33. **(a)** Velocity: $s'(t) = -2\cos t$ m/sec

Speed: $|s'(t)| = |2\cos t|$ m/sec

Acceleration: $s''(t) = 2\sin t$ m/sec^2

Jerk: $s'''(t) = 2\cos t$ m/sec^3

(b) Velocity: $-2\cos\frac{\pi}{4} = -\sqrt{2}$ m/sec

Speed: $|-\sqrt{2}| = \sqrt{2}$ m/sec

Acceleration: $2\sin\frac{\pi}{4} = \sqrt{2}$ m/sec^2

Jerk: $2\cos\frac{\pi}{4} = \sqrt{2}$ m/sec^3

(c) The body starts at 2, goes to 0 and then oscillates between 0 and 4.

Speed:

Greatest when $\cos t = \pm 1$ (or $t = k\pi$), at the center of the interval of motion.

Zero when $\cos t = 0$ (or $t = \frac{k\pi}{2}$, k odd), at the endpoints of the interval of motion.

Acceleration:

Greatest (in magnitude) when $\sin t = \pm 1$ (or $t = \frac{k\pi}{2}$, k odd)

Zero when $\sin t = 0$ (or $t = k\pi$)

Jerk:

Greatest (in magnitude) when $\cos t = \pm 1$ (or $t = k\pi$).

Zero when $\cos t = 0$ (or $t = \frac{k\pi}{2}$, k odd)

34. **(a)** Velocity: $s'(t) = \cos t - \sin t$ m/sec

Speed: $s'(t) = |\cos t - \sin t|$ m/sec

Acceleration: $s''(t) = -\sin t - \cos t$ m/sec^2

Jerk: $s'''(t) = -\cos t - \sin t$ m/sec^3

(b) Velocity: $\cos\frac{\pi}{4} - \sin\frac{\pi}{4} = 0$ m/sec

Speed: $|0| = 0$ m/sec

Acceleration: $-\sin\frac{\pi}{4} - \cos\frac{\pi}{4} = -\sqrt{2}$ m/sec^2

Jerk: $-\cos\frac{\pi}{4} + \sin\frac{\pi}{4} = 0$ m/sec^3

(c) The body starts at 1, goes to $\sqrt{2}$ and then oscillates between $\pm\sqrt{2}$.

Speed:

Greatest when $t = \frac{3\pi}{4} + k\pi$

Zero when $t = \frac{\pi}{4} + k\pi$

Acceleration:

Greatest (in magnitude) when $t = \frac{\pi}{4} + k\pi$

Zero when $t = \frac{3\pi}{4} + k\pi$

Jerk:

Greatest (in magnitude) when $t = \frac{3\pi}{4} + k\pi$

Zero when $t = \frac{\pi}{4} + k\pi$

35. $y' = \frac{d}{dx}\csc x = -\csc x \cot x$

$$\begin{aligned}
y'' &= \frac{d}{dx}(-\csc x \cot x)\\
&= -(\csc x)\frac{d}{dx}(\cot x) - (\cot x)\frac{d}{dx}(\csc x)\\
&= -(\csc x)(-\csc^2 x) - (\cot x)(-\csc x \cot x)\\
&= \csc^3 x + \csc x \cot^2 x
\end{aligned}$$

36. $y' = \frac{d}{d\theta}(\theta \tan\theta)$

$$\begin{aligned}
&= \theta\frac{d}{d\theta}(\tan\theta) + (\tan\theta)\frac{d}{d\theta}(\theta)\\
&= \theta\sec^2\theta + \tan\theta
\end{aligned}$$

$$\begin{aligned}
y'' &= \frac{d}{d\theta}(\theta\sec^2\theta + \tan\theta)\\
&= \theta\frac{d}{d\theta}[(\sec\theta)(\sec\theta)] + (\sec^2\theta)\frac{d}{d\theta}(\theta) + \frac{d}{d\theta}(\tan\theta)\\
&= \theta\left[(\sec\theta)\frac{d}{d\theta}(\sec\theta) + (\sec\theta)\frac{d}{d\theta}(\sec\theta)\right] + \sec^2\theta + \sec^2\theta\\
&= 2\theta\sec^2\theta\tan\theta + 2\sec^2\theta\\
&= (2\theta\tan\theta + 2)(\sec^2\theta)
\end{aligned}$$

or, writing in terms of sines and cosines,

$$= \frac{2 + 2\theta\tan\theta}{\cos^2\theta}$$

$$= \frac{2\cos\theta + 2\theta\sin\theta}{\cos^3\theta}$$

37. Continuous:

Note that $g(0) = \lim_{x\to 0^+} g(x) = \lim_{x\to 0^+} \cos x = \cos(0) = 1$, and $\lim_{x\to 0^-} g(x) = \lim_{x\to 0^-}(x + b) = b$. We require $\lim_{x\to 0^-} g(x) = g(0)$, so $b = 1$. The function is continuous if $b = 1$.

Differentiable:

For $b = 1$, the left-hand derivative is 1 and the right-hand derivative is $-\sin(0) = 0$, so the function is not differentiable. For other values of b, the function is discontinuous at $x = 0$ and there is no left-hand derivative. So, there is no value of b that will make the function differentiable at $x = 0$.

38. Observe the pattern:

$\frac{d}{dx}\cos x = -\sin x$ $\quad \frac{d^5}{dx^5}\cos x = -\sin x$

$\frac{d^2}{dx^2}\cos x = -\cos x$ $\quad \frac{d^6}{dx^6}\cos x = -\cos x$

$\frac{d^3}{dx^3}\cos x = \sin x$ $\quad \frac{d^7}{dx^7}\cos x = \sin x$

$\frac{d^4}{dx^4}\cos x = \cos x$ $\quad \frac{d^8}{dx^8}\cos x = \cos x$

Continuing the pattern, we see that

$\frac{d^n}{dx^n}\cos x = \sin x$ when $n = 4k + 3$ for any whole number k.

Since $999 = 4(249) + 3$, $\frac{d^{999}}{dx^{999}}\cos x = \sin x$.

39. Observe the pattern:

$\frac{d}{dx}\sin x = \cos x$ $\quad \frac{d^5}{dx^5}\sin x = \cos x$

$\frac{d^2}{dx^2}\sin x = -\sin x$ $\quad \frac{d^6}{dx^6}\sin x = -\sin x$

$\frac{d^3}{dx^3}\sin x = -\cos x$ $\quad \frac{d^7}{dx^7}\sin x = -\cos x$

$\frac{d^4}{dx^4}\sin x = \sin x$ $\quad \frac{d^8}{dx^8}\sin x = \sin x$

Continuing the pattern, we see that

$\frac{d^n}{dx^n}\sin x = \cos x$ when $n = 4k+1$ for any whole number k.

Since $725 = 4(181) + 1$, $\frac{d^{725}}{dx^{725}}\sin x = \cos x$.

40. The line is tangent to the graph of $y = \sin x$ at (0, 0). Since $y'(0) = \cos(0) = 1$, the line has slope 1 and its equation is $y = x$.

41. (a) Using $y = x$, $\sin(0.12) \approx 0.12$.

(b) $\sin(0.12) \approx 0.1197122$; The approximation is within 0.0003 of the actual value.

42.
$$\begin{aligned}\frac{d}{dx}\sin 2x &= \frac{d}{dx}(2\sin x\cos x)\\ &= 2\frac{d}{dx}(\sin x\cos x)\\ &= 2\left[(\sin x)\frac{d}{dx}(\cos x) + (\cos x)\frac{d}{dx}(\sin x)\right]\\ &= 2[(\sin x)(-\sin x) + (\cos x)(\cos x)]\\ &= 2(\cos^2 x - \sin^2 x)\\ &= 2\cos 2x\end{aligned}$$

43.
$$\begin{aligned}\frac{d}{dx}\cos 2x &= \frac{d}{dx}[(\cos x)(\cos x) - (\sin x)(\sin x)]\\ &= \left[(\cos x)\frac{d}{dx}(\cos x) + (\cos x)\frac{d}{dx}(\cos x)\right] - \\ &\quad \left[(\sin x)\frac{d}{dx}(\sin x) + (\sin x)\frac{d}{dx}(\sin x)\right]\\ &= 2(\cos x)(-\sin x) - 2(\sin x)(\cos x)\\ &= -4\sin x\cos x\\ &= -2(2\sin x\cos x)\\ &= -2\sin 2x\end{aligned}$$

44. True. The derivative is positive at $t = \frac{3\pi}{4}$.

45. False. The velocity is negative and the speed is positive at $t = \frac{\pi}{4}$.

46. A. $\quad y = \sin x + \cos x$

$$\begin{aligned}\frac{dy}{dx_1} &= \frac{d}{dx} = \cos x - \sin x\\ y(\pi) &= \sin\pi + \cos\pi = -1\\ y'(\pi) &= \cos\pi - \sin\pi = -1\\ y &= -1(x-\pi) - 1\\ y &= -x + \pi - 1\end{aligned}$$

47. B. See 46.

$$\begin{aligned}m_2 &= -\frac{1}{m_1} = -\frac{1}{-1} = 1\\ y &= (x-\pi) - 1\end{aligned}$$

48. C. $\quad y = x\sin x$

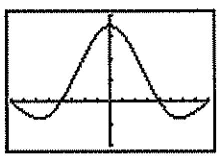
$$\begin{aligned}y' &= \sin x + x\cos x\\ y'' &= \cos x + \cos x - x\sin x\\ &= -x\sin x + 2\cos x\end{aligned}$$

49. C. $v(t) = \frac{ds}{dt} = \frac{d}{dt}(3 + \sin t)$

$v(t) = \cos t = 0$

$t = \frac{\pi}{2}$

50. (a)

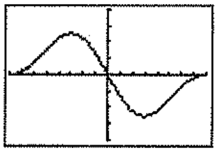


[–360, 360] by [–0.01, 0.02]

The limit is $\frac{\pi}{180}$ because this is the conversion factor for changing from degrees to radians.

50. Continued

(b)



[–360, 360] by [–0.02, 0.02]

This limit is still 0.

(c) $\frac{d}{dx}\sin x = \lim_{h\to 0}\frac{\sin(x+h)-\sin x}{h}$

$= \lim_{h\to 0}\frac{\sin x\cos h + \cos x \sin h - \sin x}{h}$

$= \lim_{h\to 0}\frac{\sin x(\cos h - 1) + \cos x \sin h}{h}$

$= \left(\lim_{h\to 0}\sin x\right)\left(\lim_{h\to 0}\frac{\cos h - 1}{h}\right) + \left(\lim_{h\to 0}\cos x\right)\left(\lim_{h\to 0}\frac{\sin h}{h}\right)$

$= (\sin x)(0) + (\cos x)\left(\frac{\pi}{180}\right)$

$= \frac{\pi}{180}\cos x$

(d) $\frac{d}{dx}\cos x = \lim_{h\to 0}\frac{\cos(x+h)-\cos x}{h}$

$= \lim_{h\to 0}\frac{\cos x\cos h - \sin x\sin h - \cos x}{h}$

$= \lim_{h\to 0}\frac{(\cos x)(\cos h - 1) - \sin x\sin h}{h}$

$= \left(\lim_{h\to 0}\cos x\right)\left(\lim_{h\to 0}\frac{\cos h - 1}{h}\right) - \left(\lim_{h\to 0}\sin x\right)\left(\lim_{h\to 0}\frac{\sin h}{h}\right)$

$= (\cos x)(0) - (\sin x)\left(\frac{\pi}{180}\right)$

$= -\frac{\pi}{180}\sin x$

(e) $\frac{d^2}{dx^2}\sin x = \frac{d}{dx}\frac{\pi}{180}\cos x = \frac{\pi}{180}\left(-\frac{\pi}{180}\sin x\right)$

$= -\frac{\pi^2}{180^2}\sin x$

$\frac{d^3}{dx^3}\sin x = \frac{d}{dx}\left(-\frac{\pi^2}{180^2}\sin x\right) = -\frac{\pi^2}{180^2}\left(\frac{\pi}{180}\cos x\right)$

$= -\frac{\pi^3}{180^3}\cos x$

$\frac{d^2}{dx^2}\cos x = \frac{d}{dx}\left(-\frac{\pi}{180}\sin x\right) = -\frac{\pi}{180}\left(\frac{\pi}{180}\cos x\right)$

$= -\frac{\pi^2}{180^2}\cos x$

$\frac{d^3}{dx^3}\cos x = \frac{d}{dx}\left(-\frac{\pi^2}{180^2}\cos x\right) = -\frac{\pi^2}{180^2}\left(-\frac{\pi}{180}\sin x\right)$

$= \frac{\pi^3}{180^3}\sin x$

51. $\lim_{h\to 0}\frac{(\cos h - 1)}{h} = \lim_{h\to 0}\frac{(\cos h - 1)(\cos h + 1)}{h(\cos h + 1)}$

$= \lim_{h\to 0}\frac{\cos^2 h - 1}{h(\cos h + 1)}$

$= \lim_{h\to 0}\frac{-\sin^2 h}{h(\cos h + 1)}$

$= -\left(\lim_{h\to 0}\frac{\sin h}{h}\right)\left(\lim_{h\to 0}\frac{\sin h}{\cos h + 1}\right)$

$= -(1)\left(\frac{0}{2}\right) = 0$

52. $y' = \frac{d}{dx}(A\sin x + B\cos x) = A\cos x - B\sin x$

$y'' = \frac{d}{dx}(A\cos x - B\sin x) = -A\sin x - B\cos x$

Solve: $y'' - y = \sin x$

$(-A\sin x - B\cos x) - (A\sin x + B\cos x) = \sin x$

$-2A\sin x - 2B\cos x = \sin x$

At $x = \frac{\pi}{2}$, this gives $-2A = 1$, so $A = -\frac{1}{2}$.

At $x = 0$, we have $-2B = 0$, so $B = 0$.

Thus, $A = -\frac{1}{2}$ and $B = 0$.

Section 3.6 Chain Rule (pp. 148–156)

Quick Review 3.6

1. $f(g(x)) = f(x^2+1) = \sin(x^2+1)$

2. $f(g(h(x))) = f(g(7x)) = f((7x)^2+1)$
   $= \sin[(7x)^2+1] = \sin(49x^2+1)$

3. $(g \circ h)(x) = g(h(x)) = g(7x) = (7x)^2 + 1 = 49x^2 + 1$

4. $(h \circ g)(x) = h(g(x)) = h(x^2+1) = 7(x^2+1) = 7x^2 + 7$

5. $f\left(\frac{g(x)}{h(x)}\right) = f\left(\frac{x^2+1}{7x}\right) = \sin\frac{x^2+1}{7x}$

6. $\sqrt{\cos x + 2} = g(\cos x) = g(f(x))$

7. $\sqrt{3\cos^2 x + 2} = g(3\cos^2 x) = g(h(\cos x)) = g(h(f(x)))$

8. $3\cos x + 6 = 3(\cos x + 2) = 3(\sqrt{\cos x + 2})^2$
   $= h(\sqrt{\cos x + 2}) = h(g(\cos x)) = h(g(f(x)))$

9. $\cos 27x^4 = f(27x^4) = f(3(3x^2)^2) = f(h(3x^2)) = f(h(h(x)))$

10. $\cos\sqrt{2+3x^2} = \cos\sqrt{3x^2+2} = f(\sqrt{3x^2+2})$
    $= f(g(3x^2)) = f(g(h(x)))$

Section 3.6 Exercises

1. $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$

$y=\sin u \qquad u=3x+1$

$\frac{dy}{du}=\cos u \qquad \frac{du}{dx}=3$

$\frac{dy}{dx}=3\cos(3x+1)$

2. $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$

$y=\sin u \qquad u=7-5x$

$\frac{dy}{du}=\cos u \qquad \frac{du}{dx}=-5$

$\frac{dy}{dx}=-5\cos(7-5x)$

3. $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$

$y=\cos u \qquad u=\sqrt{3}x$

$\frac{dy}{du}=-\sin u \qquad \frac{du}{dx}=\sqrt{3}$

$\frac{dy}{dx}=-\sqrt{3}\sin(\sqrt{3}x)$

4. $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$

$y=\tan u \qquad u=2x-x^3$

$\frac{dy}{du}=\sec^2 u \qquad \frac{du}{dx}=2-3x^2$

$\frac{dy}{dx}=(2-3x^2)\sec^2(2x-x^3)$

5. $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$

$y=u^2 \qquad u=\frac{\sin x}{1+\cos x}$

$\frac{dy}{du}=2u \qquad \frac{du}{dx}=\frac{\sin x}{(1+\cos x)^2}$

$\frac{dy}{dx}=\frac{2\sin x}{(1+\cos x)^2}$

6. $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$

$y=5\cot u \qquad u=\frac{2}{x}$

$\frac{dy}{du}=-5\csc^2 u \qquad \frac{du}{dx}=-\frac{2}{x^2}$

$\frac{dy}{dx}=\frac{10}{x^2}\csc^2\left(\frac{2}{x}\right)$

7. $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$

$y=\cos u \qquad u=\sin x$

$\frac{dy}{du}=-\sin u \qquad \frac{du}{dx}=\cos x$

$\frac{dy}{dx}=-\sin(\sin x)\cos x$

8. $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$

$y=\sec u$

$\frac{dy}{du}=\sec u\tan u$

$u=\tan x$

$\frac{du}{dx}=\sec^2 x$

$\frac{dy}{dx}=\sec(\tan x)\tan(\tan x)\sec^2 x$

9. $\frac{ds}{dt}=\frac{d}{dt}\cos\left(\frac{\pi}{2}-3t\right)$

$=\left[-\sin\left(\frac{\pi}{2}-3t\right)\right]\frac{d}{dt}\left(\frac{\pi}{2}-3t\right)$

