4. Continued

(b) $x_{n+1} = x_n - \dfrac{f(x)}{f'(x)}$, $f(x) = x^2 - 26 = 0$

$$x_2 = 5 - \frac{(5)^2 - 26}{2(5)} = 5.1$$

(c) $f(x) = \sqrt[3]{x}$

$$x = 3$$

$$f'(27) = \frac{1}{3}(27)^{-2/3} = \frac{1}{27}$$

$$\sqrt{26} = 3 + \frac{1}{27}(26 - 27)$$

$$\sqrt{26} = 2.963$$

Chapter 4 Review (pp. 256–260)

1. $y = x\sqrt{2-x}$

$$y' = x\left(\frac{1}{2\sqrt{2-x}}\right)(-1) + (\sqrt{2-x})(1)$$
$$= \frac{-x + 2(2-x)}{2\sqrt{2-x}}$$
$$= \frac{4-3x}{2\sqrt{2-x}}$$

The first derivative has a zero at $\frac{4}{3}$.

Critical point value: $x = \frac{4}{3}$ $y = \frac{4\sqrt{6}}{9} \approx 1.09$

Endpoint values: $x = -2$ $y = -4$

$x = 2$ $y = 0$

The global maximum value is $\frac{4\sqrt{6}}{9}$ at $x = \frac{4}{3}$, and the global minimum value is -4 at $x = -2$.

2. Since y is a cubic function with a positive leading coefficient, we have $\lim_{x\to-\infty} y = -\infty$ and $\lim_{x\to\infty} y = \infty$. There are no global extrema.

3. $y' = (x^2)(e^{1/x^2})(-2x^{-3}) + (e^{1/x^2})(2x)$

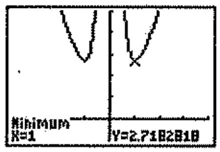
$$= 2e^{1/x^2}\left(-\frac{1}{x} + x\right)$$
$$= \frac{2e^{1/x^2}(x-1)(x+1)}{x}$$

Intervals	$x < -1$	$-1 < x < 0$	$0 < x < 1$	$x > 1$
Sign of y'	−	+	−	+
Behavior of y	Decreasing	Increasing	Decreasing	Increasing

$$y'' = \frac{d}{dx}[2e^{1/x^2}(-x^{-1} + x)]$$
$$= (2e^{1/x^2})(x^{-2} + 1) + (-x^{-1} + x)(2e^{1/x^2})(-2x^{-3})$$
$$= (2e^{1/x^2})(x^{-2} + 1 + 2x^{-4} - 2x^{-2})$$
$$= \frac{2e^{1/x^2}(x^4 - x^2 + 2)}{x^4}$$
$$= \frac{2e^{1/x^2}[(x^2 - 0.5)^2 + 1.75]}{x^4}$$

The second derivative is always positive (where defined), so the function is concave up for all $x \neq 0$.

Graphical support:



[−4, 4] by [−1, 5]

(a) $[-1, 0)$ and $[1, \infty)$

(b) $(-\infty, -1]$ and $(0, 1]$

(c) $(-\infty, 0)$ and $(0, \infty)$

(d) None

(e) Local (and absolute) minima at $(1, e)$ and $(-1, e)$

(f) None

4. Note that the domain of the function is $[-2, 2]$.

$$y' = x\left(\frac{1}{2\sqrt{4-x^2}}\right)(-2x) + (\sqrt{4-x^2})(1)$$
$$= \frac{-x^2 + (4-x^2)}{\sqrt{4-x^2}}$$
$$= \frac{4-2x^2}{\sqrt{4-x^2}}$$

Intervals	$-2 < x < -\sqrt{2}$	$-\sqrt{2} < x < \sqrt{2}$	$\sqrt{2} < x < 2$
Sign of y'	−	+	−
Behavior of y	Decreasing	Increasing	Decreasing

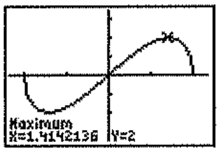
$$y'' = \frac{(\sqrt{4-x^2})(-4x) - (4-2x^2)\left(\frac{1}{2\sqrt{4-x^2}}\right)(-2x)}{4-x^2}$$
$$= \frac{2x(x^2-6)}{(4-x^2)^{3/2}}$$

Note that the values $x = \pm\sqrt{6}$ are not zeros of y'' because they fall outside of the domain.

