{f(0+h) - f(0)}{h} = \frac{f(h)}{h}$ is shown.



$[-4, 4]$ by $[-3, 3]$

The left- and right-hand limits are $-\infty$ and ∞ , respectively. Since they are not the same, the curve does not have a vertical tangent at $x = 0$. No.

47. This function has a tangent with slope zero at the origin.

It is sandwiched between two functions, $y = x^2$ and $y = -x^2$, both of which have slope zero at the origin.

Looking at the difference quotient,

$$-h \leq \frac{f(0+h) - f(0)}{h} \leq h,$$

so the Sandwich Theorem tells us the limit is 0.

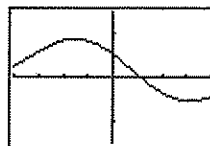
48. This function does not have a tangent line at the origin. As the function oscillates between $y = x$ and $y = -x$ infinitely often near the origin, there are an infinite number of difference quotients (secant line slopes) with a value of 1 and with a value of -1 . Thus the limit of the difference quotient doesn't exist.

The difference quotient is $\frac{f(0+h) - f(0)}{h} = \sin \frac{1}{h}$ which oscillates between 1 and -1 infinitely often near zero.

49. Let $f(x) = \sin x$. The difference quotient is

$$\frac{f(1+h) - f(1)}{h} = \frac{\sin(1+h) - \sin(1)}{h}.$$

A graph and table for the difference quotient are shown.



$[-4, 4]$ by $[-1.5, 1.5]$

X	Y
-.005	.5424
-.002	.54114
-.001	.54072
0	.53988
.001	.53888
.002	.53746
.005	.53582

$\sqrt{\text{t}} \text{ } \langle \leq \sin(1+X) - \sin(1) \rangle \leq \text{si} \dots$

Since the limit as $h \rightarrow 0$ is about 0.540, the slope of $y = \sin x$ at $x = 1$ is about 0.540.

Quick Quiz Sections 2.3 and 2.4

1. D. $\frac{f(3) - f(0)}{3 - 0} = \frac{\sqrt{3+1} - \sqrt{0+1}}{3} = \frac{2-1}{3} = \frac{1}{3}$

2. E. $f(4-h) \approx \frac{3}{4}(4) \approx 3$ where $h \rightarrow 0$

$$f(4) = 2$$

$$f(4+h) \approx -4 + 7 \approx 3$$

3. B. $\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$

$$\lim_{h \rightarrow 0} \frac{9 - (x+h)^2 - (9 - x^2)}{h}$$

$$\lim_{h \rightarrow 0} \frac{-2xh - h^2}{h} = \lim_{h \rightarrow 0} -2x - h = -2x$$

$$y = 9 - x^2 = 9 - (2)^2$$

$$y = 5$$

$$y^1 = -2(2) = -4$$

$$y = -4(x - 2) + 5$$

$$y = -4x + 13$$

$$4. (a) f(3) = 2(3) - (3)^2 \\ = 6 - 9 = -3$$

$$(b) f(3+h) = 2(3+h) - (3+h)^2 \\ = 6 + 2h - (9 + 6h + h^2) \\ = -3 - 4h - h^2$$

$$(c) \frac{f(3+h) - f(3)}{h} \\ = \frac{-3 - 4h - h^2 - (-3)}{h} \\ = \frac{-4h - h^2}{h} \\ = -4 - h$$

$$(d) -4 - h \text{ where } h \rightarrow 0 \rightarrow -4$$

Chapter 2 Review Exercises (pp. 95–97)

$$1. \lim_{x \rightarrow -2} (x^3 - 2x^2 + 1) = (-2)^3 - 2(-2)^2 + 1 = -15$$

$$2. \lim_{x \rightarrow -2} \frac{x^2 + 1}{3x^2 - 2x + 5} = \frac{(-2)^2 + 1}{3(-2)^2 - 2(-2) + 5} = \frac{5}{21}$$

3. No limit, because the expression $\sqrt{1-2x}$ is undefined for values of x near 4.

4. No limit, because the expression $\sqrt[4]{9-x^2}$ is undefined for values of x near 5.

$$5. \lim_{x \rightarrow 0} \frac{\frac{1}{2+x} - \frac{1}{2}}{x} = \lim_{x \rightarrow 0} \frac{2 - (2+x)}{2x(2+x)} = \lim_{x \rightarrow 0} \frac{-x}{2x(2+x)} \\ = \lim_{x \rightarrow 0} \left(-\frac{1}{2(2+x)} \right) = -\frac{1}{2(2+0)} = -\frac{1}{4}$$

$$6. \lim_{x \rightarrow \pm\infty} \frac{2x^2 + 3}{5x^2 + 7} = \lim_{x \rightarrow \pm\infty} \frac{2x^2}{5x^2} = \frac{2}{5}$$