$=\left[-\sin\left(\frac{\pi}{2}-3t\right)\right](-3)$

$=3\sin\left(\frac{\pi}{2}-3t\right)$

10. $\frac{ds}{dt}=\frac{d}{dt}[t\cos(\pi-4t)]$

$=(t)\frac{d}{dt}[\cos(\pi-4t)]+\cos(\pi-4t)\frac{d}{dt}(t)$

$=t[-\sin(\pi-4t)]\frac{d}{dt}(\pi-4t)+\cos(\pi-4t)(1)$

$=t[-\sin(\pi-4t)](-4)+\cos(\pi-4t)$

$=4t\sin(\pi-4t)+\cos(\pi-4t)$

11. $\frac{ds}{dt}=\frac{d}{dt}\left(\frac{4}{3\pi}\sin 3t+\frac{4}{5\pi}\cos 5t\right)$

$=\frac{4}{3\pi}(\cos 3t)\frac{d}{dt}(3t)+\frac{4}{5\pi}(-\sin 5t)\frac{d}{dt}(5t)$

$=\frac{4}{3\pi}(\cos 3t)(3)+\frac{4}{5\pi}(-\sin 5t)(5)$

$=\frac{4}{\pi}\cos 3t-\frac{4}{\pi}\sin 5t$

12. $\frac{ds}{dt}=\frac{d}{dt}\left[\sin\left(\frac{3\pi}{2}t\right)+\cos\left(\frac{7\pi}{4}t\right)\right]$

$=\cos\left(\frac{3\pi}{2}t\right)\frac{d}{dt}\left(\frac{3\pi}{2}t\right)-\sin\left(\frac{7\pi}{4}t\right)\frac{d}{dt}\left(\frac{7\pi}{4}t\right)$

$=\frac{3\pi}{2}\cos\left(\frac{3\pi}{2}t\right)-\frac{7\pi}{4}\sin\left(\frac{7\pi}{4}t\right)$

13. $\frac{dy}{dx}=\frac{d}{dx}(x+\sqrt{x})^{-2}=-2(x+\sqrt{x})^{-3}\frac{d}{dx}(x+\sqrt{x})$
$=-2(x+\sqrt{x})^{-3}\left(1+\frac{1}{2\sqrt{x}}\right)$

14. $\frac{dy}{dx}=\frac{d}{dx}(\csc x+\cot x)^{-1}$
$=-(\csc x+\cot x)^{-2}\frac{d}{dx}(\csc x+\cot x)$
$=-\frac{1}{(\csc x+\cot x)^2}(-\cot x\csc x-\csc^2 x)$
$=\frac{(\csc x)(\cot x+\csc x)}{(\csc x+\cot x)^2}=\frac{\csc x}{\csc x+\cot x}$

15. $\frac{dy}{dx}=\frac{d}{dx}(\sin^{-5}x-\cos^3 x)$
$=(-5\sin^{-6}x)\frac{d}{dx}(\sin x)-(3\cos^2 x)\frac{d}{dx}(\cos x)$
$=-5\sin^{-6}x\cos x+3\cos^2 x\sin x$

16. $\frac{dy}{dx}=\frac{d}{dx}[x^3(2x-5)^4]$
$=(x^3)\frac{d}{dx}(2x-5)^4+(2x-5)^4\frac{d}{dx}(x^3)$
$=(x^3)(4)(2x-5)^3\frac{d}{dx}(2x-5)+(2x-5)^4(3x^2)$
$=(x^3)(4)(2x-5)^3(2)+3x^2(2x-5)^4$
$=8x^3(2x-5)^3+3x^2(2x-5)^4$
$=x^2(2x-5)^3[8x+3(2x-5)]$
$=x^2(2x-5)^3(14x-15)$

17. $\frac{dy}{dx}=\frac{d}{dx}(\sin^3 x\tan 4x)$
$=(\sin^3 x)\frac{d}{dx}(\tan 4x)+(\tan 4x)\frac{d}{dx}(\sin^3 x)$
$=(\sin^3 x)(\sec^2 4x)\frac{d}{dx}(4x)+(\tan 4x)(3\sin^2 x)\frac{d}{dx}(\sin x)$
$=(\sin^3 x)(\sec^2 4x)(4)+(\tan 4x)(3\sin^2 x)(\cos x)$
$=4\sin^3 x\sec^2 4x+3\sin^2 x\cos x\tan 4x$

18. $\frac{dy}{dx}=\frac{d}{dx}(4\sqrt{\sec x+\tan x})$
$=4\cdot\frac{1}{2\sqrt{\sec x+\tan x}}\frac{d}{dx}(\sec x+\tan x)$
$=\frac{2}{\sqrt{\sec x+\tan x}}(\sec x\tan x+\sec^2 x)$
$=2\sec x\frac{\sec x+\tan x}{\sqrt{\sec x+\tan x}}$
$=2\sec x\sqrt{\sec x+\tan x}$

19. $\frac{dy}{dx}=\frac{d}{dx}\left(\frac{3}{\sqrt{2x+1}}\right)$
$=\frac{(\sqrt{2x+1})\frac{d}{dx}(3)-3\frac{d}{dx}(\sqrt{2x+1})}{(\sqrt{2x+1})^2}$
$=\frac{(\sqrt{2x+1})(0)-3\left(\frac{1}{2\sqrt{2x+1}}\right)\frac{d}{dx}(2x+1)}{2x+1}$
$=\frac{-3\left(\frac{1}{2\sqrt{2x+1}}\right)(2)}{2x+1}$
$=-\frac{3}{(2x+1)\sqrt{2x+1}}$
$=-3(2x+1)^{-3/2}$

20. $\frac{dy}{dx}=\frac{d}{dx}\frac{x}{\sqrt{1+x^2}}$
$=\frac{(\sqrt{1+x^2})\frac{d}{dx}(x)-x\frac{d}{dx}(\sqrt{1+x^2})}{(\sqrt{1+x^2})^2}$
$=\frac{(\sqrt{1+x^2})(1)-x\left(\frac{1}{2\sqrt{1+x^2}}\right)\frac{d}{dx}(1+x^2)}{1+x^2}$
$=\frac{\sqrt{1+x^2}-x\left(\frac{1}{2\sqrt{1+x^2}}\right)(2x)}{1+x^2}$
$=\frac{(1+x^2)-x^2}{(1+x^2)(\sqrt{1+x^2})}$
$=(1+x^2)^{-3/2}$

21. The last step here uses the identity $2\sin a\cos a=\sin 2a$.
$\frac{dy}{dx}=\frac{d}{dx}\sin^2(3x-2)$
$=2\sin(3x-2)\frac{d}{dx}\sin(3x-2)$
$=2\sin(3x-2)\cos(3x-2)\frac{d}{dx}(3x-2)$
$=2\sin(3x-2)\cos(3x-2)(3)$
$=6\sin(3x-2)\cos(3x-2)$
$=3\sin(6x-4)$

22. $\frac{dy}{dx}=\frac{d}{dx}(1+\cos 2x)^2=2(1+\cos 2x)\frac{d}{dx}(1+\cos 2x)$
$=2(1+\cos 2x)(-\sin 2x)\frac{d}{dx}(2x)$
$=2(1+\cos 2x)(-\sin 2x)(2)$
$=-4(1+\cos 2x)(\sin 2x)$

23. $\frac{dy}{dx}=\frac{d}{dx}(1+\cos^2 7x)^3$
$=3(1+\cos^2 7x)^2\frac{d}{dx}(1+\cos^2 7x)$
$=3(1+\cos^2 7x)^2(2\cos 7x)\frac{d}{dx}(\cos 7x)$
$=3(1+\cos^2 7x)^2(2\cos 7x)(-\sin 7x)\frac{d}{dx}(7x)$
$=3(1+\cos^2 7x)^2(2\cos 7x)(-\sin 7x)(7)$
$=-42(1+\cos^2 7x)^2\cos 7x\sin 7x$

24. $\frac{dy}{dx}=\frac{d}{dx}(\sqrt{\tan 5x})=\frac{1}{2\sqrt{\tan 5x}}\frac{d}{dx}\tan 5x$
$=\frac{1}{2\sqrt{\tan 5x}}(\sec^2 5x)\frac{d}{dx}(5x)$
$=\frac{1}{2\sqrt{\tan 5x}}(\sec^2 5x)(5)$
$=\frac{5\sec^2 5x}{2\sqrt{\tan 5x}}$ or $\frac{5}{2}(\tan 5x)^{-1/2}\sec^2 5x$

25. $\frac{dr}{d\theta}=\frac{d}{d\theta}\tan(2-\theta)=\sec^2(2-\theta)\frac{d}{d\theta}(2-\theta)$
$=\sec^2(2-\theta)(-1)=-\sec^2(2-\theta)$

26. $\frac{dr}{d\theta}=\frac{d}{d\theta}(\sec 2\theta\ \tan 2\theta)$
$=(\sec 2\theta)\frac{d}{d\theta}(\tan 2\theta)+(\tan 2\theta)\frac{d}{d\theta}(\sec 2\theta)$
$=(\sec 2\theta)(\sec^2 2\theta)\frac{d}{d\theta}(2\theta)+(\tan 2\theta)(\sec 2\theta\tan 2\theta)\frac{d}{d\theta}(2\theta)$
$=2\sec^3 2\theta+2\sec 2\theta\tan^2 2\theta$

27. $\frac{dr}{d\theta}=\frac{d}{d\theta}\sqrt{\theta\sin\theta}=\frac{1}{2\sqrt{\theta\sin\theta}}\frac{d}{d\theta}(\theta\sin\theta)$
$=\frac{1}{2\sqrt{\theta\sin\theta}}\left[\theta\frac{d}{d\theta}(\sin\theta)+(\sin\theta)\frac{d}{d\theta}(\theta)\right]$
$=\frac{1}{2\sqrt{\theta\sin\theta}}(\theta\cos\theta+\sin\theta)$
$=\frac{\theta\cos\theta+\sin\theta}{2\sqrt{\theta\sin\theta}}$

28. $\frac{dr}{d\theta}=\frac{d}{d\theta}(2\theta\sqrt{\sec\theta})$
$=(2\theta)\frac{d}{d\theta}(\sqrt{\sec\theta})+(\sqrt{\sec\theta})\frac{d}{d\theta}(2\theta)$
$=(2\theta)\left(\frac{1}{2\sqrt{\sec\theta}}\right)\frac{d}{d\theta}(\sec\theta)+2\sqrt{\sec\theta}$
$=(2\theta)\left(\frac{1}{2\sqrt{\sec\theta}}\right)(\sec\theta\tan\theta)+2\sqrt{\sec\theta}$
$=\theta(\sqrt{\sec\theta})(\tan\theta)+2\sqrt{\sec\theta}$
$=\sqrt{\sec\theta}(\theta\tan\theta+2)$

29. $y'=\frac{d}{dx}\tan x=\sec^2 x$
$y''=\frac{d}{dx}\sec^2 x=(2\sec x)\frac{d}{dx}(\sec x)$
$=(2\sec x)(\sec x\tan x)$
$=2\sec^2 x\tan x$

30. $y'=\frac{d}{dx}\cot x=-\csc^2 x$
$y''=\frac{d}{dx}(-\csc^2 x)=(-2\csc x)\frac{d}{dx}(\csc x)$
$=(-2\csc x)(-\csc x\cot x)$
$=2\csc^2 x\cot x$

31. $y'=\frac{d}{dx}\cot(3x-1)=-\csc^2(3x-1)\frac{d}{dx}(3x-1)$
$=-3\csc^2(3x-1)$
$y''=\frac{d}{dx}[-3\csc^2(3x-1)]$
$=-3[2\csc(3x-1)]\frac{d}{dx}\csc(3x-1)$
$=-3[2\csc(3x-1)]\bullet$
$[-\csc(3x-1)\cot(3x-1)]\frac{d}{dx}(3x-1)$
$=-3[2\csc(3x-1)][-\csc(3x-1)\cot(3x-1)](3)$
$=18\csc^2(3x-1)\cot(3x-1)$

32. $y'=\frac{d}{dx}\left[9\tan\left(\frac{x}{3}\right)\right]=9\sec^2\left(\frac{x}{3}\right)\frac{d}{dx}\left(\frac{x}{3}\right)$
$=3\sec^2\left(\frac{x}{3}\right)$
$y''=\frac{d}{dx}\left[3\sec^2\left(\frac{x}{3}\right)\right]=3\left[2\sec\left(\frac{x}{3}\right)\right]\frac{d}{dx}\sec\left(\frac{x}{3}\right)$
$=6\left[\sec\left(\frac{x}{3}\right)\right]\left[\sec\left(\frac{x}{3}\right)\tan\left(\frac{x}{3}\right)\right]\frac{d}{dx}\left(\frac{x}{3}\right)$
$=2\sec^2\left(\frac{x}{3}\right)\tan\left(\frac{x}{3}\right)$

33. $f'(u)=\frac{d}{du}(u^5+1)=5u^4$
$g'(x)=\frac{d}{dx}(\sqrt{x})=\frac{1}{2\sqrt{x}}$
$(f\circ g)'(1)=f'(g(1))g'(1)=f'(1)g'(1)=(5)\left(\frac{1}{2}\right)=\frac{5}{2}$

34. $f'(u)=\frac{d}{du}(1-u^{-1})=u^{-2}=\frac{1}{u^2}$

$g'(x)=\frac{d}{dx}(1-x)^{-1}=-(1-x)^{-2}\frac{d}{dx}(1-x)$

$=-(1-x)^{-2}(-1)=\frac{1}{(1-x)^2}$

$(f\circ g)'(-1)=f'(g(-1))g'(-1)=f'\left(\frac{1}{2}\right)g'(-1)$

$=(4)\left(\frac{1}{4}\right)=1$

35. $f'(u)=\frac{d}{du}\left(\cot\frac{\pi u}{10}\right)=-\csc^2\left(\frac{\pi u}{10}\right)\frac{d}{du}\left(\frac{\pi u}{10}\right)$

$=-\frac{\pi}{10}\csc^2\left(\frac{\pi u}{10}\right)$

$g'(x)=\frac{d}{dx}(5\sqrt{x})=\frac{5}{2\sqrt{x}}$

$(f\circ g)'(1)=f'(g(1))g'(1)=f'(5)g'(1)$

$=-\frac{\pi}{10}\left[\csc^2\left(\frac{\pi}{2}\right)\right]\left(\frac{5}{2}\right)$

$=-\frac{\pi}{10}(1)\left(\frac{5}{2}\right)=-\frac{\pi}{4}$

36. $f'(u)=\frac{d}{du}\left[u+(\cos u)^{-2}\right]$

$=1-2(\cos u)^{-3}\frac{d}{du}\cos u$

$=1+\frac{2\sin u}{\cos^3 u}$

$g'(x)=\frac{d}{dx}(\pi x)=\pi$

$(f\circ g)'\left(\frac{1}{4}\right)=f'(g(\frac{1}{4}))g'\left(\frac{1}{4}\right)$

$=f'\left(\frac{\pi}{4}\right)g'\left(\frac{1}{4}\right)$

$=\left(1+\frac{\frac{2}{\sqrt{2}}}{\left(\frac{1}{\sqrt{2}}\right)^3}\right)(\pi)$

$=5\pi$

37. $f'(u)=\frac{d}{du}\frac{2u}{u^2+1}=\frac{(u^2+1)\frac{d}{du}(2u)-(2u)\frac{d}{du}(u^2+1)}{(u^2+1)^2}$

$=\frac{(u^2+1)(2)-(2u)(2u)}{(u^2+1)^2}=\frac{-2u^2+2}{(u^2+1)^2}$

$g'(x)=\frac{d}{dx}(10x^2+x+1)=20x+1$

$(f\circ g)'(0)=f'(g(0))g'(0)=f'(1)g'(0)=(0)(1)=0$

38. $f'(u)=\frac{d}{du}\left(\frac{u-1}{u+1}\right)^2=2\left(\frac{u-1}{u+1}\right)\frac{d}{du}\left(\frac{u-1}{u+1}\right)$

$=2\left(\frac{u-1}{u+1}\right)\frac{(u+1)\frac{d}{du}(u-1)-(u-1)\frac{d}{du}(u+1)}{(u+1)^2}$

$=2\left(\frac{u-1}{u+1}\right)\frac{(u+1)-(u-1)}{(u+1)^2}=\frac{4(u-1)}{(u+1)^3}$

$g'(x)=\frac{d}{dx}(x^{-2}-1)=-2x^{-3}$

$(f\circ g)'(-1)=f'(g(-1))g'(-1)$

$=f'(0)g'(-1)$

$=(-4)(2)=-8$

39. (a) $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$

$=\frac{d}{du}(\cos u)\frac{d}{dx}(6x+2)$

$=(-\sin u)(6)$

$=-6\sin u$

$=-6\sin(6x+2)$

(b) $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$

$=\frac{d}{du}(\cos 2u)\frac{d}{dx}(3x+1)$

$=(-\sin 2u)(2)\cdot(3)$

$=-6\sin 2u$

$=-6\sin(6x+2)$

40. (a) $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$

$=\frac{d}{du}\sin(u+1)\frac{d}{dx}(x^2)$

$=\cos(u+1)(1)\cdot 2x$

$=2x\cos(u+1)$

$=2x\cos(x^2+1)$

(b) $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$

$=\frac{d}{du}(\sin u)\frac{d}{dx}(x^2+1)$

$=(\cos u)(2x)$

$=2x\cos u$

$=2x\cos(x^2+1)$

41. $\frac{dx}{dt}=\frac{d}{dt}(2\cos t)=-2\sin t$

$\frac{dy}{dt}=\frac{d}{dt}(2\sin t)=2\cos t$

$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{2\cos t}{-2\sin t}=-\cot t$

This line passes through $\left(2\cos\frac{\pi}{4},\ 2\sin\frac{\pi}{4}\right)=(\sqrt{2},\ \sqrt{2})$ and has slope $-\cot\frac{\pi}{4}=-1$. Its equation is $y=-(x-\sqrt{2})+\sqrt{2}$, or $y=-x+2\sqrt{2}$.