Intervals	$-2 < x < 0$	$0 < x < 2$
Sign of y''	+	−
Behavior of y	Concave up	Concave down

4. Continued

Graphical support:



[–2.35, 2.35] by [–3.5, 3.5]

(a) $[-\sqrt{2}, \sqrt{2}]$

(b) $[-2, -\sqrt{2}]$ and $[\sqrt{2}, 2]$

(c) $(-2, 0)$

(d) $(0, 2)$

(e) Local maxima: $(-2, 0)$, $(\sqrt{2}, 2)$

Local minima: $(2, 0)$, $(-\sqrt{2}, -2)$

Note that the extrema at $x = \pm\sqrt{2}$ are also absolute extrema.

(f) $(0, 0)$

5. $y' = 1 - 2x - 4x^3$

Using grapher techniques, the zero of y' is $x \approx 0.385$.

Intervals	$x < 0.385$	$0.385 < x$
Sign of y'	+	–
Behavior of y	Increasing	Decreasing

$y'' = -2 - 12x^2 = -2(1 + 6x^2)$

The second derivative is always negative so the function is concave down for all x.

Graphical support:

[–4, 4] by [–4, 2]

(a) Approximately $(-\infty, 0.385]$

(b) Approximately $[0.385, \infty)$

(c) None

(d) $(-\infty, \infty)$

(e) Local (and absolute) maximum at $\approx (0.385, 1.215)$

(f) None

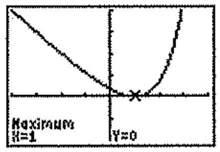
6. $y' = e^{x-1} - 1$

Intervals	$x < 1$	$1 < x$
Sign of y'	–	+
Behavior of y	Decreasing	Increasing

$y'' = e^{x-1}$

The second derivative is always positive, so the function is concave up for all x.

Graphical support:



[–4, 4] by [–2, 4]

(a) $[1, \infty)$

(b) $(-\infty, 1]$

(c) $(-\infty, \infty)$

(d) None

(e) Local (and absolute) minimum at $(1, 0)$

(f) None

7. Note that the domain is $(-1, 1)$.

$y = (1 - x^2)^{-1/4}$

$y' = -\frac{1}{4}(1 - x^2)^{-5/4}(-2x) = \frac{x}{2(1 - x^2)^{5/4}}$

Intervals	$-1 < x < 0$	$0 < x < 1$
Sign of y'	–	+
Behavior of y	Decreasing	Increasing

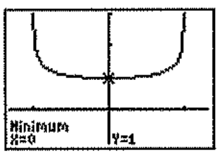
$$y'' = \frac{2(1 - x^2)^{5/4}(1) - (x)(2)\left(\frac{5}{4}\right)(1 - x^2)^{1/4}(-2x)}{4(1 - x^2)^{5/2}}$$

$$= \frac{(1 - x^2)^{1/4}[2 - 2x^2 + 5x^2]}{4(1 - x^2)^{5/2}}$$

$$= \frac{3x^2 + 2}{4(1 - x^2)^{9/4}}$$

The second derivative is always positive, so the function is concave up on its domain $(-1, 1)$.

Graphical support:



[–1.3, 1.3] by [–1, 3]

(a) $[0, 1)$

(b) $(-1, 0]$

(c) $(-1, 1)$

(d) None

(e) Local minimum at $(0, 1)$

(f) None

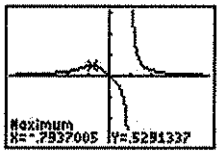
8. $y' = \frac{(x^3-1)(1)-(x)(3x^2)}{(x^3-1)^2} = \frac{2x^3+1}{(x^3-1)^2}$

Intervals	$x < -2^{-1/3}$	$-2^{-1/3} < x < 1$	$1 < x$
Sign of y'	+	−	−
Behavior of y	Increasing	Decreasing	Decreasing

$$y'' = -\frac{(x^3-1)^2(6x^2)-(2x^3+1)(2)(x^3-1)(3x^2)}{(x^3-1)^4}$$
$$= -\frac{(x^3-1)(6x^2)-(2x^3+1)(6x^2)}{(x^3-1)^3}$$
$$= \frac{6x^2(x^3+2)}{(x^3-1)^3}$$

Intervals	$x < -2^{1/3}$	$-2^{1/3} < x < 0$	$0 < x < 1$	$1 < x$
Sign of y''	+	−	−	+
Behavior of y	Concave up	Concave down	Concave down	Concave up