7. An end behavior model for $\frac{x^4 + x^3}{12x^3 + 128}$ is $\frac{x^4}{12x^3} = \frac{1}{12}x$.

Therefore

$$\lim_{x \rightarrow \infty} \frac{x^4 + x^3}{12x^3 + 128} = \lim_{x \rightarrow \infty} \frac{1}{12}x = \infty$$

$$\lim_{x \rightarrow -\infty} \frac{x^4 + x^3}{12x^3 + 128} = \lim_{x \rightarrow -\infty} \frac{1}{12}x = -\infty$$

$$8. \lim_{x \rightarrow 0} \frac{\sin 2x}{4x} = \frac{1}{2} \lim_{x \rightarrow 0} \frac{\sin 2x}{2x} = \frac{1}{2}(1) = \frac{1}{2}$$

9. Multiply the numerator and denominator by $\sin x$.

$$\lim_{x \rightarrow 0} \frac{x \csc x + 1}{x \csc x} = \lim_{x \rightarrow 0} \frac{x + \sin x}{x} = \lim_{x \rightarrow 0} \left(1 + \frac{\sin x}{x} \right) \\ = \left(\lim_{x \rightarrow 0} 1 \right) + \left(\lim_{x \rightarrow 0} \frac{\sin x}{x} \right) = 1 + 1 = 2$$

$$10. \lim_{x \rightarrow 0} e^x \sin x = e^0 \sin 0 = 1 \cdot 0 = 0$$

11. Let $x = \frac{7}{2} + h$, where h is in $\left(0, \frac{1}{2}\right)$. Then

$$\text{int}(2x-1) = \text{int} \left[2 \left(\frac{7}{2} \right) + 2h - 1 \right] = \text{int}(6+2h) = 6, \text{ because } 6 \\ + 2h \text{ is in } (6, 7).$$

$$\text{Therefore, } \lim_{x \rightarrow 7/2^+} \text{int}(2x-1) = \lim_{x \rightarrow 7/2^+} 6 = 6.$$

12. Let $x = \frac{7}{2} + h$, where h is in $\left(-\frac{1}{2}, 0\right)$. Then

$$\text{int}(2x-1) = \text{int} \left[2 \left(\frac{7}{2} \right) + 2h - 1 \right] = \text{int}(6+2h) = 5, \text{ because } 6 \\ + 2h \text{ is in } (5, 6).$$

$$\text{Therefore, } \lim_{x \rightarrow 7/2^-} \text{int}(2x-1) = \lim_{x \rightarrow 7/2^-} 5 = 5$$

13. Since $\lim_{x \rightarrow \infty} (-e^{-x}) = \lim_{x \rightarrow \infty} e^{-x} = 0$, and $-e^{-x} \leq e^{-x} \cos x \leq$

e^{-x} for all x , the Sandwich Theorem gives

$$\lim_{x \rightarrow \infty} e^{-x} \cos x = 0.$$

14. Since the expression x is an end behavior model for both

$$x + \sin x \text{ and } x + \cos x, \lim_{x \rightarrow \infty} \frac{x + \sin x}{x + \cos x} = \lim_{x \rightarrow \infty} \frac{x}{x} = 1.$$

15. Limit exists.

16. Limit exists.

17. Limit exists.

18. Limit does not exist.

19. Limit exists.

20. Limit exists.

21. Yes

22. No

23. No

24. Yes

$$25. (a) \lim_{x \rightarrow 3^-} g(x) = 1$$

$$(b) g(3) = 1.5$$

$$(c) \text{No, since } \lim_{x \rightarrow 3^-} g(x) \neq g(3).$$

(d) g is discontinuous at $x = 3$ (and at points not in the domain).

(e) Yes, the discontinuity at $x = 3$ can be removed by assigning the value 1 to $g(3)$.