42. $\frac{dx}{dt}=\frac{d}{dt}(\sin 2\pi t)=(\cos 2\pi t)\frac{d}{dt}(2\pi t)=2\pi\cos 2\pi t$

$\frac{dy}{dt}=\frac{d}{dt}(\cos 2\pi t)=(-\sin 2\pi t)\frac{d}{dt}(2\pi t)=-2\pi\sin 2\pi t$

$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{-2\pi\sin 2\pi t}{2\pi\cos 2\pi t}=-\tan 2\pi t$

The line passes through $\left(\sin\frac{2\pi}{-6},\ \cos\frac{2\pi}{-6}\right)=\left(-\frac{\sqrt{3}}{2},\ \frac{1}{2}\right)$ and has slope $-\tan\frac{2\pi}{-6}=\sqrt{3}$. Its equation is $y=\sqrt{3}\left(x+\frac{\sqrt{3}}{2}\right)+\frac{1}{2}$, or $y=\sqrt{3}x+2$.

43. $\frac{dx}{dt}=\frac{d}{dt}(\sec^2 t-1)=(2\sec t)\frac{d}{dt}(\sec t)$

$=(2\sec t)(\sec t\tan t)$

$=2\sec^2 t\tan t$

$\frac{dy}{dt}=\frac{d}{dt}\tan t=\sec^2 t$

$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{\sec^2 t}{2\sec^2 t\tan t}=\frac{1}{2}\cot t.$

The line passes through

$\left(\sec^2\left(-\frac{\pi}{4}\right)-1,\ \tan\left(-\frac{\pi}{4}\right)\right)=(1,\ -1)$ and has slope $\frac{1}{2}\cot\left(-\frac{\pi}{4}\right)=-\frac{1}{2}$. Its equation is $y=-\frac{1}{2}(x-1)-1$, or $y=-\frac{1}{2}x-\frac{1}{2}$.

44. $\frac{dx}{dt}=\frac{d}{dt}\sec t=\sec t\tan t$

$\frac{dy}{dt}=\frac{d}{dt}\tan t=\sec^2 t$

$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{\sec^2 t}{\sec t\tan t}=\frac{\sec t}{\tan t}=\frac{1}{\sin t}=\csc t$

The line passes through $\left(\sec\frac{\pi}{6},\ \tan\frac{\pi}{6}\right)=\left(\frac{2}{\sqrt{3}},\frac{1}{\sqrt{3}}\right)$ and has slope $\csc\frac{\pi}{6}=2$. Its equation is $y=2\left(x-\frac{2}{\sqrt{3}}\right)+\frac{1}{\sqrt{3}}$, or $y=2x-\sqrt{3}$.

45. $\frac{dx}{dt}=\frac{d}{dt}t=1$

$\frac{dy}{dt}=\frac{d}{dt}\sqrt{t}=\frac{1}{2\sqrt{t}}$

$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{1/(2\sqrt{t})}{1}=\frac{1}{2\sqrt{t}}$

The line passes through $\left(\frac{1}{4},\sqrt{\frac{1}{4}}\right)=\left(\frac{1}{4},\frac{1}{2}\right)$ and has slope $\frac{1}{2\sqrt{\frac{1}{4}}}=1$. Its equation is $y=1\left(x-\frac{1}{4}\right)+\frac{1}{2}$, or $y=x+\frac{1}{4}$.

46. $\frac{dx}{dt}=\frac{d}{dt}(2t^2+3)=4t$

$\frac{dy}{dt}=\frac{d}{dt}(t^4)=4t^3$

$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{4t^3}{4t}=t^2$

The line passes through $(2(-1)^2+3, (-1)^4)=(5, 1)$ and has slope $(-1)^2=1$. Its equation is $y=1(x-5)+1$, or $y=x-4$.

47. $\frac{dx}{dt}=\frac{d}{dt}(t-\sin t)=1-\cos t$

$\frac{dy}{dt}=\frac{d}{dt}(1-\cos t)=\sin t$

$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{\sin t}{1-\cos t}$

The line passes through

$\left(\frac{\pi}{3}-\sin\frac{\pi}{3},\ 1-\cos\frac{\pi}{3}\right)=\left(\frac{\pi}{3}-\frac{\sqrt{3}}{2},\ \frac{1}{2}\right)$ and has slope

$\frac{\sin\left(\frac{\pi}{3}\right)}{1-\cos\left(\frac{\pi}{3}\right)}=\sqrt{3}$. Its equation is

$y=\sqrt{3}\left(x-\frac{\pi}{3}+\frac{\sqrt{3}}{2}\right)+\frac{1}{2}$, or

$y=\sqrt{3}x+2-\frac{\pi}{\sqrt{3}}$.

48. $\frac{dx}{dt}=\frac{d}{dt}\cos t=-\sin t$

$\frac{dy}{dt}=\frac{d}{dt}(1+\sin t)=\cos t$

$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{\cos t}{-\sin t}=-\cot t$

The line passes through $\left(\cos\frac{\pi}{2},\ 1+\sin\frac{\pi}{2}\right)=(0,\ 2)$ and has slope $-\cot\left(\frac{\pi}{2}\right)=0$. Its equation is $y=2$.

49. (a) $\frac{dx}{dt}=\frac{d}{dt}(t^2+t)=2t+1$

$\frac{dy}{dt}=\frac{d}{dt}\sin t=\cos t$

$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{\cos t}{2t+1}$

(b) $\frac{d}{dt}\left(\frac{dy}{dx}\right)=\frac{d}{dt}\frac{\cos t}{2t+1}$

$=\frac{(2t+1)\frac{d}{dt}(\cos t)-(\cos t)\frac{d}{dt}(2t+1)}{(2t+1)^2}$

$=\frac{(2t+1)(-\sin t)-(\cos t)(2)}{(2t+1)^2}$

$=-\frac{(2t+1)(\sin t)+2\cos t}{(2t+1)^2}$

(c) Let $u=\frac{dy}{dx}$.

Then $\frac{du}{dt}=\frac{du}{dx}\frac{dx}{dt}$, so $\frac{du}{dx}=\frac{du}{dt}\div\frac{dx}{dt}$. Therefore,

$\frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{d}{dt}\left(\frac{dy}{dx}\right)\div\frac{dx}{dt}$

$=-\frac{(2t+1)(\sin t)+2\cos t}{(2t+1)^2}\div(2t+1)$

$=-\frac{(2t+1)(\sin t)+2\cos t}{(2t+1)^3}$

(d) The expression in part (c).

50. Since the radius passes through (0, 0) and (2cos t, 2sin$t$), it has slope given by tan t. But the slope of the tangent line is

$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{2\cos t}{-2\sin t}=-\cot t$, which is the negative reciprocal

of tan t. This means that the radius and the tangent line are perpendicular. (The preceding argument breaks down when $t=\frac{k\pi}{2}$, where k is an integer. At these values, either the radius is horizontal and the tangent line is vertical or the radius is vertical and the tangent line is horizontal, so the result still holds.)

51. $\frac{ds}{dt}=\frac{ds}{d\theta}\frac{d\theta}{dt}=\frac{d}{d\theta}(\cos\theta)\frac{d\theta}{dt}$

$=(-\sin\theta)\left(\frac{d\theta}{dt}\right)$

When $\theta=\frac{3\pi}{2}$ and $\frac{d\theta}{dt}=5$, $\frac{ds}{dt}=\left(-\sin\frac{3\pi}{2}\right)(5)=5$.

52. $\frac{dy}{dt}=\frac{dy}{dx}\frac{dx}{dt}=\frac{d}{dx}(x^2+7x-5)\frac{dx}{dt}$

$=(2x+7)\left(\frac{dx}{dt}\right)$

When $x=1$ and $\frac{dx}{dt}=\frac{1}{3}$, $\frac{dy}{dt}=[2(1)+7]\left(\frac{1}{3}\right)=3$.

53. $\frac{dy}{dx}=\frac{d}{dx}\sin\frac{x}{2}=\left(\cos\frac{x}{2}\right)\frac{d}{dx}\left(\frac{x}{2}\right)=\frac{1}{2}\cos\frac{x}{2}$

Since the range of the function $f(x)=\frac{1}{2}\cos\frac{x}{2}$ is $\left[-\frac{1}{2},\frac{1}{2}\right]$,

the largest possible value of $\frac{dy}{dx}$ is $\frac{1}{2}$.

54. $\frac{dy}{dx}=\frac{d}{dx}(\sin mx)=(\cos mx)\frac{d}{dx}(mx)=m\cos mx$

The desired line has slope $y'(0)=m\cos 0=m$ and passes through (0, 0), so its equation is $y=mx$.

55. $\frac{dy}{dx}=\frac{d}{dx}2\tan\frac{\pi x}{4}=\left(2\sec^2\frac{\pi x}{4}\right)\frac{d}{dx}\left(\frac{\pi x}{4}\right)$

$=\frac{\pi}{2}\sec^2\left(\frac{\pi x}{4}\right)$

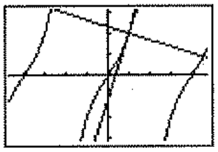
$y'(1)=\frac{\pi}{2}\sec^2\left(\frac{\pi}{4}\right)=\frac{\pi}{2}(\sqrt{2})^2=\pi.$

The tangent line has slope π and passes through $\left(1, 2\tan\frac{\pi}{4}\right)=(1, 2)$. Its equation is $y=\pi(x-1)+2$, or $y=\pi x-\pi+2$.

The normal line has slope $-\frac{1}{\pi}$ and passes through (1, 2).

Its equation is $y=-\frac{1}{\pi}(x-1)+2$, or $y=-\frac{1}{\pi}x+\frac{1}{\pi}+2$.

Graphical support:



[–4.7, 4.7] by [–3.1, 3.1]

56. (a) $\frac{d}{dx}[2f(x)]=2f'(x)$

At $x=2$, the derivative is $2f'(2)=2\left(\frac{1}{3}\right)=\frac{2}{3}$.

(b) $\frac{d}{dx}[f(x)+g(x)]=f'(x)+g'(x)$

At $x=3$, the derivative is $f'(3)+g'(3)=2\pi+5$.

(c) $\frac{d}{dx}[f(x)\cdot g(x)]=f(x)g'(x)+g(x)f'(x)$

At $x=3$, the derivative is

$f(3)g'(3)+g(3)f'(3)=(3)(5)+(-4)(2\pi)$
$=15-8\pi.$

(d) $\frac{d}{dx}\frac{f(x)}{g(x)}=\frac{g(x)f'(x)-f(x)g'(x)}{[g(x)]^2}$

At $x=2$, the derivative is

$$\frac{g(2)f'(2)-f(2)g'(2)}{[g(2)]^2}=\frac{(2)\left(\frac{1}{3}\right)-(8)(-3)}{(2)^2}$$

$$=\frac{\frac{74}{3}}{4}=\frac{37}{6}.$$

(e) $\frac{d}{dx}f(g(x))=f'(g(x))g'(x)$

At $x=2$, the derivative is

$$f'(g(2))g'(2)=f'(2)g'(2)=\left(\frac{1}{3}\right)(-3)=-1.$$

(f) $\frac{d}{dx}\sqrt{f(x)}=\frac{1}{2\sqrt{f(x)}}\frac{d}{dx}f(x)=\frac{f'(x)}{2\sqrt{f(x)}}$

At $x=2$, the derivative is

$$\frac{f'(2)}{2\sqrt{f(2)}}=\frac{\frac{1}{3}}{2\sqrt{8}}=\frac{1}{6(2\sqrt{2})}=\frac{1}{12\sqrt{2}}.$$

(g) $\frac{d}{dx}\frac{1}{g^2(x)}=\frac{d}{dx}[g(x)]^{-2}=-2[g(x)]^{-3}\frac{d}{dx}g(x)=-\frac{2g'(x)}{[g(x)]^3}$

At $x=3$, the derivative is

$$-\frac{2g'(3)}{[g(3)]^3}=-\frac{2(5)}{(-4)^3}=-\frac{10}{-64}=\frac{5}{32}.$$

(h) $\frac{d}{dx}\sqrt{f^2(x)+g^2(x)}=\frac{1}{2\sqrt{f^2(x)+g^2(x)}}\frac{d}{dx}[f^2(x)+g^2(x)]$

$$=\frac{1}{2\sqrt{f^2(x)+g^2(x)}}[2f(x)\frac{d}{dx}f(x)+2g(x)\frac{d}{dx}g(x)]$$

$$=\frac{f(x)f'(x)+g(x)g'(x)}{\sqrt{f^2(x)+g^2(x)}}$$

At $x=2$, the derivative is

$$\frac{f(2)f'(2)+g(2)g'(2)}{\sqrt{f^2(2)+g^2(2)}}=\frac{(8)\left(\frac{1}{3}\right)+(2)(-3)}{\sqrt{8^2+2^2}}$$

$$=\frac{-\frac{10}{3}}{\sqrt{68}}=-\frac{\frac{10}{3}}{2\sqrt{17}}=-\frac{5}{3\sqrt{17}}$$

57. $\frac{d}{dx}\cos(x°)=\frac{d}{dx}\cos\left(\frac{\pi x}{180}\right)=-\frac{\pi}{180}\sin\left(\frac{\pi x}{180}\right)=-\frac{\pi}{180}\sin(x°)$

58. (a) $\frac{d}{dx}[5f(x)-g(x)]=5f'(x)-g'(x)$

At $x=1$, the derivative is

$$5f'(1)-g'(1)=5\left(-\frac{1}{3}\right)-\left(-\frac{8}{3}\right)=1.$$

(b) $\frac{d}{dx}f(x)g^3(x)=f(x)\frac{d}{dx}g^3(x)+g^3(x)\frac{d}{dx}f(x)$

$=f(x)[3g^2(x)]\frac{d}{dx}g(x)+g^3(x)f'(x)$

$=3f(x)g^2(x)g'(x)+g^3(x)f'(x)$

At $x=0$, the derivative is $3f(0)g^2(0)g'(0)+g^3(0)f'(0)$

$=3(1)(1)^2\left(\frac{1}{3}\right)+(1)^3(5)=6.$

58. Continued

(c) $\frac{d}{dx}\frac{f(x)}{g(x)+1} = \frac{[g(x)+1]\frac{d}{dx}f(x) - f(x)\frac{d}{dx}[g(x)+1]}{[g(x)+1]^2}$

$= \frac{[g(x)+1]f'(x) - f(x)g'(x)}{[g(x)+1]^2}$

At $x = 1$, the derivative is

$$\frac{[g(1)+1]f'(1) - f(1)g'(1)}{[g(1)+1]^2} = \frac{(-4+1)\left(-\frac{1}{3}\right) - (3)\left(-\frac{8}{3}\right)}{(-4+1)^2}$$

$$= \frac{9}{9} = 1.$$

(d) $\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$

At $x = 0$, the derivative is

$$f'(g(0))g'(0) = f'(1)g'(0) = \left(-\frac{1}{3}\right)\left(\frac{1}{3}\right) = -\frac{1}{9}.$$

(e) $\frac{d}{dx}g(f(x)) = g'(f(x))f'(x)$

At $x = 0$, the derivative is

$$g'(f(0))f'(0) = g'(1)f'(0) = \left(-\frac{8}{3}\right)(5) = -\frac{40}{3}$$

(f) $\frac{d}{dx}[g(x)+f(x)]^{-2} = -2[g(x)+f(x)]^{-3}\frac{d}{dx}[g(x)+f(x)]$

$= -\frac{2[g'(x)+f'(x)]}{[g(x)+f(x)]^3}$

At $x = 1$, the derivative is

$$-\frac{2[g'(1)+f'(1)]}{[g(1)+f(1)]^3} = -\frac{2\left(-\frac{8}{3}-\frac{1}{3}\right)}{(-4+3)^3} = -\frac{-6}{-1} = -6.$$

(g) $\frac{d}{dx}[f(x+g(x))] = f'(x+g(x))\frac{d}{dx}[x+g(x)]$

$= f'(x+g(x))(1+g'(x))$

At $x = 0$, the derivative is

$$f'(0+g(0))(1+g'(0)) = f'(0+1)\left(1+\frac{1}{3}\right)$$

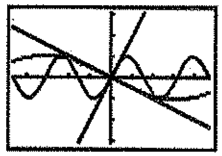
$$= f'(1)\left(\frac{4}{3}\right)$$

$$= \left(-\frac{1}{3}\right)\left(\frac{4}{3}\right) = -\frac{4}{9}.$$

59. For $y = \sin 2x$, $y' = (\cos 2x)\frac{d}{dx}(2x) = 2\cos 2x$ and the slope at the origin is 2.

For $y = -\sin\frac{x}{2}$, $y' = \left(-\cos\frac{x}{2}\right)\frac{d}{dx}\left(\frac{x}{2}\right) = -\frac{1}{2}\cos\frac{x}{2}$ and the slope at the origin is $-\frac{1}{2}$. Since the slopes of the two tangent lines are 2 and $-\frac{1}{2}$, the lines are perpendicular and the curves are orthogonal.

A graph of the two curves along with the tangents $y = 2x$ and $y = -\frac{1}{2}x$ is shown.



[−4.7, 4.7] by [−3.1, 3.1]

60. Because the symbols $\frac{dy}{dx}, \frac{dy}{du}$, and $\frac{du}{dx}$ are not fractions. The individual symbols dy, dx, and du do not have numerical values.