Graphical support:



[−4.7, 4.7] by [−3.1, 3.1]

(a) $(-\infty, -2^{-1/3}] \approx (-\infty, -0.794]$

(b) $[-2^{-1/3}, 1) \approx [-0.794, 1)$ and $(1, \infty)$

(c) $(-\infty, -2^{-1/3}) \approx (-\infty, -1.260)$ and $(1, \infty)$

(d) $(-2^{-1/3}, 1) \approx (-1.260, 1)$

(e) Local minimum at

$$\left(-2^{-1/3}, \frac{2}{3}\cdot 2^{-1/3}\right) \approx (-0.794, 0.529)$$

(f) $\left(-2^{1/3}, \frac{1}{3}\cdot 2^{1/3}\right) \approx (-1.260, 0.420)$

9. Note that the domain is $[-1, 1]$.

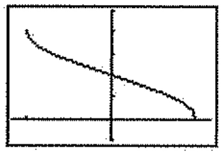
$$y' = -\frac{1}{\sqrt{1-x^2}}$$

Since y' is negative on $(-1, 1)$ and y is continuous, y is decreasing on its domain $[-1, 1]$.

$$y'' = \frac{d}{dx}[-(1-x^2)^{-1/2}]$$
$$= \frac{1}{2}(1-x^2)^{-3/2}(-2x) = -\frac{x}{(1-x^2)^{3/2}}$$

Intervals	$-1 < x < 0$	$0 < x < 1$
Sign of y''	+	−
Behavior of y	Concave up	Concave down

Graphical support:



$[-1.175, 1.175]$ by $\left[-\frac{\pi}{4}, \frac{5\pi}{4}\right]$

(a) None

(b) $[-1, 1]$

(c) $(-1, 0)$

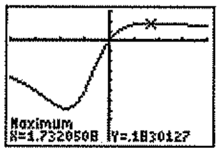
(d) $(0, 1)$

(e) Local (and absolute) maximum at $(-1, \pi)$; local (and absolute) minimum at $(1, 0)$

(f) $\left(0, \frac{\pi}{2}\right)$

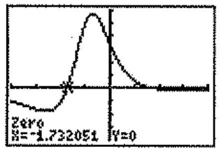
10. This problem can be solved graphically by using NDER to obtain the graphs shown below.

y



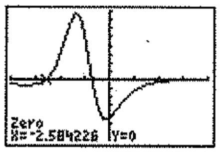
[−4, 4] by [−1, 0.3]

y'



[−4, 4] by [−0.4, 0.6]

y''



[−4, 4] by [−0.7, 0.8]

An alternative approach using a combination of algebraic and graphical techniques follows. Note that the denominator of y is always positive because it is equivalent to $(x+1)^2+2$.

$$y' = \frac{(x^2+2x+3)(1)-(x)(2x+2)}{(x^2+2x+3)^2}$$
$$= \frac{-x^2+3}{(x^2+2x+3)^2}$$

Intervals	$x < -\sqrt{3}$	$-\sqrt{3} < x < \sqrt{3}$	$\sqrt{3} < x$
Sign of y'	−	+	−
Behavior of y	Decreasing	Increasing	Decreasing

$$y'' = \frac{(x^2+2x+3)^2(-2x)-(-x^2+3)(2)(x^2+2x+3)(2x+2)}{(x^2+2x+3)^4}$$
$$= \frac{(x^2+2x+3)(-2x)-2(2x+2)(-x^2+3)}{(x^2+2x+3)^3}$$
$$= \frac{2x^3-18x-12}{(x^2+2x+3)^3}$$

10. Continued

Using graphing techniques, the zeros of $2x^3 - 18x - 12$ (and hence of y'') are at $x \approx -2.584$, $x \approx -0.706$, and $x \approx 3.290$.