26. (a) $\lim_{x \rightarrow 1^-} k(x) = 1.5$

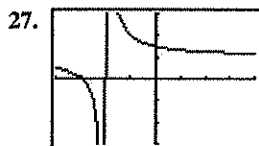
(b) $\lim_{x \rightarrow 1^+} k(x) = 0$

(c) $k(1) = 0$

(d) No, since $\lim_{x \rightarrow 1} k(x) \neq k(1)$

(e) k is discontinuous at $x = 1$ (and at points not in the domain).

(f) No, the discontinuity at $x = 1$ is not removable because the one-sided limits are different.

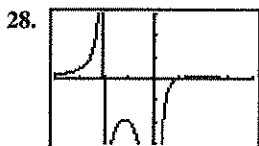


$[-4, 4]$ by $[-3, 3]$

(a) Vertical asymptote: $x = -2$

(b) Left-hand limit = $\lim_{x \rightarrow -2^-} \frac{x+3}{x+2} = -\infty$

Right-hand limit: $\lim_{x \rightarrow -2^+} \frac{x+3}{x+2} = \infty$



$[-4, 4]$ by $[-3, 3]$

(a) Vertical asymptotes: $x = 0, x = -2$

(b) At $x = 0$:

Left-hand limit = $\lim_{x \rightarrow 0^-} \frac{x-1}{x^2(x+2)} = -\infty$

Right-hand limit = $\lim_{x \rightarrow 0^+} \frac{x-1}{x^2(x+2)} = -\infty$

At $x = -2$:

Left-hand limit = $\lim_{x \rightarrow -2^-} \frac{x-1}{x^2(x+2)} = \infty$

Right-hand limit = $\lim_{x \rightarrow -2^+} \frac{x-1}{x^2(x+2)} = -\infty$

29. (a) At $x = -1$:

Left-hand limit = $\lim_{x \rightarrow -1^-} f(x) = \lim_{x \rightarrow -1^-} (1) = 1$

Right-hand limit = $\lim_{x \rightarrow -1^+} f(x) = \lim_{x \rightarrow -1^+} (-x) = 1$

At $x = 0$:

Left-hand limit = $\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} (-x) = 0$

Right-hand limit = $\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} (-x) = 0$

At $x = 1$:

Left-hand limit = $\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} (-x) = -1$

Right-hand limit = $\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (1) = 1$

(b) At $x = -1$: Yes, the limit is 1.

At $x = 0$: Yes, the limit is 0.

At $x = 1$: No, the limit doesn't exist because the two one-sided limits are different.

(c) At $x = -1$: Continuous because $f(-1) =$ the limit.

At $x = 0$: Discontinuous because $f(0) \neq$ the limit.

At $x = 1$: Discontinuous because the limit does not exist.

30. (a) Left-hand limit = $\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} |x^3 - 4x|$
 $= |(1)^3 - 4(1)| = |-3| = 3$

Right-hand limit = $\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (x^2 - 2x - 2)$
 $= (1)^2 - 2(1) - 2 = -3$

(b) No, because the two one-sided limits are different.

(c) Every place except for $x = 1$

(d) At $x = 1$

31. Since $f(x)$ is a quotient of polynomials, it is continuous and its points of discontinuity are the points where it is undefined, namely $x = -2$ and $x = 2$.

32. There are no points of discontinuity, since $g(x)$ is continuous and defined for all real numbers.

33. (a) End behavior model: $\frac{2x}{x^2}$, or $\frac{2}{x}$

(b) Horizontal asymptote: $y = 0$ (the x -axis)

34. (a) End behavior model: $\frac{2x^2}{x^2}$, or 2

(b) Horizontal asymptote: $y = 2$

35. (a) End behavior model: $\frac{x^3}{x}$, or x^2

(b) Since the end behavior model is quadratic, there are no horizontal asymptotes.

36. (a) End behavior model: $\frac{x^4}{x^3}$, or x

(b) Since the end behavior model represents a nonhorizontal line, there are no horizontal asymptotes.

37. (a) Since $\lim_{x \rightarrow \infty} \frac{x+e^x}{e^x} = \lim_{x \rightarrow \infty} \left(\frac{x}{e^x} + 1 \right) = 1$, a right end behavior model is e^x .

(b) Since $\lim_{x \rightarrow -\infty} \frac{x+e^x}{x} = \lim_{x \rightarrow -\infty} \left(1 + \frac{e^x}{x} \right) = 1$, a left end behavior model is x .