61. Velocity: $s'(t) = -2\pi bA\sin(2\pi bt)$

acceleration: $s''(t) = -4\pi^2 b^2 A\cos(2\pi bt)$

jerk: $s'''(t) = 8\pi^3 b^3 A\sin(2\pi bt)$

The velocity, amplitude, and jerk and proportional to b, b^2, and b^3 respectively. If the frequency b is doubled, then the amplitude of the velocity is doubled, the amplitude of the acceleration is quadrupled, and the amplitude of the jerk is multiplied by 8.

62. (a) $y'(t) = \frac{d}{dt}37\sin\left[\frac{2\pi}{365}(x-101)\right] + \frac{d}{dt}(25)$

$= 37\cos\left[\frac{2\pi}{365}(x-101)\right] \cdot \frac{d}{dx}\left[\frac{2\pi}{365}(x-101)\right] + 0$

$= 37\cos\left[\frac{2\pi}{365}(x-101)\right] \cdot \frac{2\pi}{365}$

$= \frac{74\pi}{365}\cos\left[\frac{2\pi}{365}(x-101)\right]$

Since $\cos u$ is greatest when $u = 0$, $\pm 2\pi$, and so on, $y'(t)$ is greatest when $\frac{2\pi}{365}(x-101) = 0$, or $x = 101$. The temperature is increasing the fastest on day 101 (April 11).

(b) The rate of increase is

$y'(101) = \frac{74\pi}{365} \approx 0.637$ degrees per day.

63. Velocity: $s'(t) = \frac{d}{dt}\sqrt{1+4t} = \frac{1}{2\sqrt{1+4t}}\frac{d}{dt}(1+4t)$

$= \frac{4}{2\sqrt{1+4t}} = \frac{2}{\sqrt{1+4t}}$

At $t = 6$, the velocity is $\frac{2}{\sqrt{1+4(6)}} = \frac{2}{5}$ m/sec

63. Continued

Acceleration: $s''(t) = \frac{d}{dt}\frac{2}{\sqrt{1+4t}}$

$$= \frac{(\sqrt{1+4t})\frac{d}{dt}(2) - 2\frac{d}{dt}\sqrt{1+4t}}{(\sqrt{1+4t})^2}$$

$$= \frac{-2\left(\frac{1}{2\sqrt{1+4t}}\right)\frac{d}{dt}(1+4t)}{1+4t}$$

$$= \frac{\frac{-4}{\sqrt{1+4t}}}{1+4t} = -\frac{4}{(1+4t)^{3/2}}$$

At $t = 6$, the acceleration is $-\frac{4}{[1+4(6)]^{3/2}} = -\frac{4}{125}$ m/sec^2

64. Acceleration $= \frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} = \left(\frac{dv}{ds}\right)(v) = \left[\frac{d}{ds}(k\sqrt{s})\right](k\sqrt{s})$

$$= \left(\frac{k}{2\sqrt{s}}\right)(k\sqrt{s}) = \frac{k^2}{2}, \text{ a constant.}$$

65. Note that this Exercise concerns itself with the slowing down caused by the earth's atmosphere, *not* the acceleration caused by gravity.

Given: $v = \frac{k}{\sqrt{s}}$

Acceleration $= \frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} = \left(\frac{dv}{ds}\right)(v) = (v)\left(\frac{dv}{ds}\right)$

$$= \left(\frac{k}{\sqrt{s}}\right)\frac{d}{ds}\frac{k}{\sqrt{s}}$$

$$= \left(\frac{k}{\sqrt{s}}\right)\left(\frac{\sqrt{s}\frac{d}{ds}(k) - k\frac{d}{ds}\sqrt{s}}{(\sqrt{s})^2}\right)$$

$$= \left(\frac{k}{\sqrt{s}}\right)\left(\frac{\frac{-k}{(2\sqrt{s})}}{s}\right)$$

$$= -\frac{k^2}{2s^2}, s \geq 0$$

Thus, the acceleration is inversely proportional to s^2.

66. Acceleration $= \frac{dv}{dt} = \frac{df(x)}{dt} = \frac{df(x)}{dx}\frac{dx}{dt} = f'(x)f(x)$

67. $\frac{dT}{du} = \frac{dT}{dL}\frac{dL}{du} = \left(\frac{d}{dL}2\pi\sqrt{\frac{L}{g}}\right)(kL)$

$$= \left(2\pi\frac{1}{2\sqrt{\frac{L}{g}}}\right)\left(\frac{d}{dL}\frac{L}{g}\right)(kL)$$

$$= \left(\frac{\pi}{\sqrt{\frac{L}{g}}}\right)\left(\frac{1}{g}\right)(kL) = k\pi\sqrt{\frac{L}{g}} = \frac{kT}{2}$$

68. No, this does not contradict the Chain Rule. The Chain Rule states that if two functions are differentiable at the appropriate points, then their composite must also be differentiable. It does not say: If a composite is differentiable, then the functions which make up the composite must all be differentiable.

69. Yes. Note that $\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$. If the graph of $y = f(g(x))$ has a horizontal tangent at $x = 1$, then $f'(g(1))g'(1) = 0$, so either $g'(1) = 0$ or $f'(g(1)) = 0$. This means that either the graph of $y = g(x)$ has a horizontal tangent at $x = 1$, or the graph of $y = f(u)$ has a horizontal tangent at $u = g(1)$.

70. False. See example 8.

71. False. It is +1.

72. E. $\frac{dy}{dx} = \frac{d}{dx}\tan(4x)$

$y = \tan u \qquad u = 4x$

$\frac{dy}{du} = \sec^2 u \qquad \frac{du}{dx} = 4$

$\frac{dy}{dx} = 4\sec^2(4x)$

73. C. $\frac{dy}{dx} = \frac{d}{dx}\cos^2(x^3 + x^2)$

$y = \cos^2 u \qquad u = x^3 + x^2$

$\frac{dy}{du} = -2\sin u\cos u \qquad \frac{du}{dx} = 3x^2 + 2x$

$\frac{dy}{dx} = -2(3x^2 + 2x)\cos(3x^2 + 2x)\sin(3x^2 + 2x)$

74. A. $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dy}{dt}}$

$\frac{dy}{dt} = \frac{d}{dt}(-1 + \sin t)$
$= \cos t$

$\frac{dx}{dt} = \frac{d}{dt}(t - \cos t)$
$= 1 + \sin t$

$\frac{\frac{dy}{dt}}{\frac{dy}{dt}} = \frac{\cos t}{1 + \sin t}$

$\left.\frac{dy}{dx}\right|_{t=0} = \frac{\cos 0}{1 + \sin 0} = 1$

$x(0) = 0 - \cos 0 = -1$

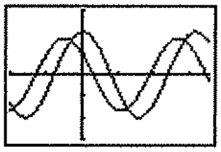
$y(0) = -1 + \sin 0 = -1$

$y = 1(x - (-1) + (-1)) = x$

75. B. See problem 74.

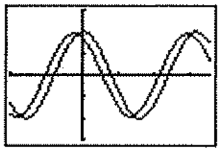
$$\frac{dy}{dx} = \frac{\cos t}{1+\sin t} = 0$$
$$\cos t = 0$$
$$t = \frac{\pi}{2}$$

76. For $h = 1$:



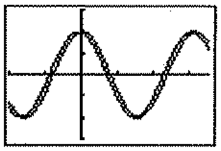
$[-2, 3.5]$ by $[-3, 3]$

For $h = 0.5$:



$[-2, 3.5]$ by $[-3, 3]$

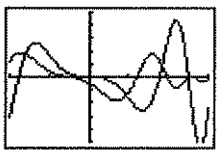
For $h = 0.2$:



$[-2, 3.5]$ by $[-3, 3]$

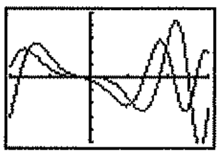
As $h \to 0$, the second curve (the difference quotient) approaches the first ($y = 2\cos 2x$). This is because $2\cos 2x$ is the derivative of $\sin 2x$, and the second curve is the difference quotient used to define the derivative of $\sin 2x$. As $h \to 0$, the difference quotient expression should be approaching the derivative.

77. For $h = 1$:



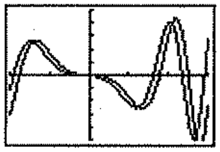
$[-2, 3]$ by $[-5, 5]$

For $h = 0.3$:



$[-2, 3]$ by $[-5, 5]$

For $h = 0.3$:



$[-2, 3]$ by $[-5, 5]$

As $h \to 0$, the second curve (the difference quotient) approaches the first ($y = -2x\sin(x^2)$). This is because $-2x\sin(x^2)$ is the derivative of $\cos(x^2)$, and the second curve is the difference quotient used to define the derivative of $\cos(x^2)$. As $h \to 0$, the difference quotient expression should be approaching the derivative.

78. (a) Let $f(x) = |x|$.

Then

$$\frac{d}{dx}|u| = \frac{d}{dx}f(u) = f'(u)\frac{du}{dx} = \left(\frac{d}{dx}|u|\right)\left(\frac{du}{dx}\right) = \frac{u}{|u|}u'.$$

The derivative of the absolute value function is +1 for positive values, −1 for negative values, and undefined at 0. So $f'(u) = \begin{cases} -1, & u < 0 \\ 1, & u > 0. \end{cases}$

But this is exactly how the expression $\frac{u}{|u|}$ evaluates.

(b) $f'(x) = \left[\frac{d}{dx}(x^2-9)\right] \cdot \frac{x^2-9}{|x^2-9|} = \frac{(2x)(x^2-9)}{|x^2-9|}$

$$g'(x) = \frac{d}{dx}(|x|\sin x)$$
$$= |x|\frac{d}{dx}(\sin x) + (\sin x)\frac{d}{dx}|x|$$
$$= |x|\cos x + \frac{x\sin x}{|x|}$$

Note: The expression for $g'(x)$ above is undefined at $x = 0$, but actually

$$g'(0) = \lim_{h\to 0}\frac{g(0+h)-g(0)}{h} = \lim_{h\to 0}\frac{|h|\sin h}{h} = 0.$$

Therefore, we may express the derivative as

$$g'(x) = \begin{cases} |x|\cos x + \dfrac{x\sin x}{|x|}, & x \neq 0 \\ 0, & x = 0. \end{cases}$$

79. $\frac{dG}{dx} = \frac{d}{dx}\sqrt{uv} = \frac{d}{dx}\sqrt{x(x+c)} = \frac{d}{dx}\sqrt{x^2+cx}$

$$= \frac{1}{2\sqrt{x^2+cx}}\frac{d}{dx}(x^2+cx) = \frac{2x+c}{2\sqrt{x^2+cx}} = \frac{x+(x+c)}{2\sqrt{x(x+c)}}$$
$$= \frac{u+v}{2\sqrt{uv}} = \frac{A}{G}$$

Quick Quiz Sections 3.4-3.6

1. B. $y = \sin^4 u \qquad u = 3x$

$$\frac{dy}{du} = 4\sin^3 u \cos u \quad \frac{du}{dx} = 3$$
$$\frac{dy}{dx} = 12\sin^3(3x)\cos(3x)$$

2. A. $y = \cos x + \tan x$
$y' = -\sin x + \sec^2 x$
$y'' = -\cos x + 2\sec^2 x \tan x$

3. C. $x = 3\sin t \qquad y = 2\cos t$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$
$$\frac{dy}{dt} = \frac{d}{dt}(2\cos t) = -2\sin t$$
$$\frac{dx}{dt} = \frac{d}{dt}(3\sin t) = 3\cos t$$
$$\frac{dy}{dx} = \frac{-2\sin t}{3\cos t} = -\frac{2}{3}\tan t.$$

4. **(a)** $s(0) = -(0)^2 + (0) + 2$
$s(0) = 2$
$(0, 2)$

(b) $-t^2 + t + 2 > 0$
$(t+1)(-t+2) > 0$
$-1 < t < \frac{1}{2}$ but $t < 0$ not real
$0 \le t < \frac{1}{2}$

(c) $t > \frac{1}{2}$

(d) $v(t) = \frac{ds}{dt} = \frac{d}{dt}(-t^2 + t + 2)$
$= -2t + 1$

(e) $a(t) = \frac{dv}{dt} = \frac{d}{dt}(-2t + 1)$
$= -2$

(f) $t = \frac{1}{2}$

Section 3.7 Implicit Differentiation (pp. 157–164)

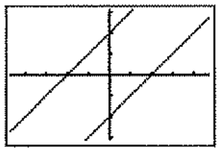
Exploration 1 An Unexpected Derivative

1. $2x - 2y - 2xy' + 2yy' = 0$. Solving for y', we find that $\frac{dy}{dx} = 1$ (provided $y \ne x$).

2. With a constant derivative of 1, the graph would seem to be a line with slope 1.

3. Letting $x = 0$ in the original equation, we find that $y = \pm 2$. This would seem to indicate that this equation defines two lines implicitly, both with slope 1. The two lines are $y = x + 2$ and $y = x - 2$.

4. Factoring the original equation, we have
$[(x - y) - 2][(x - y) + 2] = 0$
$\therefore x - y - 2 = 0$ or $x - y + 2 = 0$
$\therefore y = x - 2$ or $y = x + 2$.
The graph is shown below.

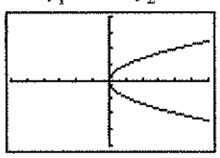


[−4.7, 4.7] by [−3.1, 3.1]

5. At each point (x, y) on either line, $\frac{dy}{dx} = 1$. The condition $y \ne x$ is true because both lines are parallel to the line $y = x$. The derivative is surprising because it does not depend on x or y, but there are no inconsistencies.

Quick Review 3.7

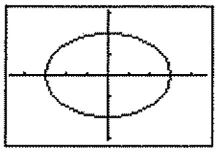
1. $x - y^2 = 0$
$x = y^2$
$\pm\sqrt{x} = y$
$y_1 = \sqrt{x},\ y_2 = -\sqrt{x}$



[-6, 6] by [-4, 4]

2. $4x^2+9y^2=36$

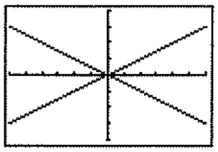
$$9y^2=36-4x^2$$
$$y^2=\frac{36-4x^2}{9}=\frac{4}{9}(9-x^2)$$
$$y=\pm\frac{2}{3}\sqrt{9-x^2}$$
$$y_1=\frac{2}{3}\sqrt{9-x^2},\ y_2=-\frac{2}{3}\sqrt{9-x^2}$$



[-4.7, 4.7] by [-3.1, 3.1]

3. $x^2-4y^2=0$

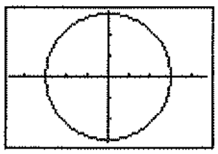
$$(x+2y)(x-2y)=0$$
$$y=\pm\frac{x}{2}$$
$$y_1=\frac{x}{2},\ y_2=-\frac{x}{2}$$



[-6, 6] by [-4, 4]

4. $x^2+y^2=9$

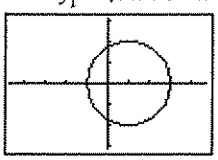
$$y^2=9-x^2$$
$$y=\pm\sqrt{9-x^2}$$
$$y_1=\sqrt{9-x^2},\ y_2=-\sqrt{9-y^2}$$



[-4.7, 4.7] by [-3.1, 3.1]

5. $x^2+y^2=2x+3$

$$y^2=2x+3-x^2$$
$$y=\pm\sqrt{2x+3-x^2}$$
$$y_1=\sqrt{2x+3-x^2},\ y^2=-\sqrt{2x+3-x^2}$$



[-4.7, 4.7] by [-3.1, 3.1]

6. $x^2y'-2xy=4x-y$

$$x^2y'=4x-y+2xy$$
$$y'=\frac{4x-y+2xy}{x^2}$$

7. $y'\sin x-x\cos x=xy'+y$

$$y'\sin x-xy'=y+x\cos x$$
$$(\sin x-x)y'=y+x\cos x$$
$$y'=\frac{y+x\cos x}{\sin x-x}$$

8. $x(y^2-y')=y'(x^2-y)$

$$xy^2=y'(x^2-y+x)$$
$$y'=\frac{xy^2}{x^2-y+x}$$

9. $\sqrt{x}(x-\sqrt[3]{x}=x^{1/2}(x-x^{1/3})$

$$=x^{1/2}x-x^{1/2}x^{1/3}$$
$$=x^{3/2}-x^{5/6}$$

10. $\frac{x+\sqrt[3]{x^2}}{\sqrt{x^3}}=\frac{x+x^{2/3}}{x^{3/2}}$

$$=\frac{x}{x^{3/2}}+\frac{x^{2/3}}{x^{3/2}}$$
$$=x^{-1/2}+x^{-5/6}$$

Section 3.7 Exercises

1. $x^2y+xy^2=6$

$$\frac{d}{dx}(x^2y)+\frac{d}{dx}(xy^2)=\frac{d}{dx}(6)$$
$$x^2\frac{dy}{dx}+y(2x)+x(2y)\frac{dy}{dx}+y^2(1)=0$$
$$x^2\frac{dy}{dx}+2xy\frac{dy}{dx}=-(2xy+y^2)$$
$$(2xy+x^2)\frac{dy}{dx}=-(2xy+y^2)$$
$$\frac{dy}{dx}=-\frac{2xy+y^2}{2xy+x^2}$$