Intervals	$(-\infty, -2.584)$	$(-2.584, -0.706)$	$(-0.706, 3.290)$	$(3.290, \infty)$
Sign of y''	−	+	−	+
Behavior of y	Concave down	Concave up	Concave down	Concave up

(a) $[-\sqrt{3}, \sqrt{3}]$

(b) $(-\infty, -\sqrt{3}]$ and $[\sqrt{3}, \infty)$

(c) Approximately $(-2.584, -0.706)$ and $(3.290, \infty)$

(d) Approximately $(-\infty, -2.584)$ and $(-0.706, 3.290)$

(e) Local maximum at $\left(\sqrt{3}, \frac{\sqrt{3}-1}{4}\right)$

$\approx (1.732, 0.183)$;

local minimum at $\left(-\sqrt{3}, \frac{-\sqrt{3}-1}{4}\right)$

$\approx (-1.732, -0.683)$

(f) $\approx(-2.584, -0.573)$, $(-0.706, -0.338)$, and $(3.290, 0.161)$

11. For $x > 0$, $y' = \frac{d}{dx} \ln x = \frac{1}{x}$

For $x < 0$: $y' = \frac{d}{dx}\ln(-x) = \frac{1}{-x}(-1) = \frac{1}{x}$

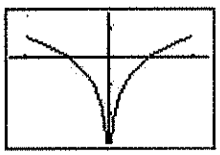
Thus $y' = \frac{1}{x}$ for all x in the domain.

Intervals	$(-2, 0)$	$(0, 2)$
Sign of y'	−	+
Behavior of y	Decreasing	Increasing

$y'' = -x^{-2}$

The second derivative always negative, so the function is concave down on each open interval of its domain.

Graphical support:



$[-2.35, 2.35]$ by $[-3, 1.5]$

(a) $(0, 2]$

(b) $[-2, 0)$

(c) None

(d) $(-2, 0)$ and $(0, 2)$

(e) Local (and absolute) maxima at $(-2, \ln 2)$ and $(2, \ln 2)$

(f) None

12. $y' = 3\cos 3x - 4\sin 4x$

Using graphing techniques, the zeros of y' in the domain $0 \le x \le 2\pi$ are $x \approx 0.176$, $x \approx 0.994$, $x = \frac{\pi}{2} \approx 1.57$, $x \approx 2.148$, and $x \approx 2.965$, $x \approx 3.834$, $x = \frac{3\pi}{2}$, $x \approx 5.591$

Intervals	$0<x<0.176$	$0.176 < x < 0.994$	$0.994 < x < \frac{\pi}{2}$	$\frac{\pi}{2} < x < 2.148$	$2.148 < x < 2.965$
Sign of y'	+	−	+	−	+
Behavior of y	Increasing	Decreasing	Increasing	Decreasing	Increasing

Intervals	$2.965 < x < 3.834$	$3.834 < x < \frac{3\pi}{2}$	$\frac{3\pi}{2} < x < 5.591$	$5.591 < x < 2\pi$
Sign of y'	−	+	−	+
Behavior of y	Decreasing	Increasing	Decreasing	Increasing

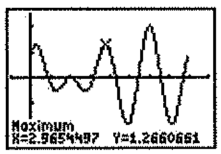
$y'' = -9 \sin 3x - 16 \cos 4x$

Using graphing techniques, the zeros of y'' in the domain $0 \le x \le 2\pi$ are $x \approx 0.542$, $x \approx 1.266$, $x \approx 1.876$, $x \approx 2.600$, $x \approx 3.425$, $x \approx 4.281$, $x \approx 5.144$ and $x \approx 6.000$.

Intervals	$0 < x < 0.542$	$0.542 < x < 1.266$	$1.266 < x < 1.876$	$1.876 < x < 2.600$	$2.600 < x < 3.425$
Sign of y''	−	+	−	+	−
Behavior of y	Concave down	Concave up	Concave down	Concave up	Concave down

Intervals	$3.425 < x < 4.281$	$4.281 < x < 5.144$	$5.144 < x < 6.000$	$6.00 < x < 2\pi$
Sign of y''	+	−	+	−
Behavior of y	Concave up	Concave down	Concave up	Concave down

Graphical support:



$\left[-\frac{\pi}{4}, \frac{9\pi}{4}\right]$ by $[-2.5, 2.5]$

12. Continued

(a) Approximately [0, 0.176], $\left[0.994, \frac{\pi}{2}\right]$, [2.148, 2.965], $\left[3.834, \frac{3\pi}{2}\right]$, and $\left[5.591, 2\pi\right]$

(b) Approximately [0.176, 0.994], $\left[\frac{\pi}{2}, 2.148\right]$, [2.965, 3.834], and $\left[\frac{3\pi}{2}, 5.591\right]$

(c) Approximately (0.542, 1.266), (1.876, 2.600), (3,425, 4.281), and (5.144, 6.000)

(d) Approximately (0, 0.542), (1.266, 1.876), (2.600, 3.425), (4.281, 5.144), and (6000, 2π)

(e) Local maxima at $\approx$ (0.176, 1.266), $\left(\frac{\pi}{2}, 0\right)$ and (2.965, 1.266), $\left(\frac{3\pi}{2}, 2\right)$, and $(2\pi, 1)$; local minima at $\approx$ (0, 1), (0.994, −0.513), (2.148, −0.513), (3.834, −1.806), and (5.591, −1.806) Note that the local extrema at $x \approx 3.834$, $x = \frac{3\pi}{2}$, and $x \approx 5.591$ are also extrema.