38. (a, b) Note that $\lim_{x \rightarrow \pm\infty} \left(-\frac{1}{\ln|x|} \right) = \lim_{x \rightarrow \pm\infty} \left(\frac{1}{\ln|x|} \right) = 0$ and

$$-\frac{1}{\ln|x|} < \frac{\sin x}{\ln|x|} < \frac{1}{\ln|x|} \text{ for all } x \neq 0.$$

Therefore, the Sandwich Theorem gives

$$\lim_{x \rightarrow \pm\infty} \frac{\sin x}{\ln|x|} = 0. \text{ Hence}$$

$$\lim_{x \rightarrow \pm\infty} \frac{\ln|x| + \sin x}{\ln|x|} = \lim_{x \rightarrow \pm\infty} \left(1 + \frac{\sin x}{\ln|x|} \right) = 1 + 0 = 1,$$

so $\ln|x|$ is both a right end behavior model and a left end behavior model.

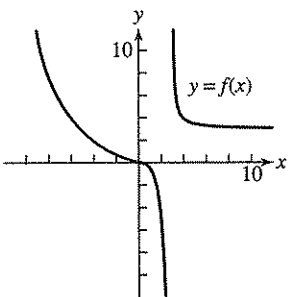
39. $\lim_{x \rightarrow 3} f(x) = \lim_{x \rightarrow 3} \frac{x^2 + 2x - 15}{x - 3} = \lim_{x \rightarrow 3} \frac{(x-3)(x+5)}{x-3}$
 $= \lim_{x \rightarrow 3} (x+5) = 3+5 = 8.$

Assign the value $k = 8$.

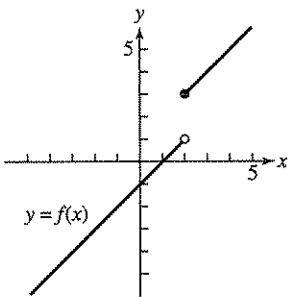
40. $\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} \frac{\sin x}{2x} = \frac{1}{2} \lim_{x \rightarrow 0} \frac{\sin x}{x} = \frac{1}{2}(1) = \frac{1}{2}$

Assign the value $k = \frac{1}{2}$.

41. One possible answer:



42. One possible answer:



43. $\frac{f(\pi/2) - f(0)}{\pi/2 - 0} = \frac{2 - 1}{\pi/2} = \frac{2}{\pi}$

44. $\lim_{h \rightarrow 0} \frac{V(a+h) - V(a)}{h} = \lim_{h \rightarrow 0} \frac{\frac{1}{3}\pi(a+h)^2 H - \frac{1}{3}\pi a^2 H}{h}$
 $= \frac{1}{3}\pi H \lim_{h \rightarrow 0} \frac{a^2 + 2ah + h^2 - a^2}{h}$
 $= \frac{1}{3}\pi H \lim_{h \rightarrow 0} (2a + h)$
 $= \frac{1}{3}\pi H(2a)$
 $= \frac{2}{3}\pi aH$

45. $\lim_{h \rightarrow 0} \frac{S(a+h) - S(a)}{h} = \lim_{h \rightarrow 0} \frac{6(a+h)^2 - 6a^2}{h}$
 $= \lim_{h \rightarrow 0} \frac{6a^2 + 12ah + 6h^2 - 6a^2}{h}$
 $= \lim_{h \rightarrow 0} (12a + 6h)$
 $= 12a$

46. $\lim_{h \rightarrow 0} \frac{y(a+h) - y(a)}{h}$
 $= \lim_{h \rightarrow 0} \frac{[(a+h)^2 - (a+h) - 2] - (a^2 - a - 2)}{h}$
 $= \lim_{h \rightarrow 0} \frac{a^2 + 2ah + h^2 - a - h - 2 - a^2 + a + 2}{h}$
 $= \lim_{h \rightarrow 0} \frac{2ah + h^2 - h}{h}$
 $= \lim_{h \rightarrow 0} (2a + h - 1)$
 $= 2a - 1$

47. (a) $\lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \rightarrow 0} \frac{[(1+h)^2 - 3(1+h)] - (-2)}{h}$
 $= \lim_{h \rightarrow 0} \frac{1 + 2h + h^2 - 3 - 3h + 2}{h}$
 $= \lim_{h \rightarrow 0} (-1 + h)$
 $= -1$

(b) The tangent at P has slope -1 and passes through $(1, -2)$.

$$y = -1(x-1) - 2$$

$$y = -x - 1$$

(c) The normal at P has slope 1 and passes through $(1, -2)$.