2. $x^3+y^3=18xy$

$$\frac{d}{dx}(x^3)+\frac{d}{dx}(y^3)=\frac{d}{dx}(18xy)$$
$$3x^2+3y^2\frac{dy}{dx}=18x\frac{dy}{dx}+18y(1)$$
$$3y^2\frac{dy}{dx}-18x\frac{dy}{dx}=18y-3x^2$$
$$(3y^2-18x)\frac{dy}{dx}=18y-3x^2$$
$$\frac{dy}{dx}=\frac{18y-3x^2}{3y^2-18x}$$
$$\frac{dy}{dx}=\frac{6y-x^2}{y^2-6x}$$

3. $$y^2=\frac{x-1}{x+1}$$
$$\frac{d}{dx}y^2=\frac{d}{dx}\frac{x-1}{x+1}$$
$$2y\frac{dy}{dx}=\frac{(x+1)(1)-(x-1)(1)}{(x+1)^2}$$
$$2y\frac{dy}{dx}=\frac{2}{(x+1)^2}$$
$$\frac{dy}{dx}=\frac{1}{y(x+1)^2}$$

4. $$x^2=\frac{x-y}{x+y}$$
$$\frac{d}{dx}(x^2)=\frac{d}{dx}\frac{x-y}{x+y}$$
$$2x=\frac{(x+y)\left(1-\frac{dy}{dx}\right)-(x-y)\left(1+\frac{dy}{dx}\right)}{(x+y)^2}$$
$$2x=\frac{\left[x-x\frac{dy}{dx}+y-y\frac{dy}{dx}\right]-\left[x+x\frac{dy}{dx}-y-y\frac{dy}{dx}\right]}{(x+y)^2}$$
$$2x=\frac{2y-2x\frac{dy}{dx}}{(x+y)^2}$$
$$x(x+y)^2=y-x\frac{dy}{dx}$$
$$x\frac{dy}{dx}=y-x(x+y)^2$$
$$\frac{dy}{dx}=\frac{y-x(x+y)^2}{x}=\frac{y}{x}-(x+y)^2$$

Alternate solution:
$$x^2=\frac{x-y}{x+y}$$
$$x^2(x+y)=x-y$$
$$x^3+x^2y=x-y$$
$$\frac{d}{dx}(x^3)+\frac{d}{dx}(x^2y)=\frac{d}{dx}(x)-\frac{d}{dx}(y)$$
$$3x^2+x^2\frac{dy}{dx}+y(2x)=1-\frac{dy}{dx}$$
$$(x^2+1)\frac{dy}{dx}=1-3x^2-2xy$$
$$\frac{dy}{dx}=\frac{1-3x^2-2xy}{x^2+1}$$

5. $$x=\tan y$$
$$\frac{d}{dx}(x)=\frac{d}{dx}(\tan y)$$
$$1=\sec^2 y\frac{dy}{dx}$$
$$\frac{dy}{dx}=\frac{1}{\sec^2 y}=\cos^2 y$$

6. $$x=\sin y$$
$$\frac{d}{dx}(x)=\frac{d}{dx}(\sin y)$$
$$1=\cos y\frac{dy}{dx}$$
$$\frac{dy}{dx}=\frac{1}{\cos y}=\sec y$$

7. $$x+\tan xy=0$$
$$\frac{d}{dx}(x)+\frac{d}{dx}(\tan xy)=\frac{d}{dx}(0)$$
$$1+\sec^2(xy)\frac{d}{dx}(xy)=0$$
$$1+(\sec^2 xy)[x\frac{dy}{dx}+(y)\,(1)]=0$$
$$(\sec^2 xy)\,(x)\frac{dy}{dx}=-1-(\sec^2 xy)(y)$$
$$\frac{dy}{dx}=\frac{-1-y\sec^2 xy}{x\sec^2 xy}$$
$$\frac{dy}{dx}=-\frac{1}{x}\cos^2 xy-\frac{y}{x}$$

8. $$x+\sin y=xy$$
$$\frac{d}{dx}(x)+\frac{d}{dx}(\sin y)=\frac{d}{dx}(xy)$$
$$1+(\cos y)\frac{dy}{dx}=x\frac{dy}{dx}+(y)(1)$$
$$(\cos y-x)\frac{dy}{dx}=-1+y$$
$$\frac{dy}{dx}=\frac{-1+y}{\cos y-x}=\frac{1-y}{x-\cos y}$$

9. $$\frac{d}{dx}(x^2+y^2)=\frac{d}{dx}(13)$$
$$2x+2y\frac{dy}{dx}=0$$
$$\frac{dy}{dx}=-\frac{x}{y},\ -\frac{-2}{3}=\frac{2}{3}$$

10. $$\frac{d}{dx}(x^2+y^2)=\frac{d}{dx}(9)$$
$$2x+2y\frac{dy}{dx}=0$$
$$\frac{dy}{dx}=-\frac{x}{y},\ -\frac{0}{3}=0$$

11. $$\frac{d}{dx}((x-1)^2+(y-1)^2)=\frac{d}{dx}(13)$$
$$2(x-1)1+(2(y-1)1)\frac{dy}{dx}=0$$
$$\frac{dy}{dx}=-\frac{x-1}{y-1},-\frac{3-1}{4-1}=-\frac{2}{3}$$

12. $$\frac{d}{dx}((x+2)^2+(y+3)^2)=\frac{d}{dx}25$$
$$2(x+2)1+(2(y+3)1)\frac{dy}{dx}=0$$
$$\frac{dy}{dx}=-\frac{x+2}{y+3},-\frac{1+2}{-7+3}=\frac{3}{4}$$

13. $$\frac{d}{dx}(x^2y-xy^2)=\frac{d}{dx}(4)$$
$$2xy-y^2+(2xy-x^2)\frac{dy}{dx}=0$$
$$\frac{dy}{dx}=-\frac{2xy-y^2}{2xy-x^2},$$

defined at every point except where $x=0$ or $y=\frac{x}{2}$.

14. $$\frac{d}{dx}(x)=\frac{d}{dx}(\cos y)$$
$$1=-\sin y\frac{dy}{dx}$$
$$\frac{dy}{dx}=-\frac{1}{\sin y},$$
defined everywhere.

15. $$\frac{d}{dx}(x^3+y^3)=\frac{d}{dx}(xy)$$
$$3x^2+3y^2\frac{dy}{dx}=y+x\frac{dx}{dy}$$
$$3x^2-y=(x-3y^2)\frac{dy}{dx}$$
$$\frac{dy}{dx}=\frac{3x^2-y}{x-3y^2},$$

defined everywhere except where $y^2=\frac{x}{3}$

16. $$\frac{d}{dx}(x^2+4xy+4y^2-3x)=\frac{d}{dx}(6)$$
$$2x+4y-3+(4x+8y)\frac{dy}{dx}=0$$
$$\frac{dy}{dx}=\frac{3-2x-4y}{4x+8y},$$

defined everywhere except where $y=-\frac{1}{2}x$

17. $$x^2+xy-y^2=1$$
$$\frac{d}{dx}(x^2)+\frac{d}{dx}(xy)-\frac{d}{dx}(y^2)=\frac{d}{dx}(1)$$
$$2x+x\frac{dy}{dx}+(y)(1)-2y\frac{dy}{dx}=0$$
$$(x-2y)\frac{dy}{dx}=-2x-y$$
$$\frac{dy}{dx}=\frac{-2x-y}{x-2y}=\frac{2x+y}{2y-x}$$

Slope at (2, 3): $\frac{2(2)+3}{2(1)-2}=\frac{7}{4}$

(a) Tangent: $y=\frac{7}{4}(x-2)+3$ or $y=\frac{7}{4}x-\frac{1}{2}$

(b) Normal: $y=-\frac{4}{7}(x-2)+3$ or $y=-\frac{4}{7}x+\frac{29}{7}$

18. $$x^2+y^2=25$$
$$\frac{d}{dx}(x^2)+\frac{d}{dx}(y^2)=\frac{d}{dx}(25)$$
$$2x+2y\frac{dy}{dx}=0$$
$$\frac{dy}{dx}=-\frac{x}{y}$$

Slope at $(3,-4)$: $-\frac{3}{-4}=\frac{3}{4}$

(a) Tangent: $y=\frac{3}{4}(x-3)+(-4)$ or $y=\frac{3}{4}x-\frac{25}{4}$

(b) Normal: $y=-\frac{4}{3}(x-3)+(-4)$ or $y=-\frac{4}{3}x$

19. $$x^2y^2=9$$
$$\frac{d}{dx}(x^2y^2)=\frac{d}{dx}(9)$$
$$(x^2)(2y)\frac{dy}{dx}+(y^2)(2x)=0$$
$$2x^2y\frac{dy}{dx}=-2xy^2$$
$$\frac{dy}{dx}=-\frac{2xy^2}{2x^2y}=-\frac{y}{x}$$

Slope at $(-1, 3)$: $-\frac{3}{-1}=3$

(a) Tangent: $y=3(x+1)+3$ or $y=3x+6$

(b) Normal: $y=-\frac{1}{3}(x+1)+3$ or $y=-\frac{1}{3}x+\frac{8}{3}$

20. $$y^2-2x-4y-1=0$$

$$\frac{d}{dx}(y^2)-\frac{d}{dx}(2x)-\frac{d}{dx}(4y)-\frac{d}{dx}(1)=\frac{d}{dx}(0)$$

$$2y\frac{dy}{dx}-2-4\frac{dy}{dx}-0=0$$

$$(2y-4)\frac{dy}{dx}=2$$

$$\frac{dy}{dx}=\frac{1}{y-2}$$

Slope at $(-2, 1)$: $\frac{1}{1-2}=-1$

(a) Tangent: $y=-(x+2)+1$ or $y=-x-1$

(b) Normal: $y=1(x+2)+1$ or $y=x+3$

21. $$6x^2+3xy+2y^2+17y-6=0$$

$$\frac{d}{dx}(6x^2)+\frac{d}{dx}(3xy)+\frac{d}{dx}(2y^2)+\frac{d}{dx}(17y)-\frac{d}{dx}(6)=\frac{d}{dx}(0)$$

$$12x+3x\frac{dy}{dx}+(3y)(1)+4y\frac{dy}{dx}+17\frac{dy}{dx}-0=0$$

$$3x\frac{dy}{dx}+4y\frac{dy}{dx}+17\frac{dy}{dx}=-12x-3y$$

$$(3x+4y+17)\frac{dy}{dx}=-12x-3y$$

$$\frac{dy}{dx}=\frac{-12x-3y}{3x+4y+17}$$

Slope at $(-1, 0)$: $\frac{-12(-1)-3(0)}{3(-1)+4(0)+17}=\frac{12}{14}=\frac{6}{7}$

(a) Tangent: $y=\frac{6}{7}(x+1)+0$ or $y=\frac{6}{7}x+\frac{6}{7}$

(b) Normal: $y=-\frac{7}{6}(x+1)+0$ or $y=-\frac{7}{6}x-\frac{7}{6}$

22. $$x^2-\sqrt{3}xy+2y^2=5$$

$$\frac{d}{dx}(x^2)-\sqrt{3}\frac{d}{dx}(xy)+2\frac{d}{dx}(y^2)=\frac{d}{dx}(5)$$

$$2x-\sqrt{3}(x)\frac{dy}{dx}-\sqrt{3}(y)(1)+4y\frac{dy}{dx}=0$$

$$(-x\sqrt{3}+4y)\frac{dy}{dx}=y\sqrt{3}-2x$$

$$\frac{dy}{dx}=\frac{y\sqrt{3}-2x}{-x\sqrt{3}+4y}$$

Slope at $(\sqrt{3}, 2)$: $\frac{2\sqrt{3}-2\sqrt{3}}{-\sqrt{3}\sqrt{3}+4(2)}=0$

(a) Tangent: $y=2$

(b) Normal: $x=\sqrt{3}$

23. $$2xy+\pi\sin y=2\pi$$

$$2\frac{d}{dx}(xy)+\pi\frac{d}{dx}(\sin y)=\frac{d}{dx}(2\pi)$$

$$2x\frac{dy}{dx}+2y(1)+\pi\cos y\frac{dy}{dx}=0$$

$$(2x+\pi\cos y)\frac{dy}{dx}=-2y$$

$$\frac{dy}{dx}=-\frac{2y}{2x+\pi\cos y}$$

Slope at $\left(1,\frac{\pi}{2}\right)$: $-\frac{2(\pi/2)}{2(1)+\pi\cos(\pi/2)}=-\frac{\pi}{2}$

(a) Tangent: $y=-\frac{\pi}{2}(x-1)+\frac{\pi}{2}$ or $y=-\frac{\pi}{2}x+\pi$

(b) Normal: $y=\frac{2}{\pi}(x-1)+\frac{\pi}{2}$ or $y=\frac{2}{\pi}x-\frac{2}{\pi}+\frac{\pi}{2}$

24. $x\sin 2y=y\cos 2x$

$$\frac{d}{dx}(x\sin 2y)=\frac{d}{dy}(y\cos 2x)$$

$$(x)(\cos 2y)(2)\frac{dy}{dx}+(\sin 2y)(1)=$$

$$(y)(-\sin 2x)(2)+(\cos 2x)\left(\frac{dy}{dx}\right)$$

$$(2x\cos 2y)\frac{dy}{dx}-(\cos 2x)\frac{dy}{dx}=-2y\sin 2x-\sin 2y$$

$$\frac{dy}{dx}=-\frac{2y\sin 2x+\sin 2y}{2x\cos 2y-\cos 2x}$$

Slope at $\left(\frac{\pi}{4},\frac{\pi}{2}\right)$: $-\frac{2\left(\frac{\pi}{2}\right)\sin\left(\frac{\pi}{2}\right)+\sin(\pi)}{2\left(\frac{\pi}{4}\right)\cos(\pi)-\cos\left(\frac{\pi}{2}\right)}$

$$=-\frac{(\pi)(1)+0}{\left(\frac{\pi}{2}\right)(-1)-0}=2$$

(a) Tangent: $y=2\left(x-\frac{\pi}{4}\right)+\frac{\pi}{2}$ or $y=2x$

(b) Normal: $y=-\frac{1}{2}\left(x-\frac{\pi}{4}\right)+\frac{\pi}{2}$ or $y=-\frac{1}{2}x+\frac{5\pi}{8}$

25. $$y=2\sin(\pi x-y)$$

$$\frac{dy}{dx}=\frac{d}{dx}2\sin(\pi x-y)$$

$$\frac{dy}{dx}=2\cos(\pi x-y)\left(\pi-\frac{dy}{dx}\right)$$

$$[1+2\cos(\pi x-y)]\frac{dy}{dx}=2\pi\cos(\pi x-y)$$

$$\frac{dy}{dx}=\frac{2\pi\cos(\pi x-y)}{1+2\cos(\pi x-y)}$$

Slope at $(1, 0)$: $\frac{2\pi\cos\pi}{1+2\cos\pi}=\frac{2\pi(-1)}{1+2(-1)}=2\pi$

(a) Tangent: $y=2\pi(x-1)+0$ or $y=2\pi x-2\pi$

(b) Normal: $y=-\frac{1}{2\pi}(x-1)+0$ or $y=-\frac{x}{2\pi}+\frac{1}{2\pi}$

26. $x^2\cos^2 y - \sin y = 0$

$$\frac{d}{dx}(x^2\cos^2 y) - \frac{d}{dx}(\sin y) = \frac{d}{dx}(0)$$

$$(x^2)(2\cos y)(-\sin y)\left(\frac{dy}{dx}\right) + (\cos^2 y)(2x) - (\cos y)\frac{dy}{dx} = 0$$

$$-(2x^2\cos y\sin y + \cos y)\frac{dy}{dx} = -2x\cos^2 y$$

$$\frac{dy}{dx} = \frac{2x\cos^2 y}{\cos y + 2x^2\cos y\sin y} = \frac{2x\cos y}{1+2x^2\sin y}$$

Slope at $(0, \pi)$: $\dfrac{2(0)\cos\pi}{1+2(0)^2\sin\pi} = 0$

(a) Tangent: $y = \pi$

(b) Normal: $x = 0$

27. $x^2 + y^2 = 1$

$$\frac{d}{dx}(x^2) + \frac{d}{dx}(y^2) = \frac{d}{dx}(1)$$

$$2x + 2yy' = 0$$

$$2yy' = -2x$$

$$y' = -\frac{x}{y}$$

$$y'' = \frac{d}{dx}\left(-\frac{x}{y}\right)$$

$$= -\frac{(y)(1)-(x)(y')}{y^2}$$

$$= -\frac{y - x\left(-\frac{x}{y}\right)}{y^2}$$

$$= -\frac{x^2+y^2}{y^3}$$

Since our original equation was $x^2 + y^2 = 1$, we may substitute 1 for $x^2 + y^2$, giving $y'' = -\dfrac{1}{y^3}$.

28. $x^{2/3} + y^{2/3} = 1$

$$\frac{d}{dx}(x^{2/3}) + \frac{d}{dx}(y^{2/3}) = \frac{d}{dx}(1)$$

$$\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}y' = 0$$

$$y' = -\frac{x^{-1/3}}{y^{-1/3}} = -\left(\frac{y}{x}\right)^{1/3}$$

$$y'' = \frac{d}{dx}\left[-\left(\frac{y}{x}\right)^{1/3}\right]$$

$$= -\frac{1}{3}\left(\frac{y}{x}\right)^{-2/3}\frac{d}{dx}\left(\frac{y}{x}\right)$$

$$= -\frac{1}{3}\left(\frac{y}{x}\right)^{-2/3}\frac{xy'-(y)(1)}{x^2}$$

$$= -\frac{1}{3}\frac{-(x)\left(\frac{y}{x}\right)^{1/3} - y}{x^{4/3}y^{2/3}}$$

$$= \frac{1}{3}\frac{x^{2/3}y^{1/3} + y}{x^{4/3}y^{2/3}}$$

$$= \frac{x^{2/3}+y^{2/3}}{3x^{4/3}y^{1/3}}$$

Since our original equation was $x^{2/3} + y^{2/3} = 1$, we may substitute 1 for $x^{2/3} + y^{2/3}$, giving $y'' = \dfrac{1}{3x^{4/3}y^{1/3}}$.