(f) $\approx$ (0.542, 0.437), (1.266, −0.267), (1.876, −0.267), (2.600, 0.437), (3.425, −0.329), (4.281, 0.120), (5.144, 0.120), and (6.000, −0.329)

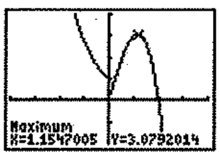
13. $y' = \begin{cases} -e^{-x}, & x < 0 \\ 4 - 3x^2, & x > 0 \end{cases}$

Intervals	$x < 0$	$0 < x < \frac{2}{\sqrt{3}}$	$\frac{2}{\sqrt{3}} < x$
Sign of	−	+	−
Behavior of y	Decreasing	Increasing	Decreasing

$y'' = \begin{cases} e^{-x}, & x > 0 \\ -6x, & x < 0 \end{cases}$

Intervals	$x < 0$	$0 < x$
Sign of y''	+	−
Behavior of y	Concave up	Concave down

Graphical support:



[−4, 4] by [−2, 4]

(a) $\left(0, \frac{2}{\sqrt{3}}\right]$

(b) $(-\infty, 0]$ and $\left[\frac{2}{\sqrt{3}}, \infty\right)$

(c) $(-\infty, 0)$

(d) $(0, \infty)$

(e) Local maximum at $\left(\frac{2}{\sqrt{3}}, \frac{16}{3\sqrt{3}}\right) \approx (1.155, 3.079)$

(f) None. Note that there is no point of inflection at $x = 0$ because the derivative is undefined and no tangent line exists at this point.

14. $y' = -5x^4 + 7x^2 + 10x + 4$

Using graphing techniques, the zeros of y' are $x \approx -0.578$ and $x \approx -1.692$.

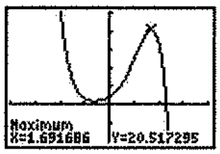
Intervals	$x < -0.578$	$-0.578 < x < 1.692$	$1.692 < x$
Sign of y'	−	+	−
Behavior of y	Decreasing	Increasing	Decreasing

$y'' = -20x^3 + 14x + 10$

Using graphing techniques, the zeros of y'' is $x \approx 1.079$.

Intervals	$x < 1.079$	$1.079 < x$
Sign of y''	+	−
Behavior of y	Concave up	Concave down

Graphical support:



[−4, 4] by [−10, 25]

(a) Approximately [−0.578, 1.692]

(b) Approximately $(-\infty, -0.578]$ and $[1.692, \infty)$

(c) Approximately $(-\infty, 1.079)$

(d) Approximately $(1.079, \infty)$

(e) Local maximum at $\approx$ (1.692, 20.517); local minimum at $\approx$ (−0.578, 0.972)

(f) $\approx$ (1.079, 13.601)

15. $y = 2x^{4/5} - x^{9/5}$

$$y' = \frac{8}{5}x^{-1/5} - \frac{9}{5}x^{4/5} = \frac{8-9x}{5\sqrt[5]{x}}$$

Intervals	$x < 0$	$0 < x < \frac{8}{9}$	$\frac{8}{9} < x$
Sign of y'	−	+	−
Behavior of y	Decreasing	Increasing	Decreasing

$$y'' = -\frac{8}{25}x^{-6/5} - \frac{36}{25}x^{-1/5} = \frac{4(2+9x)}{25x^{6/5}}$$

Intervals	$x < -\frac{2}{9}$	$-\frac{2}{9} < x < 0$	$0 < x$
Sign of y''	+	−	−
Behavior of y	Concave up	Concave down	Concave down

Graphical support:

[−4, 4] by [−3, 3]