$$y = 1(x-1) - 2$$

$$y = x - 3$$

48. At $x = a$, the slope of the curve is

$$\begin{aligned}\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} &= \lim_{h \rightarrow 0} \frac{[(a+h)^2 - 3(a+h)] - (a^2 - 3a)}{h} \\ &= \lim_{h \rightarrow 0} \frac{a^2 + 2ah + h^2 - 3a - 3h - a^2 + 3a}{h} \\ &= \lim_{h \rightarrow 0} \frac{2ah - 3h + h^2}{h} \\ &= \lim_{h \rightarrow 0} (2a - 3 + h) \\ &= 2a - 3\end{aligned}$$

The tangent is horizontal when $2a - 3 = 0$, at

$$a = \frac{3}{2} \left(\text{or } x = \frac{3}{2} \right). \text{ Since } f\left(\frac{3}{2}\right) = -\frac{9}{4}, \text{ the point where this}$$

occurs is $\left(\frac{3}{2}, -\frac{9}{4}\right)$.

49. (a) $p(0) = \frac{200}{1 + 7e^{-0.1(0)}} = \frac{200}{8} = 25$

Perhaps this is the number of bears placed in the reserve when it was established.

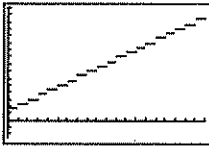
(b) $\lim_{t \rightarrow \infty} p(t) = \lim_{t \rightarrow \infty} \frac{200}{1 + 7e^{-0.1t}} = \frac{200}{1} = 200$

(c) Perhaps this is the maximum number of bears which the reserve can support due to limitations of food, space, or other resources. Or, perhaps the number is capped at 200 and excess bears are moved to other locations.

50. (a) $f(x) = \begin{cases} 3.20 - 1.35 \text{int}(-x+1), & 0 < x \leq 20 \\ 0, & x = 0 \end{cases}$

(Note that we cannot use the formula $f(x) = 3.20 + 1.35 \text{int } x$, because it gives incorrect results when x is an integer.)

(b)



$[0, 20]$ by $[-5, 32]$

f is discontinuous at integer values of x : $0, 1, 2, \dots, 19$.

51. (a)

$[5, 20]$ by $[15000, 18000]$

(b) $PQ_1 = \frac{17,019 - 15,487}{2003 - 1998} = \frac{1532}{5} = 306.4$

$PQ_2 = \frac{17,019 - 15,759}{2003 - 1999} = \frac{1260}{4} = 315$

$PQ_3 = \frac{17,019 - 15,983}{2003 - 2000} = \frac{1036}{3} = 345.3$

$PQ_4 = \frac{17,019 - 16,355}{2003 - 2001} = \frac{664}{2} = 332$

$PQ_5 = \frac{17,019 - 16,692}{2003 - 2002} = \frac{327}{1} = 327$

(c) We use the average rate of change in the population from 2002 to 2003 which is 327,000

(d) $y = 309.457x + 12966.533$, rate of change is 309 thousand because rate of change of a linear function is its slope.

52. Let $A = \lim_{x \rightarrow c} f(x)$ and $B = \lim_{x \rightarrow c} g(x)$. Then $A + B = 2$ and

$A - B = 1$. Adding, we have $2A = 3$, so $A = \frac{3}{2}$, whence

$\frac{3}{2} + B = 2$, which gives $B = \frac{1}{2}$. Therefore, $\lim_{x \rightarrow c} f(x) = \frac{3}{2}$

and $\lim_{x \rightarrow c} g(x) = \frac{1}{2}$.

53. (a) $x - 9 \neq 0$

All x not equal to -3 or 3 .

(b) $x = -3, x = 3$

(c) $\lim_{x \rightarrow \infty} \frac{x}{|x^2 - 9|} = 0$
 $y = 0$

(d) When $x - 9 \rightarrow 0, f(x) \rightarrow \infty$.

$x = -3$ and $x = 3$ are discontinuous.