29. $y^2 = x^2 + 2x$

$$\frac{d}{dx}(y^2) = \frac{d}{dx}(x^2) + \frac{d}{dx}(2x)$$

$$2yy' = 2x + 2$$

$$y' = \frac{2x+2}{2y} = \frac{x+1}{y}$$

$$y'' = \frac{d}{dx}\left(\frac{x+1}{y}\right)$$

$$= \frac{(y)(1)-(x+1)y'}{y^2}$$

$$= \frac{y-(x+1)\left(\frac{x+1}{y}\right)}{y^2}$$

$$= \frac{y^2-(x+1)^2}{y^3}$$

Since our original equation was $y^2 = x^2 + 2x$, we may write $y^2 - (x+1)^2 = (x^2+2x) - (x^2+2x+1) = -1$, which gives $y = -\dfrac{1}{y^3}$.

30. $y^2 + 2y = 2x + 1$

$$\frac{d}{dx}(y^2+2y) = \frac{d}{dx}(2x+1)$$

$$(2y+2)y' = 2$$

$$y' = \frac{1}{y+1}$$

30. Continued

$$y'' = \frac{d}{dx}\frac{1}{y+1}$$
$$= -(y+1)^{-2}y'$$
$$= -(y+1)^{-2}\left(\frac{1}{y+1}\right)$$
$$= -\frac{1}{(y+1)^3}$$

31. $\frac{dy}{dx} = \frac{d}{dx}x^{9/4} = \frac{9}{4}x^{(9/4)-1} = \frac{9}{4}x^{5/4}$

32. $\frac{dy}{dx} = \frac{d}{dx}x^{-3/5} = -\frac{3}{5}x^{(-3/5)-1} = -\frac{3}{5}x^{-8/5}$

33. $\frac{dy}{dx} = \frac{d}{dx}\sqrt[3]{x} = \frac{d}{dx}x^{1/3} = \frac{1}{3}x^{(1/3)-1} = \frac{1}{3}x^{-2/3}$

34. $\frac{dy}{dx} = \frac{d}{dx}\sqrt[4]{x} = \frac{d}{dx}x^{1/4} = \frac{1}{4}x^{(1/4)-1} = \frac{1}{4}x^{-3/4}$

35. $\frac{dy}{dx} = \frac{d}{dx}(2x+5)^{-1/2} = -\frac{1}{2}(2x+5)^{(-1/2)-1}\frac{d}{dx}(2x+5)$

$$= -\frac{1}{2}(2x+5)^{-3/2}(2) = -(2x+5)^{-3/2}$$

36. $\frac{dy}{dx} = \frac{d}{dx}(1-6x)^{2/3}$

$$= \frac{2}{3}(1-6x)^{(2/3)-1}\frac{d}{dx}(1-6x)$$
$$= \frac{2}{3}(1-6x)^{-1/3}(-6)$$
$$= -4(1-6x)^{-1/3}$$

37. $\frac{dy}{dx} = \frac{d}{dx}\left(x\sqrt{x^2+1}\right)$

$$= x\frac{d}{dx}\sqrt{x^2+1} + \sqrt{x^2+1}\frac{d}{dx}(x)$$
$$= x\frac{d}{dx}(x^2+1)^{1/2} + (x^2+1)^{1/2}$$
$$= x \cdot \frac{1}{2}(x^2+1)^{-1/2}(2x) + (x^2+1)^{1/2}$$
$$= x^2(x^2+1)^{-1/2} + (x^2+1)^{1/2}$$

Note: This answer is equivalent to $\frac{2x^2+1}{\sqrt{x^2+1}}$.

38. $\frac{dy}{dx} = \frac{d}{dx}\frac{x}{\sqrt{x^2+1}} = \frac{(x^2+1)^{1/2}\frac{d}{dx}x - x\frac{d}{dx}(x^2+1)^{1/2}}{x^2+1}$

$$= \frac{(x^2+1)^{1/2} - x \cdot \frac{1}{2}(x^2+1)^{-1/2}(2x)}{x^2+1}$$
$$= \frac{x^2+1-x^2}{(x^2+1)(x^2+1)^{1/2}}$$
$$= \frac{1}{(x^2+1)^{3/2}}$$
$$= (x^2+1)^{-3/2}$$

39. $\frac{dy}{dx} = \frac{d}{dx}(1-x^{1/2})^{1/2}$

$$= \frac{1}{2}(1-x^{1/2})^{-1/2}\frac{d}{dx}(1-x^{1/2})$$
$$= \frac{1}{2}(1-x^{1/2})^{-1/2}\left(-\frac{1}{2}x^{-1/2}\right)$$
$$= -\frac{1}{4}(1-x^{1/2})^{-1/2}x^{-1/2}$$

40. $\frac{dy}{dx} = \frac{d}{dx}3(2x^{-1/2}+1)^{-1/3}$

$$= -(2x^{-1/2}+1)^{-4/3}\frac{d}{dx}(2x^{-1/2}+1)$$
$$= -(2x^{-1/2}+1)^{-4/3}(-x^{-3/2})$$
$$= x^{-3/2}(2x^{-1/2}+1)^{-4/3}$$

41. $\frac{dy}{dx} = \frac{d}{dx}3(\csc x)^{3/2}$

$$= \frac{9}{2}(\csc x)^{1/2}\frac{d}{dx}(\csc x)$$
$$= \frac{9}{2}(\csc x)^{1/2}(-\csc x \cot x)$$
$$= -\frac{9}{2}(\csc x)^{3/2}\cot x$$

42. $\frac{dy}{dx} = \frac{d}{dx}[\sin(x+5)]^{5/4}$

$$= \frac{5}{4}[\sin(x+5)]^{1/4}\frac{d}{dx}\sin(x+5)$$
$$= \frac{5}{4}[\sin(x+5)]^{1/4}\cos(x+5)$$

43. (a) If $f(x) = \frac{3}{2}x^{2/3} - 3$, then

$f'(x) = x^{-1/3}$ and $f''(x) = -\frac{1}{3}x^{-4/3}$

which contradicts the given equation $f''(x) = x^{-1/3}$.

(b) If $f(x) = \frac{9}{10}x^{5/3} - 7$, then

$f'(x) = \frac{3}{2}x^{2/3}$ and $f''(x) = x^{-1/3}$, which matches the given equation.

(c) Differentiating both sides of the given equation

$f''(x) = x^{-1/3}$ gives $f'''(x) = -\frac{1}{3}x^{-4/3}$, so it *must* be true that $f'''(x) = -\frac{1}{3}x^{-4/3}$.

(d) If $f'(x) = \frac{3}{2}x^{2/3} + 6$, then $f''(x) = x^{-1/3}$, which matches the given equation.

Conclusion: (b), (c), and (d) could be true.

44. (a) If $g'(t) = 4\sqrt[4]{t} - 4$, then

$$g''(t) = \frac{d}{dx}(4t^{1/4} - 4) = t^{-3/4} = \frac{1}{t^{3/4}},$$ which matches the given equation.

(b) Differentiating both sides of the given equation $g''(t) = \frac{1}{t^{3/4}} = t^{-3/4}$ gives $g'''(t) = -\frac{3}{4}t^{-7/4}$, which is not consistent with $g'''(t) = -\frac{4}{\sqrt[4]{t}}$.

(c) If $g(t) = t - 7 + \frac{16}{5}t^{5/4}$, then

$g'(t) = 1 + 4t^{1/4}$ and $g''(t) = t^{-3/4} = \frac{1}{t^{3/4}}$, which matches the given equation.

(d) If $g'(t) = \frac{1}{4}t^{1/4}$, then $g''(t) = \frac{1}{16}t^{-3/4}$, which contradicts the given equation.

Conclusion: (a) and (c) could be true.

45. (a)

$$y^4 = y^2 - x^2$$
$$\frac{d}{dx}(y^4) = \frac{d}{dx}(y^2) - \frac{d}{dx}x^2$$
$$4y^3\frac{dy}{dx} = 2y\frac{dy}{dx} - 2x$$
$$(4y^3 - 2y)\frac{dy}{dx} = -2x$$
$$\frac{dy}{dx} = \frac{-2x}{4y^3 - 2y} = \frac{x}{y - 2y^3}$$

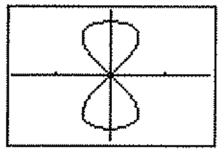
At $\left(\frac{\sqrt{3}}{4}, \frac{\sqrt{3}}{2}\right)$:

$$\text{Slope} = \frac{\frac{\sqrt{3}}{4}}{\frac{\sqrt{3}}{2} - 2\left(\frac{\sqrt{3}}{2}\right)^3}$$
$$= \frac{\frac{\sqrt{3}}{4}}{\frac{\sqrt{3}}{2} - \frac{3\sqrt{3}}{4}} \cdot \frac{\frac{4}{\sqrt{3}}}{\frac{4}{\sqrt{3}}} = \frac{1}{2-3} = -1$$

At $\left(\frac{\sqrt{3}}{4}, \frac{1}{2}\right)$:

$$\text{Slope} = \frac{\frac{\sqrt{3}}{4}}{\frac{1}{2} - 2\left(\frac{1}{2}\right)^3} = \frac{\frac{\sqrt{3}}{4}}{\frac{1}{2} - \frac{1}{4}} \cdot \frac{4}{4} = \frac{\sqrt{3}}{1} = \sqrt{3}$$

(b)



[−1.8, 1.8] by [−1.2, 1.2]
Parameter interval: $-1 \le t \le 1$

46. (a)

$$y^2(2 - x) = x^3$$
$$\frac{d}{dx}\left[y^2(2-x)\right] = \frac{d}{dx}(x^3)$$
$$(y^2)(-1) + (2-x)(2y)\frac{dy}{dx} = 3x^2$$
$$2y(2-x)\frac{dy}{dx} = 3x^2 + y^2$$
$$\frac{dy}{dx} = \frac{3x^2 + y^2}{2y(2-x)}$$

Slope at (1, 1): $\frac{3(1)^2 + (1)^2}{2(1)(2-1)} = \frac{4}{2} = 2$

Tangent: $y = 2(x - 1) + 1$ or $y = 2x - 1$

Normal: $y = -\frac{1}{2}(x-1) + 1$ or $y = -\frac{1}{2}x + \frac{3}{2}$

(b) One way is to graph the equations $y = \pm\sqrt{\frac{x^3}{2-x}}$.

47. (a) $(-1)^3(1)^2 = \cos(\pi)$ is true since both sides equal -1.

(b)

$$x^3y^2 = \cos(\pi y)$$
$$\frac{d}{dx}(x^3y^2) = \frac{d}{dx}\cos(\pi y)$$
$$(x^3)(2y)\frac{dy}{dx} + (y^2)(3x^2) = (-\sin \pi y)(\pi)\frac{dy}{dx}$$
$$(2x^3y + \pi \sin \pi y)\frac{dy}{dx} = -3x^2y^2$$
$$\frac{dy}{dx} = -\frac{3x^2y^2}{2x^3y + \pi \sin \pi y}$$

Slope at $(-1, 1)$: $-\frac{3(-1)^2(1)}{2(-1)^3(1) + \pi \sin \pi} = \frac{-3}{-2} = \frac{3}{2}$

The slope of the tangent line is $\frac{3}{2}$.

48. (a) When $x = 2$, we have $y^3 - 2y = -1$, or $y^3 - 2y + 1 = 0$. Clearly, $y = 1$ is one solution, and we may factor $y^3 - 2y + 1$ as $(y-1)(y^2 + y - 1)$. The solutions of $y^2 + y - 1 = 0$ are $y = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(-1)}}{2(1)} = \frac{-1 \pm \sqrt{5}}{2}$.

Hence, there are three possible y-values: 1, $\frac{-1-\sqrt{5}}{2}$, and $\frac{-1+\sqrt{5}}{2}$.

48. Continued

(b) $$y^3 - xy = -1$$

$$\frac{d}{dx}(y^3) - \frac{d}{dx}(xy) = \frac{d}{dx}(-1)$$

$$3y^2y' - xy' - (y)(1) = 0$$

$$(3y^2 - x)y' = y$$

$$y' = \frac{y}{3y^2 - x}$$

$$y'' = \frac{d}{dx}\frac{y}{3y^2 - x}$$

$$= \frac{(3y^2 - x)(y') - (y)(6yy' - 1)}{(3y^2 - x)^2}$$

$$= \frac{y - xy' - 3y^2y'}{(3y^2 - x)^2}$$

Since we are working with numerical information, there is no need to write a general expression for y'' in terms of x and y.

To evaluate $f'(2)$, evaluate the expression for y' using $x = 2$ and $y = 1$:

$$f'(2) = \frac{1}{3(1)^2 - 2} = \frac{1}{1} = 1$$

To evaluate $f''(2)$, evaluate the expression for y'' using $x = 2$, $y = 1$, and $y' = 1$:

$$f''(2) = \frac{(1) - 2(1) - 3(1)^2(1)}{[3(1)^2 - 2]^2} = \frac{-4}{1} = -4$$

49. Find the two points:

The curve crosses the x-axis when $y = 0$, so the equation becomes $x^2 + 0x + 0 = 7$, or $x^2 = 7$. The solutions are $x = \pm\sqrt{7}$, so the points are $(\pm\sqrt{7}, 0)$.

Show tangents are parallel:

$$x^2 + xy + y^2 = 7$$

$$\frac{d}{dx}(x^2) + \frac{d}{dx}(xy) + \frac{d}{dx}(y^2) = \frac{d}{dx}(7)$$

$$2x + x\frac{dy}{dx} + (y)(1) + 2y\frac{dy}{dx} = 0$$

$$(x + 2y)\frac{dy}{dx} = -(2x + y)$$

$$\frac{dy}{dx} = -\frac{2x + y}{x + 2y}$$

Slope at $(\sqrt{7}, 0)$: $-\dfrac{2\sqrt{7}) + 0}{\sqrt{7} + 2(0)} = -2$

Slope at $(-\sqrt{7}, 0)$: $-\dfrac{2(-\sqrt{7}) + 0}{-\sqrt{7} + 2(0)} = -2$

The tangents at these points are parallel because they have the same slope. The common slope is -2.

50. $$x^2 + xy + y^2 = 7$$

$$\frac{d}{dx}(x^2) + \frac{d}{dx}(xy) + \frac{d}{dx}(y^2) = \frac{d}{dx}(7)$$

$$2x + x\frac{dy}{dx} + (y)(1) + 2y\frac{dy}{dx} = 0$$

$$(x + 2y)\frac{dy}{dx} = -(2x + y)$$

$$\frac{dy}{dx} = -\frac{2x + y}{x + 2y}$$

(a) The tangent is parallel to the x-axis when $\dfrac{dy}{dx} = -\dfrac{2x + y}{x + 2y} = 0$, or $y = -2x$.

Substituting $-2x$ for y in the original equation, we have

$$x^2 + xy + y^2 = 7$$

$$x^2 + (x)(-2x) + (-2x)^2 = 7$$

$$x^2 - 2x^2 + 4x^2 = 7$$

$$3x^2 = 7$$

$$x = \pm\sqrt{\frac{7}{3}}$$

The points are $\left(-\sqrt{\frac{7}{3}}, 2\sqrt{\frac{7}{3}}\right)$ and $\left(\sqrt{\frac{7}{3}}, -2\sqrt{\frac{7}{3}}\right)$.

(b) Since x and y are interchangeable in the original equation, $\dfrac{dx}{dy}$ can be obtained by interchanging x and y in the expression for $\dfrac{dy}{dx}$. That is, $\dfrac{dx}{dy} = -\dfrac{2y + x}{x + 2y}$. The tangent is parallel to the y-axis when $\dfrac{dx}{dy} = 0$, or $x = -2y$. Substituting $-2y$ for x in the original equation, we have:

$$x^2 + xy + y^2 = 7$$

$$(-2y)^2 + (-2y)(y) + y^2 = 7$$

$$4y^2 - 2y^2 + y^2 = 7$$

$$3y^2 = 7$$

$$y = \pm\sqrt{\frac{7}{3}}$$

The points are $\left(-2\sqrt{\frac{7}{3}}, \sqrt{\frac{7}{3}}\right)$ and $\left(2\sqrt{\frac{7}{3}}, -\sqrt{\frac{7}{3}}\right)$.

Note that these are the same points that would be obtained by interchanging x and y in the solution to part (a).

51. First curve:

$$2x^2 + 3y^2 = 5$$

$$\frac{d}{dx}(2x^2) + \frac{d}{dx}(3y^2) = \frac{d}{dx}(5)$$

$$4x + 6y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{4x}{6y} = -\frac{2x}{3y}$$

51. Continued

Second curve:

$$y^2 = x^3$$
$$\frac{d}{dx}y^2 = \frac{d}{dx}x^3$$
$$2y\frac{dy}{dx} = 3x^2$$
$$\frac{dy}{dx} = \frac{3x^2}{2y}$$

At (1, 1), the slopes are $-\frac{2}{3}$ and $\frac{3}{2}$ respectively.