(a) $\left[0, \frac{8}{9}\right]$

(b) $(-\infty, 0]$ and $\left[\frac{8}{9}, \infty\right)$

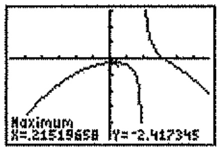
(c) $\left(-\infty, -\frac{2}{9}\right)$

(d) $\left(-\frac{2}{9}, 0\right)$ and $(0, \infty)$

(e) Local maximum at $\left(\frac{8}{9}, \frac{10}{9} \cdot \left(\frac{8}{9}\right)^{4/5}\right) \approx (0.889, 1.011)$; local minimum at $(0, 0)$

(f) $\left(-\frac{2}{9}, \frac{20}{9} \cdot \left(-\frac{2}{9}\right)^{4/5}\right) \approx \left(-\frac{2}{9}, 0.667\right)$

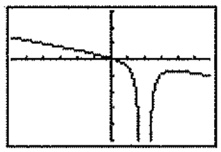
16. We use a combination of analytic and grapher techniques to solve this problem. Depending on the viewing windows chosen, graphs obtained using NDER may exhibit strange behavior near $x = 2$ because, for example, NDER $(y, 2) \approx 5{,}000{,}000$ while y' is actually undefined at $x = 2$. The graph of $y = \frac{5-4x+4x^2-x^3}{x-2}$ is shown below.



[−5.875, 5.875] by [−50, 30]

$$y' = \frac{(x-2)(-4+8x-3x^2)-(5-4x+4x^2-x^3)(1)}{(x-2)^2}$$
$$= \frac{-2x^3+10x^2-16x+3}{(x-2)^2}$$

The graph of y' is shown below.



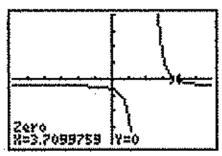
[−5.875, 5.875] by [−50, 30]

The zero of y' is $x \approx 0.215$.

Intervals	$x < 0.215$	$0.215 < x < 2$	$2 < x$
Sign of y'	+	−	−
Behavior of y	Increasing	Decreasing	Decreasing

$$y'' = \frac{(x-2)^2(-6x^2+20x-16)-(-2x^3+10x^2-16x+3)(2)(x-2)}{(x-2^4)}$$
$$= \frac{(x-2)(-6x^2+20x-16)-2(-2x^3+10x^2-16x+3)}{(x-2)^3}$$
$$= \frac{-2(x^3-6x^2+12x-13)}{(x-2)^3}$$

The graph of y'' is shown below.



[−5.875, 5.875] by [−20, 20]

The zero of $x^3 - 6x^2 + 12x - 13$ (and hence of y'') is $x \approx 3.710$.

Intervals	$x < 2$	$2 < x < 3.710$	$3.710 < x$
Sign of y''	−	+	−
Behavior of y	Concave down	Concave up	Concave down

16. Continued

(a) Approximately $(-\infty, 0.215]$

(b) Approximately $[0.215, 2)$ and $(2, \infty)$

(c) Approximately $(2, 3.710)$

(d) $(-\infty, 2)$ and approximately $(3.710, \infty)$

(e) Local maximum at $\approx (0.215, -2.417)$

(f) $\approx (3.710, -3.420)$

17. $y' = 6(x+1)(x-2)^2$

Intervals	$x < -1$	$-1 < x < 2$	$2 < x$
Sign of y'	−	+	+
Behavior of y	Decreasing	Increasing	Increasing

$$\begin{aligned} y'' &= 6(x+1)(2)(x-2) + 6(x-2)^2(1) \\ &= 6(x-2)[(2x+2)+(x-2)] \\ &= 18x(x-2) \end{aligned}$$

Intervals	$x < 0$	$0 < x < 2$	$2 < x$
Sign of y''	+	−	+
Behavior of y	Concave up	Concave down	Concave up

(a) There are no local maxima.

(b) There is a local (and absolute) minimum at $x = -1$.

(c) There are points of inflection at $x = 0$ and at $x = 2$.

18. $y' = 6(x+1)(x-2)$

Intervals	$x < -1$	$-1 < x < 2$	$2 < x$
Sign of y'	+	−	+
Behavior of y	Increasing	Decreasing	Increasing

$$y'' = \frac{d}{dx}6(x^2 - x - 2) = 6(2x-1)$$

Intervals	$x < \frac{1}{2}$	$\frac{1}{2} < x$
Sign of y''	−	+
Behavior of y	Concave down	Concave up

(a) There is a local maximum at $x = -1$.