(e) Yes. It is continuous at every point in its domain.

$$\begin{aligned} 54. \text{ (a) } f(2) &= x^2 - a^2 - x \\ &= (2)^2 - a^2 - 2 \\ &= 4 - 2a^2 \end{aligned}$$

$$\begin{aligned} \text{ (b) } f(2) &= 4 - 2x^2 \\ &= 4 - 2(2)^2 \\ &= 4 - 8 = -4 \end{aligned}$$

(c) For $x \neq 2$, f is continuous. For $x = 2$, we have

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^+} f(x) = f(2) = -4 \text{ as long as } a = \pm 2.$$

$$55. \text{ (a) } g(x) = \frac{x^3}{x^2} = x$$

$$\begin{aligned} \text{ (b) } \frac{f(x)}{g(x)} &= \frac{x^3 - 2x^2 + 1}{x^2 + 3} = \frac{1}{x} \\ &= \frac{x^3 - 2x^2 + 1}{x^3 + 3x} \\ &= \frac{x^3}{x^3} = 1 \end{aligned}$$

Chapter 3

Derivatives

Section 3.1 Derivative of a Function

(pp. 99–108)

Exploration 1 Reading the Graphs

- The graph in Figure 3.3b represents the rate of change of the depth of the water in the ditch with respect to time. Since y is measured in inches and x is measured in days, the derivative $\frac{dy}{dx}$ would be measured in inches per day. Those are the units that should be used along the y -axis in Figure 3.3b.
- The water in the ditch is 1 inch deep at the start of the first day and rising rapidly. It continues to rise, at a gradually decreasing rate, until the end of the second day, when it achieves a maximum depth of 5 inches. During days 3, 4, 5, and 6, the water level goes down, until it reaches a depth of 1 inch at the end of day 6. During the seventh day it rises again, almost to a depth of 2 inches.
- The weather appears to have been wettest at the beginning of day 1 (when the water level was rising fastest) and driest at the end of day 4 (when the water level was declining the fastest).
- The highest point on the graph of the derivative shows where the water is rising the fastest, while the lowest point (most negative) on the graph of the derivative shows where the water is declining the fastest.

5. The y -coordinate of point C gives the maximum depth of the water level in the ditch over the 7-day period, while the x -coordinate of C gives the time during the 7-day period that the maximum depth occurred. The derivative of the function changes sign from positive to negative at C , indicating that this is when the water level stops rising and begins falling.

6. Water continues to run down sides of hills and through underground streams long after the rain has stopped falling. Depending on how much high ground is located near the ditch, water from the first day's rain could still be flowing into the ditch several days later. Engineers responsible for flood control of major rivers must take this into consideration when they predict when floodwaters will "crest," and at what levels.

Quick Review 3.1

$$\begin{aligned} 1. \lim_{h \rightarrow 0} \frac{(2+h)^2 - 4}{4} &= \lim_{h \rightarrow 0} \frac{(4+4h+h^2)-4}{h} \\ &= \lim_{h \rightarrow 0} 4+h \\ &= 4+0 = 4 \end{aligned}$$

$$2. \lim_{x \rightarrow 2^+} \frac{x+3}{2} = \frac{2+3}{2} = \frac{5}{2}$$

$$3. \text{ Since } \frac{|y|}{y} = -1 \text{ for } y < 0, \lim_{y \rightarrow 0^-} \frac{|y|}{y} = -1.$$

$$\begin{aligned} 4. \lim_{x \rightarrow 4} \frac{2x-8}{\sqrt{x}-2} &= \lim_{x \rightarrow 4} \frac{2(\sqrt{x}+2)(\sqrt{x}-2)}{\sqrt{x}-2} \\ &= \lim_{h \rightarrow 4} 2(\sqrt{x}+2) = 2(\sqrt{4}+2) = 8 \end{aligned}$$

5. The vertex of the parabola is at $(0, 1)$. The slope of the line through $(0, 1)$ and another point $(h, h^2 + 1)$ on the parabola is $\frac{(h^2 + 1) - 1}{h - 0} = h$. Since $\lim_{h \rightarrow 0} h = 0$, the slope of the line tangent to the parabola at its vertex is 0.

6. Use the graph of f in the window $[-6, 6]$ by $[-4, 4]$ to find that $(0, 2)$ is the coordinate of the high point and $(2, -2)$ is the coordinate of the low point. Therefore, f is increasing on $(-\infty, 0]$ and $[2, \infty)$.

$$7. \lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (x-1)^2 = (1-1)^2 = 0$$

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} (x+2) = 1+2 = 3$$

$$8. \lim_{h \rightarrow 0^+} f(1+h) = \lim_{x \rightarrow 1^+} f(x) = 0$$

9. No, the two one-sided limits are different (see Exercise 7).

10. No, f is discontinuous at $x = 1$ because $\lim_{x \rightarrow 1} f(x)$ does not exist.