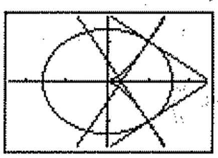
At (1, −1), the slopes are $\frac{2}{3}$ and $-\frac{3}{2}$ respectively. In both cases, the tangents are perpendicular. To graph the curves and normal lines, we may use the following parametric equations for $-\pi \le t \le \pi$:

First curve: $x = \sqrt{\frac{5}{2}}\cos t,\ y = \sqrt{\frac{5}{3}}\sin t$

Second curve: $x = \sqrt[3]{t^2},\ y = t$

Tangents at (1, 1): $x = 1 + 3t, y = 1 - 2t$
$x = 1 + 2t,\ y = 1 + 3t$

Tangents at (1, −1): $x = 1 + 3t, y = -1 + 2t$
$x = 1 + 2t,\ y = -1 - 3t$



[−2.4, 2.4] by [−1.6, 1.6]

52. $v(t) = s'(t) = \frac{d}{dt}(4 + 6t)^{3/2} = \frac{3}{2}(4 + 6t)^{1/2}(6)$

$= 9(4 + 6t)^{1/2}$

$a(t) = v'(t) = \frac{d}{dt}[9(4 + 6t)^{1/2}]$

$= \frac{9}{2}(4 + 6t)^{-1/2}(6) = 27(4 + 6t)^{-1/2}$

At $t = 2$, the velocity is $v(2) = 36$ m/sec and the acceleration is $a(2) = \frac{27}{4}\text{ m/sec}^2$.

53. Acceleration $= \frac{dv}{dt} = \frac{d}{dt}[8(s - t)^{1/2} + 1]$

$= 4(s - t)^{-1/2}\left(\frac{ds}{dt} - 1\right)$

$= 4(s - t)^{-1/2}\ (v - 1)$

$= 4(s - t)^{-1/2}\ [(8(s - t)^{1/2} + 1) - 1]$

$= 32(s - t)^{-1/2}\ (s - t)^{1/2}$

$= 32\text{ ft/sec}^2$

54.

$$y^4 - 4y^2 = x^4 - 9x^2$$
$$\frac{d}{dx}(y^4) - \frac{d}{dx}(4y^2) = \frac{d}{dx}(x^4) - \frac{d}{dx}(9x^2)$$
$$4y^3\frac{dy}{dx} - 8y\frac{dy}{dx} = 4x^3 - 18x$$
$$\frac{dy}{dx} = \frac{4x^3 - 18x}{4y^3 - 8y} = \frac{2x^3 - 9x}{2y^3 - 4y}$$

Slope at (3, 2): $\frac{2(3)^3 - 9(3)}{2(2)^3 - 4(2)} = \frac{27}{8}$

Slope at (−3, 2): $\frac{2(-3)^3 - 9(-3)}{2(2)^3 - 4(2)} = -\frac{27}{8}$

Slope at (−3, −2): $\frac{2(-3)^3 - 9(-3)}{2(-2)^3 - 4(-2)} = \frac{27}{8}$

Slope at (3, −2): $\frac{2(3)^3 - 9(3)}{2(-2)^3 - 4(-2)} = -\frac{27}{8}$

55. (a)

$$x^3 + y^3 - 9xy = 0$$
$$\frac{d}{dx}(x^3) + \frac{d}{dx}(y^3) - 9\frac{d}{dx}(xy) = \frac{d}{dx}(0)$$
$$3x^2 + 3y^2\frac{dy}{dx} - 9x\frac{dy}{dx} - 9(y)(1) = 0$$
$$(3y^2 - 9x)\frac{dy}{dx} = 9y - 3x^2$$
$$\frac{dy}{dx} = \frac{9y - 3x^2}{3y^2 - 9x} = \frac{3y - x^2}{y^2 - 3x}$$

Slope at (4, 2): $\frac{3(2) - (4)^2}{(2)^2 - 3(4)} = \frac{-10}{-8} = \frac{5}{4}$

Slope at (2, 4): $\frac{3(4) - (2)^2}{(4)^2 - 3(2)} = \frac{8}{10} = \frac{4}{5}$

(b) The tangent is horizontal when

$\frac{dy}{dx} = \frac{3y - x^2}{y^2 - 3x} = 0$, or $y = \frac{x^2}{3}$.

Substituting $\frac{x^2}{3}$ for y in the original equation, we have:

$$x^3 + y^3 - 9xy = 0$$
$$x^3 + \left(\frac{x^2}{3}\right)^3 - 9x\left(\frac{x^2}{3}\right) = 0$$
$$x^3 + \frac{x^6}{27} - 3x^3 = 0$$
$$\frac{x^3}{27}(x^3 - 54) = 0$$
$$x = 0 \text{ or } x = \sqrt[3]{54} = 3\sqrt[3]{2}$$

At $x = 0$, we have $y = \frac{0^2}{3} = 0$, which gives the point (0, 0), which is the origin. At $x = 3\sqrt[3]{2}$, we have $y = \frac{1}{3}(3\sqrt[3]{2})^2 = \frac{1}{3}(9\sqrt[3]{4}) = 3\sqrt[3]{4}$, so the point other than the origin is $(3\sqrt[3]{2}, 3\sqrt[3]{4})$ or approximately (3.780, 4.762).

55. Continued

(c) The equation $x^3 + y^3 - 9xy$ is not affected by interchanging x and y, so its graph is symmetric about the line $y = x$ and we may find the desired point by interchanging the x-value and the y-value in the answer to part (b). The desired point is $(3\sqrt[3]{4}, 3\sqrt[3]{2})$ or approximately (4.762, 3.780).

56.
$$x^2 + 2xy - 3y^2 = 0$$
$$\frac{d}{dx}(x^2) + 2\frac{d}{dx}(xy) - \frac{d}{dx}(3y^2) = \frac{d}{dx}(0)$$
$$2x + 2x\frac{dy}{dx} + 2(y)(1) - 6y\frac{dy}{dx} = 0$$
$$(2x - 6y)\frac{dy}{dx} = -2x - 2y$$
$$\frac{dy}{dx} = \frac{-2x - 2y}{2x - 6y} = \frac{x + y}{3y - x}$$

At (1, 1) the curve has slope $\frac{1+1}{3(1)-1} = \frac{2}{2} = 1$, so the normal line is $y = -1(x - 1) + 1$ or $y = -x + 2$.

Substituting $-x + 2$ for y in the original equation, we have:
$$x^2 + 2xy - 3y^2 = 0$$
$$x^2 + 2x(-x + 2) - 3(-x + 2)^2 = 0$$
$$x^2 - 2x^2 + 4x - 3(x^2 - 4x + 4) = 0$$
$$-4x^2 + 16x - 12 = 0$$
$$-4(x - 1)(x - 3) = 0$$
$$x = 1 \text{ or } x = 3$$

Since the given point (1,1) had $x = 1$, we choose $x = 3$ and so $y = -(3) + 2 = -1$. The desired point is (3, −1).

57.
$$xy + 2x - y = 0$$
$$\frac{d}{dx}(xy) + \frac{d}{dx}(2x) - \frac{d}{dx}(y) = \frac{d}{dx}(0)$$
$$x\frac{dy}{dx} + (y)(1) + 2 - \frac{dy}{dx} = 0$$
$$(x - 1)\frac{dy}{dx} = -2 - y$$
$$\frac{dy}{dx} = \frac{-2 - y}{x - 1} = \frac{2 + y}{1 - x}$$

Since the slope of the line $2x + y = 0$ is −2, we wish to find points where the normal has slope −2, that is, where the tangent has slope $\frac{1}{2}$. Thus, we have
$$\frac{2 + y}{1 - x} = \frac{1}{2}$$
$$2(2 + y) = 1 - x$$
$$4 + 2y = 1 - x$$
$$x = -2y - 3$$

Substituting $-2y - 3$ in the original equation, we have:
$$xy + 2x - y = 0$$
$$(-2y - 3)y + 2(-2y - 3) - y = 0$$
$$-2y^2 - 8y - 6 = 0$$
$$-2(y + 1)(y + 3) = 0$$
$$y = -1 \text{ or } y = -3$$

At $y = -1, x = -2y - 3 = 2 - 3 = -1$.
At $y = -3: x = -2y - 3 = 6 - 3 = 3$.

The desired points are (−1, −1) and (3, −3).
Finally, we find the desired normals to the curve, which are the lines of slope −2 passing through each of these points. At (−1, −1), the normal line is $y = -2(x + 1) - 1$ or $y = -2x - 3$. At (3, −3), the normal line is $y = -2(x - 3) - 3$ or $y = -2x + 3$.

58.
$$x = y^2$$
$$\frac{d}{dx}(x) = \frac{d}{dx}(y^2)$$
$$1 = 2y\frac{dy}{dx}$$
$$\frac{dy}{dx} = \frac{1}{2y}$$

The normal line at (x, y) has slope $-2y$. Thus, the normal line at (b^2, b) is $y = -2b(x - b^2) + b$, or $y = -2bx + 2b^3 + b$.

This line intersects the x-axis at $x = \frac{2b^3 + b}{2b} = b^2 + \frac{1}{2}$, which is the value of a and must be greater than $\frac{1}{2}$ if $b \neq 0$.

The two normals at $(b^2, \pm b)$ will be perpendicular when they have slopes ±1, which gives $-2y = \pm 1$ or $y = \pm\frac{1}{2}\left(\text{or } b = \pm\frac{1}{2}\right)$. The corresponding value of a is $b^2 + \frac{1}{2} = \left(\frac{1}{2}\right)^2 + \frac{1}{2} = \frac{3}{4}$. Thus, the two nonhorizontal normals are perpendicular when $a = \frac{3}{4}$.

59. False.
$$\frac{d}{dx}(xy^2 + x) = \frac{d}{dx}(1)$$
$$y^2 + 1 + 2xy\frac{dy}{dx} = 0$$
$$\frac{dy}{dx} = \frac{-1 - y}{2xy}, \quad \frac{-1 - 1}{2\left(\frac{1}{2}\right)1} = -2$$

60. True. By the power rule.
$$y = (x)^{1/3}$$
$$\frac{dy}{dx} = \frac{d}{dx}(x)^{1/3} = \frac{1}{3}x^{2/3} = \frac{1}{3x^{2/3}}$$

61. A. $\frac{d}{dx}(x^2 - xy + y^2) = \frac{d}{dx}(1)$

$$2x - y + (-x + 2y)\frac{dy}{dx} = 0$$
$$\frac{dy}{dx} = \frac{y-2x}{2y-x}$$

62. A. $\frac{d^2y}{dx} = \frac{d}{dx}\left(\frac{y-2x}{2y-x}\right) = \frac{d}{dx}(0)$

$$= \frac{-3y+3x}{(x-2y)^2}(y')$$
$$= -\frac{6}{(2y-x)^3}$$

63. E. $\frac{d}{dx}(y) = \frac{d}{dx}x^{3/4}$

$$\frac{dy}{dx} = \frac{3}{4}x^{-1/4} = \frac{3}{4x^{1/4}}$$

64. C. $\frac{d}{dx}(y^2 - x^2) = \frac{d}{dx}(1)$

$$-2x + 2y\frac{dy}{dx} = 0$$
$$\frac{dy}{dx} = \frac{x}{y} = \frac{1}{\sqrt{2}}$$

65. **(a)** $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$b^2x^2 + a^2y^2 = a^2b^2$$
$$\frac{d}{dx}(b^2x^2) + \frac{d}{dx}(a^2y^2) = \frac{d}{dx}(a^2b^2)$$
$$2b^2x + 2a^2y\frac{dy}{dx} = 0$$
$$\frac{dy}{dx} = -\frac{2b^2x}{2a^2y} = \frac{b^2x}{a^2y}$$

The slope at (x_1, y_1) is $-\frac{b^2x_1}{a^2y_1}$.

The tangent line is $y - y_1 = -\frac{b^2x_1}{a^2y_1}(x - x_1)$. This gives:

$$a^2y_1y - a^2y_1^2 = -b^2x_1x + b^2x_1^2$$
$$a^2y_1y + b^2x_1x = a^2y_1^2 + b^2x_1^2.$$

But $a^2y_1^2 + b^2x_1^2 = a^2b^2$ since (x_1, y_1) is on the ellipse. Therefore, $a^2y_1y + b^2x_1x = a^2b^2$, and dividing by a^2b^2 gives $\frac{x_1x}{a^2} + \frac{y_1y}{b^2} = 1$.

(b) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$$b^2x^2 - a^2y^2 = a^2b^2$$
$$\frac{d}{dx}(b^2x^2) - \frac{d}{dx}(a^2y^2) = \frac{d}{dx}(a^2b^2)$$
$$2b^2x - 2a^2y\frac{dy}{dx} = 0$$
$$\frac{dy}{dx} = \frac{-2b^2x}{-2a^2y} = \frac{b^2x}{a^2y}$$

The slope at (x_1, y_1) is $\frac{b^2x_1}{a^2y_1}$.

The tangent line is $y - y_1 = -\frac{b^2x_1}{a^2y_1}(x - x_1)$.

This gives:

$$a^2y_1y - a^2y_1^2 = b^2x_1x - b^2x_1^2$$
$$b^2x_1^2 - a^2y_1^2 = b^2x_1x - a^2y_1y$$

But $b^2x_1^2 - a^2y_1^2 = a^2b^2$ since (x_1, y_1) is on the hyperbola. Therefore, $b^2x_1x - a^2y_1y = a^2b^2$, and dividing by a^2b^2 gives $\frac{x_1x}{a^2} - \frac{y_1y}{b^2} = 1$.

66. **(a)** Solve for y:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$
$$-\frac{y^2}{b^2} = -\frac{x^2}{a^2} + 1$$
$$y^2 = \frac{b^2}{a^2}(x^2 - a^2)$$
$$y = \pm\frac{b}{a}\sqrt{x^2 - a^2}$$

(b) $\lim_{x\to\infty}\frac{f(x)}{g(x)} = \lim_{x\to\infty}\frac{\frac{b}{a}\sqrt{x^2-a^2}}{\frac{b}{a}|x|}$

$$= \lim_{x\to\infty}\frac{\sqrt{x^2-a^2}}{\sqrt{x^2}}$$
$$= \lim_{x\to\infty}\sqrt{1-\frac{a^2}{x^2}} = 1$$

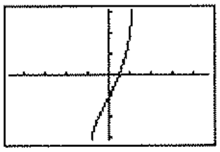
(c) $\lim_{x\to\infty}\frac{f(x)}{g(x)} = \lim_{x\to\infty}\frac{-\frac{b}{a}\sqrt{x^2-a^2}}{-\frac{b}{a}|x|}$

$$= \lim_{x\to\infty}\frac{\sqrt{x^2-a^2}}{\sqrt{x^2}}$$
$$= \lim_{x\to\infty}\sqrt{1-\frac{a^2}{x^2}} = 1$$

Section 3.8 Derivatives of Inverse Trigonometric Functions (pp. 165–171)

Exploration 1 Finding a derivative on an Inverse Graph Geometrically

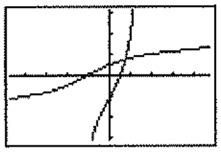
1. The graph is shown at the right. It appears to be a one-to-one function



[−4.7, 4.7] by [−3.1, 3.1]

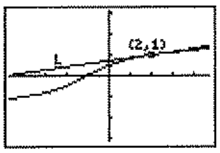
2. $f'(x) = 5x^4 + 2$. The fact that this function is always positive enables us to conclude that f is everywhere increasing, and hence one-to-one.

3. The graph of f^{-1} is shown to the right, along with the graph of f. The graph of f^{-1} is obtained from the graph of f by reflecting it in the line $y = x$.



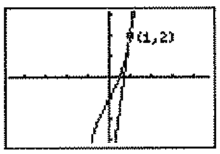
[−4.7, 4.7] by [−3.1, 3.1]

4. The line L is tangent to the graph of f^{-1} at the point $(2, 1)$.



[−4.7, 4.7] by [−3.1, 3.1]

5. The reflection of line L is tangent to the graph of f at the point



[−4.7, 4.7] by [−3.1, 3.1]

6. The reflection of the line L is the tangent line to the graph of $y = x^5 + 2x - 1$ at the point $(1, 2)$. The slope is $\frac{dy}{dx}$ at $x = 1$, which is 7.

7. The slope of L is the reciprocal of the slope of its reflection $\left(\text{since } \frac{\Delta y}{\Delta x} \text{ gets reflected to become } \frac{\Delta x}{\Delta y}\right)$. It is $\frac{1}{7}$.