(b) There is a local maximum at $x = 2$.

(c) There is a point of inflection at $x = \frac{1}{2}$.

19. Since $\frac{d}{dx}\left(-\frac{1}{4}x^{-4} - e^{-x}\right) = x^{-5} + e^{-x}$,

$$f(x) = -\frac{1}{4}x^{-4} - e^{-x} + C.$$

20. Since $\frac{d}{dx}\sec x = \sec x \tan x, f(x) = \sec x + C.$

21. Since $\frac{d}{dx}\left(2\ln x + \frac{1}{3}x^3 + x\right) = \frac{2}{x} + x^2 + 1$,

$$f(x) = 2\ln x + \frac{1}{3}x^3 + x + C.$$

22. Since $\frac{d}{dx}\left(\frac{2}{3}x^{3/2} + 2x^{1/2}\right) = \sqrt{x} + \frac{1}{\sqrt{x}}$,

$$f(x) = \frac{2}{3}x^{3/2} + 2x^{1/2} + C.$$

23. $f(x) = -\cos x + \sin x + C$

$$\begin{aligned} f(\pi) &= 3 \\ 1 + 0 + C &= 3 \\ C &= 2 \\ f(x) &= -\cos x + \sin x + 2 \end{aligned}$$

24.

$$\begin{aligned} f(x) &= \frac{3}{4}x^{4/3} + \frac{1}{3}x^3 + \frac{1}{2}x^2 + x + C \\ f(1) &= 0 \\ \frac{3}{4} + \frac{1}{3} + \frac{1}{2} + 1 + C &= 0 \\ C &= -\frac{31}{12} \\ f(x) &= \frac{3}{4}x^{4/3} + \frac{1}{3}x^3 + \frac{1}{2}x^2 + x - \frac{31}{12} \end{aligned}$$

25. $v(t) = s'(t) = 9.8t + 5$

$$\begin{aligned} s(t) &= 4.9t^2 + 5t + C \\ s(0) &= 10 \\ C &= 10 \\ s(t) &= 4.9t^2 + 5t + 10 \end{aligned}$$

26. $a(t) = v'(t) = 32$

$$\begin{aligned} v(t) &= 32t + C_1 \\ v(0) &= 20 \\ C_1 &= 20 \\ v(t) &= s'(t) = 32t + 20 \\ s(t) &= 16t^2 + 20t + C_2 \\ s(0) &= 5 \\ C_2 &= 5 \\ s(t) &= 16t^2 + 20t + 5 \end{aligned}$$

27. $f(x) = \tan x$

$f'(x) = \sec^2 x$

$$L(x) = f\left(-\frac{\pi}{4}\right) + f'\left(-\frac{\pi}{4}\right)\left[x - \left(-\frac{\pi}{4}\right)\right]$$
$$= \tan\left(-\frac{\pi}{4}\right) + \sec^2\left(-\frac{\pi}{4}\right)\left(x + \frac{\pi}{4}\right)$$
$$= -1 + 2\left(x + \frac{\pi}{4}\right)$$
$$= 2x + \frac{\pi}{2} - 1$$

28. $f(x) = \sec x$

$f'(x) = \sec x \tan x$

$$L(x) = f\left(\frac{\pi}{4}\right) + f'\left(\frac{\pi}{4}\right)\left(x - \frac{\pi}{4}\right)$$
$$= \sec\left(\frac{\pi}{4}\right) + \sec\left(\frac{\pi}{4}\right)\tan\left(\frac{\pi}{4}\right)\left(x - \frac{\pi}{4}\right)$$
$$= \sqrt{2} + \sqrt{2}(1)\left(x - \frac{\pi}{4}\right)$$
$$= \sqrt{2}x - \frac{\pi\sqrt{2}}{4} + \sqrt{2}$$

29. $f(x) = \dfrac{1}{1 + \tan x}$

$$f'(x) = -(1 + \tan x)^{-2}(\sec^2 x)$$
$$= -\frac{1}{\cos^2 x(1 + \tan x)^2}$$
$$= -\frac{1}{(\cos x + \sin x)^2}$$
$$L(x) = f(0) + f'(0)(x - 0)$$
$$= 1 - 1(x - 0)$$
$$= -x + 1$$

30. $f(x) = e^x + \sin x$

$$f'(x) = e^x + \cos x$$
$$L(x) = f(0) + f'(0)(x - 0)$$
$$= 1 + 2(x - 0)$$
$$= 2x + 1$$

31. The global minimum value of $\frac{1}{2}$ occurs at $x = 2$.

32. **(a)** The values of y' and y'' are both negative where the graph is decreasing and concave down, at T.