8. $\frac{1}{7}$

Quick Review 3.8

1. Domain: $[-1, 1]$

Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

At 1: $\frac{\pi}{2}$

2. Domain: $[-1, 1]$

Range: $[0, \pi]$

At $1 : 0$

3. Domain: all reals

Range: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

At $1 : \frac{\pi}{4}$

4. Domain: $(-\infty, -1] \cup [1, \infty)$

Range: $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$

At 1:0

5. Domain: all reals
Range: all reals
At 1:1

6. $f(x) = y = 3x - 8$

$$y + 8 = 3x$$
$$x = \frac{y+8}{3}$$

Interchange x and y:

$$y = \frac{x+8}{3}$$
$$f^{-1}(x) = \frac{x+8}{3}$$

7. $f(x) = y = \sqrt[3]{x+5}$

$$y^3 = x + 5$$
$$x = y^3 - 5$$

Interchange x and y:

$$y = x^3 - 5$$
$$f^{-1}(x) = x^3 - 5$$

8. $f(x) = y = \frac{8}{x}$

$$x = \frac{8}{y}$$

Interchange x and y:

$$y = \frac{8}{x}$$
$$f^{-1}(x) = \frac{8}{x}$$

9. $f(x)=y=\frac{3x-2}{x}$

$xy=3x-2$

$(y-3)x=-2$

$x=\frac{-2}{y-3}=\frac{2}{3-y}$

Interchange x and y:

$y=\frac{2}{3-x}$

$f^{-1}(x)=\frac{2}{3-x}$

10. $f(x)=y=\arctan\frac{x}{3}$

$\tan y=\frac{x}{3},\ -\frac{\pi}{2}<y<\frac{\pi}{2}$

$x=3\tan y,\ -\frac{\pi}{2}<y<\frac{\pi}{2}$

Interchange x and y:

$y=3\tan x,\ -\frac{\pi}{2}<x<\frac{\pi}{2}$

$f^{-1}(x)=3\tan x,\ -\frac{\pi}{2}<x<\frac{\pi}{2}$

Section 3.8 Exercises

1. $\frac{dy}{dx}=\frac{d}{dx}\cos^{-1}(x^2)=-\frac{1}{\sqrt{1-(x^2)^2}}\frac{d}{dx}(x^2)$

$=-\frac{1}{\sqrt{1-x^4}}(2x)=-\frac{2x}{\sqrt{1-x^4}}$

2. $\frac{dy}{dx}=\frac{d}{dx}\cos^{-1}\left(\frac{1}{x}\right)=-\frac{1}{\sqrt{1-\left(\frac{1}{x}\right)^2}}\frac{d}{dx}\left(\frac{1}{x}\right)$

$=-\frac{1}{\sqrt{1-\left(\frac{1}{x}\right)^2}}\left(-\frac{1}{x^2}\right)=\frac{1}{|x|\sqrt{x^2-1}}$

3. $\frac{dy}{dt}=\frac{d}{dt}\sin^{-1}\sqrt{2}t=\frac{1}{\sqrt{1-(\sqrt{2}t)^2}}\frac{d}{dt}(\sqrt{2}t)=\frac{\sqrt{2}}{\sqrt{1-2t^2}}$

4. $\frac{dy}{dt}=\frac{d}{dt}\sin^{-1}(1-t)=\frac{1}{\sqrt{1-(1-t)^2}}\frac{d}{dt}(1-t)$

$=-\frac{1}{\sqrt{2t-t^2}}$

5. $\frac{dy}{dt}=\frac{d}{dt}\sin^{-1}\left(\frac{3}{t^2}\right)=\frac{1}{\sqrt{1-\left(\frac{3}{t^2}\right)^2}}\frac{d}{dt}\left(\frac{3}{t^2}\right)$

$=\frac{1}{\sqrt{1-\frac{9}{t^4}}}\left(-\frac{6}{t^3}\right)=-\frac{6}{t\sqrt{t^4-9}}$

6. $\frac{dy}{ds}=\frac{d}{ds}(s\sqrt{1-s^2})+\frac{d}{ds}(\cos^{-1}s)$

$=(s)\left(\frac{1}{2\sqrt{1-s^2}}\right)(-2s)+(\sqrt{1-s^2})(1)-\frac{1}{\sqrt{1-s^2}}$

$=-\frac{s^2}{\sqrt{1-s^2}}+\sqrt{1-s^2}-\frac{1}{\sqrt{1-s^2}}$

$=-\frac{-s^2+(1-s^2)-1}{\sqrt{1-s^2}}$

$=-\frac{2s^2}{\sqrt{1-s^2}}$

7. $\frac{dy}{dx}=\frac{d}{dx}(x\sin^{-1}x)+\frac{d}{dx}(\sqrt{1-x^2})$

$=(x)\left(\frac{1}{\sqrt{1-x^2}}\right)+(\sin^{-1}x)(1)+\frac{1}{2\sqrt{1-x^2}}(-2x)$

$=\sin^{-1}x$

8. $\frac{dy}{dx}=\frac{d}{dx}[\sin^{-1}(2x)]^{-1}$

$=-[\sin^{-1}(2x)]^{-2}\frac{d}{dx}\sin^{-1}(2x)$

$=-[\sin^{-1}(2x)]^{-2}\frac{1}{\sqrt{1-4x^2}}(2)$

$=-\frac{2}{[\sin^{-1}(2x)]^2\sqrt{1-4x^2}}$

9. $x(t)=\sin^{-1}\left(\frac{t}{4}\right)$

$y=\sin^{-1}u \qquad u=\frac{t}{4}$

$\sin y=u \qquad \frac{du}{dt}=\frac{1}{4}$

$\frac{d}{du}(\sin y)=\frac{d}{du}u$

$\cos y\frac{dy}{du}=1$

$\frac{dy}{du}=\frac{1}{\cos y}$

$\frac{d}{du}(\sin^{-1}u)=\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$

$v(t)=\frac{1}{4\sqrt{1-t^2/16}}$

$v(3)=\frac{1}{4\sqrt{1-9/16}}=\frac{\sqrt{7}}{7}$

10. See Problem 9.

$u=\frac{\sqrt{t}}{4}$

$\frac{du}{dt}=\frac{1}{8\sqrt{t}}$

$v(t)=\frac{1}{8\sqrt{t}\sqrt{1-t/16}}$

$v(4)=\frac{1}{8\sqrt{4}\sqrt{1-4/16}}=\frac{\sqrt{3}}{24}$

11. $x(t)=\tan^{-1}t$

$$y=\tan^{-1}t$$
$$\frac{d}{dt}\tan y=\frac{d}{dt}t$$
$$\sec^2 y\frac{dy}{dx}=1$$
$$\frac{dy}{dx}=\frac{1}{\sec^2 y}=\frac{1}{1+(\tan y)^2}=\frac{1}{1+t^2}$$
$$x(2)=\frac{1}{1+2^2}=\frac{1}{5}$$

12. See problem 11.

$$x(t)=\frac{2t}{1+t^2}$$
$$x(1)=\frac{2(1)}{1+(1)^2}=1$$

13. $$\frac{dy}{ds}=\frac{d}{ds}\sec^{-1}(2s+1)=\frac{1}{|2s+1|\sqrt{(2s+1)^2-1}}\frac{d}{ds}(2s+1)=\frac{1}{|2s+1|\sqrt{4s^2+4s}}(2)=\frac{1}{|2s+1|\sqrt{s^2+s}}$$

14. $$\frac{dy}{ds}=\frac{d}{ds}\sec^{-1}5s=\frac{1}{|5s|\sqrt{(5s)^2-1}}\frac{d}{ds}(5s)=\frac{1}{|s|\sqrt{25s^2-1}}$$

15. $$\frac{dy}{dx}=\frac{d}{dx}\csc^{-1}(x^2+1)=-\frac{1}{|x^2+1|\sqrt{(x^2+1^2-1}}\frac{d}{dx}(x^2+1)=-\frac{2x}{(x^2+1)\sqrt{x^4+2x^2}}=-\frac{2}{(x^2+1)\sqrt{x^2+2}}$$

Note that the condition $x>0$ is required in the last step.

16. $$\frac{dy}{dx}=\frac{d}{dx}\csc^{-1}\left(\frac{x}{2}\right)=-\frac{1}{\left|\frac{x}{2}\right|\sqrt{\left(\frac{x}{2}\right)^2-1}}\frac{d}{dx}\left(\frac{x}{2}\right)=-\frac{2}{|x|\sqrt{x^2-4}}$$

17. $$\frac{dy}{dt}=\frac{d}{dt}\sec^{-1}\left(\frac{1}{t}\right)=\frac{1}{\left|\frac{1}{t}\right|\sqrt{\left(\frac{1}{t}\right)^2-1}}\frac{d}{dt}\left(\frac{1}{t}\right)=\frac{1}{\left|\frac{1}{t}\right|\sqrt{\left(\frac{1}{t}\right)^2-1}}\left(-\frac{1}{t^2}\right)=-\frac{1}{\sqrt{1-t^2}}$$

Note that the condition $t>0$ is required in the last step.

18. $$\frac{dy}{dt}=\frac{d}{dt}\cot^{-1}\sqrt{t}=-\frac{1}{1+(\sqrt{t})^2}\frac{d}{dt}\sqrt{t}=-\frac{1}{2\sqrt{t}(t+1)}$$

19. $$\frac{dy}{dt}=-\frac{d}{dt}\cot^{-1}\sqrt{t-1}=-\frac{1}{1+(\sqrt{t-1})^2}\frac{d}{dt}\sqrt{t-1}=-\left(\frac{1}{1+t-1}\right)\left(\frac{1}{2\sqrt{t-1}}\right)=-\frac{1}{2t\sqrt{t-1}}$$

20. $$\frac{dy}{ds}=\frac{d}{ds}\sqrt{s^2-1}-\frac{d}{ds}\sec^{-1}s=\frac{1}{2\sqrt{s^2-1}}(2s)-\frac{1}{|s|\sqrt{s^2-1}}=\frac{s|s|-1}{|s|\sqrt{s^2-1}}$$

21. $$\frac{dy}{dx}=\frac{d}{dx}(\tan^{-1}\sqrt{x^2-1})+\frac{d}{dx}(\csc^{-1}x)=\frac{1}{1+(\sqrt{x^2-1})^2}\frac{d}{dx}(\sqrt{x^2-1})-\frac{1}{|x|\sqrt{x^2-1}}$$
$$=\frac{1}{x^2}\frac{1}{2\sqrt{x^2-1}}(2x)-\frac{1}{|x|\sqrt{x^2-1}}=\frac{1}{x\sqrt{x^2-1}}-\frac{1}{|x|\sqrt{x^2-1}}=0$$

Note that the condition $x>1$ is required in the last step.

22. $$\frac{dy}{dx}=\frac{d}{dx}\left(\cot^{-1}\frac{1}{x}\right)-\frac{d}{dx}(\tan^{-1}x)=-\frac{1}{1+\left(\frac{1}{x^2}\right)}\frac{d}{dx}\left(\frac{1}{x}\right)-\frac{1}{1+x^2}$$
$$=\left(-\frac{1}{1+\frac{1}{x^2}}\right)\left(-\frac{1}{x^2}\right)-\frac{1}{1+x^2}=\frac{1}{x^2+1}-\frac{1}{1+x^2}=0,\ x\neq 0$$

The condition $x\neq 0$ is required because the original function was undefined when $x\neq 0$.

23.
$$y = \sec^{-1} x$$
$$\frac{dy}{dx}\sec y = \frac{d}{dx}x$$
$$\sec y \ \tan y\frac{dy}{dx} = 1$$
$$\frac{dy}{dx} = \frac{1}{\sec y \ \tan y}$$
$$\frac{dy}{dx} = \frac{1}{x\sqrt{x^2-1}}$$
$$y'(2) = \frac{1}{2\sqrt{4-1}} = \frac{1}{2\sqrt{3}} = 0.289$$
$$y(2) = \sec^{-1} 2 = 2.203$$
$$y = 0.289(x-2)+2.203$$
$$y = 0.289x+1.625$$

24.
$$y = \tan^{-1}(x)$$
$$\frac{d}{dx}(\tan y) = \frac{d}{dx}x$$
$$\sec^2 y\frac{dy}{dx} = 1$$
$$\frac{dy}{dx} = \frac{1}{\sec^2 y}$$
$$= \frac{1}{1+(\tan y)^2}$$
$$= \frac{1}{1+x^2}$$
$$y'(2) = \frac{1}{1+2^2}$$
$$y'(2) = \frac{1}{5}$$
$$y(2) = \tan^{-1}(x) = 1.107$$
$$y = \frac{1}{5}(x-2)+1.107$$
$$y = \frac{1}{5}x+0.707$$

25. $y = \sin^{-1}\left(\frac{x}{4}\right)$

$$y = \sin^{-1} u \qquad u = \frac{x}{4}$$
$$\sin y = u \qquad \frac{du}{dx} = \frac{1}{4}$$
$$\frac{d}{du}(\sin y) = \frac{d}{d}u$$
$$\cos y\frac{dy}{du} = 1$$
$$\frac{dy}{du} = \frac{1}{\cos y}$$
$$\frac{d}{dx}\sin^{-1} u = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$$
$$y' = \frac{1}{4\sqrt{1-\frac{x^2}{16}}}$$
$$y'(3) = \frac{1}{4\sqrt{1-\frac{9}{16}}}$$
$$y'(3) = 0.378$$
$$y(3) = \sin^{-1}\left(\frac{3}{4}\right) = 0.848$$
$$y = 0.378(x-3)+0.848$$
$$y = 0.378x-0.286$$

26. See problem 24.
$$u = x^2$$
$$\frac{du}{dx} = 2x$$
$$\frac{1}{1+u^2}\frac{du}{dx}$$
$$\frac{2x}{1+x^2}$$
$$= \frac{2(1)}{1+(1)^2}$$
$$= 1$$
$$y(1) = \tan^{-1}(1)^2$$
$$y(1) = 0.785$$
$$y = 1(x-1)+0.785$$
$$y = x-0.215$$

27. (a) Since $\frac{dy}{dx} = \sec^2 x$, the slope at $\left(\frac{\pi}{4}, 1\right)$ is $\sec^2\left(\frac{\pi}{4}\right) = 2.$

The tangent line is given by

$y = 2\left(x-\frac{\pi}{4}\right)+1$, or $y = 2x = \frac{\pi}{2}+1$.

(b) Since $\frac{dy}{dx} = \frac{1}{1+x^2}$, the slope at $\left(1, \frac{\pi}{4}\right)$ is $\frac{1}{1+1^2} = \frac{1}{2}$.

The tangent line is given by $x \neq 0$.

28. (a) Note that $f'(x) = 5x^4+6x^2+1$. Thus $f(1) = 3$ and $f'(1) = 12$.

(b) Since the graph of $y = f(x)$ includes the point (1, 3) and the slope of the graph is 12 at this point, the graph of $y = f^{-1}(x)$ will include (3, 1) and the slope will be $\frac{1}{12}$.

Thus, $f^{-1}(3) = 1$ and $(f^{-1})'(3) = \frac{1}{12}$. (We have assumed that $f^{-1}(x)$ is defined and differentiable at $x = 3$. This is true by Theorem 3, because $f'(x) = 5x^2+6x^2+1$, which is never zero.)

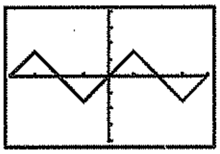
29. **(a)** Note that $f'(x) = -\sin x + 3$, which is always between 2 and 4. Thus f is differentiable at every point on the interval $(-\infty, \infty)$ and $f'(x)$ is never zero on this interval, so f has a differentiable inverse by Theorem 3.

(b) $f(0) = \cos 0 + 3(0) = 1$;
$f'(0) = -\sin 0 + 3 = 3$

(c) Since the graph of $y = f(x)$ includes the point (0, 1) and the slope of the graph is 3 at this point, the graph of $y = f^{-1}(x)$ will include (1, 0) and the slope will be $\frac{1}{3}$,

Thus, $f^{-1}(1) = 0$ and $(f^{-1})'(1) = \frac{1}{3}$.

30.



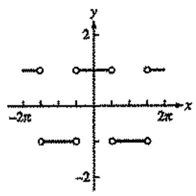
$[-2\pi, 2\pi]$ by $[-4, 4]$

(a) All reals

(b) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(c) At the points $x = k\frac{\pi}{2}$, where k is an odd integer.

(d)



(e) $f'(x) = \frac{d}{dx}\sin^{-1}(\sin x)$

$$= \frac{1}{\sqrt{1-\sin^2 x}}\frac{d}{dx}\sin x$$

$$= \frac{\cos x}{\sqrt{1-\sin^2 x}}$$

which is ± 1 depending on whether $\cos x$ is positive or negative.

31. **(a)** $v(t) = \frac{dx}{dt} = \frac{1}{1+t^2}$ which is always positive.

(b) $a(t) = \frac{dv}{dt} = -\frac{2t}{(1+t^2)^2}$ which is always negative.

(c) $\frac{\pi}{2}$

32. $\frac{d}{dx}\cos^{-1}(x) = \frac{d}{dx}\left(\frac{\pi}{2} - \sin^{-1} x\right)$

$$= 0 - \frac{d}{dx}\sin^{-1}(x)$$

$$= -\frac{1}{\sqrt{1-x^2}}$$

33. $\frac{d}{dx}\cot^{-1} x = \frac{d}{dx}\left(\frac{\pi}{2} - \tan^{-1}(x)\right)$

$$= 0 - \frac{d}{dx}\tan^{-1}(x)$$

$$= -\frac{1}{1+x^2}$$

34. $\frac{d}{dx}\csc^{-1}(x) = \frac{d}{dx}\left(\frac{\pi}{2} - \sec^{-1}(x)\right)$

$$= 0 - \frac{d}{dx}\sec^{-1}(x)$$

$$= -\frac{1}{|x|\sqrt{x^2-1}}$$

35. True. By definition of the Function.

36. False. The domain is all real numbers.

37. E. $\frac{d}{dx}\sin^{-1}\left(\frac{x}{2}\right)$

$$y = \sin^{-1} x \qquad u = \frac{x}{2}$$

$$\sin y = x \qquad \frac{du}{dx} = \frac{1}{2}$$

$$\frac{d}{dx}\sin y = \frac{d}{dx}x$$

$$\cos y\frac{dy}{dx} = 1$$

$$\frac{d}{dx} = \frac{1}{\cos y}$$

$$\frac{d}{dx}\sin^{-1} u = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$$

$$\frac{d}{dx}\sin^{-1} u = \frac{1}{\sqrt{1-\frac{x}{2}}}\frac{1}{2}$$

$$= \frac{1}{\sqrt{4-x^2}}$$

38. D. $\frac{d}{dx}\tan^{-1}(3x)$

$$y = \tan^{-1} u \qquad u = 3x$$

$$\frac{d}{du}\tan y = \frac{d}{du}u \qquad \frac{du}{dx} = 3$$

$$\sec^2 y\frac{dy}{du} = 1$$

$$\frac{dy}{dx} = \frac{1}{\sec^2 y}$$

$$= \frac{1}{1+(\tan y)^2}$$

$$= \frac{1}{1+u^2}\frac{du}{dx}$$

$$= \frac{3}{1+9x^2}$$