(b) The value of y' is negative and the value of y'' is positive where the graph is decreasing and concave up, at P.

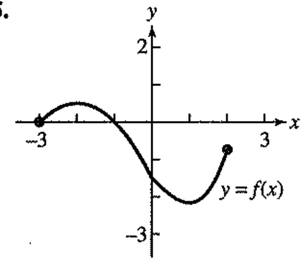
33. **(a)** The function is increasing on the interval $(0, 2]$.

(b) The function is decreasing on the interval $[-3, 0)$.

(c) The local extreme values occur only at the endpoints of the domain. A local maximum value of 1 occurs at $x = -13$, and a local maximum value of 3 occurs at $x = 2$.

34. The 24th day

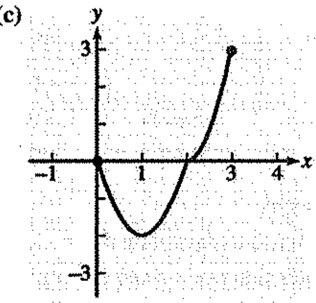
35.



36. **(a)** We know that f is decreasing on [0, 1] and increasing on [1, 3], the absolute minimum value occurs at $x = 1$ and the absolute maximum value occurs at an endpoint. Since $f(0) = 0$, $f(1) = -2$, and $f(3) = 3$, the absolute minimum value is -2 at $x = 1$ and the absolute maximum value is 3 at $x = 3$.

(b) The concavity of the graph does not change. There are no points of inflection.

(c)



37. **(a)** $f(x)$ is continuous on [0. 5, 3] and differentiable on (0. 5, 3).

(b) $f'(x) = (x)\left(\frac{1}{x}\right) + (\ln x)(1) = 1 + \ln x$

Using $a = 0.5$ and $b = 3$, we solve as follows.

$$f'(c) = \frac{f(3) - f(0.5)}{3 - 0.5}$$
$$1 + \ln c = \frac{3\ln 3 - 0.5\ln 0.5}{2.5}$$
$$\ln c = \frac{\ln\left(\frac{3^3}{0.5^{0.5}}\right)}{2.5} - 1$$
$$\ln c = 0.4\ln(27\sqrt{2}) - 1$$
$$c = e^{-1}(27\sqrt{2})^{0.4}$$
$$c = e^{-1}\sqrt[5]{1458} \approx 1.579$$

(c) The slope of the line is

$m = \dfrac{f(b) - f(a)}{b - a} = 0.4\ln(27\sqrt{2})0.2\ln 1458$, and the line passes through $(3, 3\ln 3)$. Its equation is $y = 0.2(\ln 1458)(x - 3) + 3\ln 3$, or approximately $y = 1.457x - 1.075$.

37. Continued

(d) The slope of the line is $m = 0.2\ln 1458$, and the line passes through

$(c, f(c)) = (e^{-1}\sqrt[5]{1458},\ e^{-1}\sqrt[5]{1458}(-1+0.2\ \ln\ 1458))$
$\approx (1.579,\ 0.722)$.

Its equation is

$y = 0.2(\ln 1458)(x - c) + f(c)$,

$y = 0.2\ln 1458(x - e^{-1}\sqrt[5]{1458})$
$+e^{-1}\sqrt[5]{1458}(-1+0.2\ln 1458)$,

$y = 0.2(\ln 1458)x - e^{-1}\sqrt[5]{1458}$,

or approximately $y = 1.457x - 1.579$.

38. (a) $v(t) = s'(t) = 4 - 6t - 3t^2$

(b) $a(t) = v'(t) = -6 - 6t$

(c) The particle starts at position 3 moving in the positive direction, but decelerating. At approximately $t = 0.528$, it reaches position 4.128 and changes direction, beginning to move in the negative direction. After that, it continues to accelerate while moving in the negative direction.

39. (a) $L(x) = f(0) + f'(0)(x - 0)$
$= -1 + 0(x - 0) = -1$

(b) $f(0.1) \approx L(0.1) = -1$

(c) Greater than the approximation in (b), since $f'(x)$ is actually positive over the interval (0, 0.1) and the estimate is based on the derivative being 0.

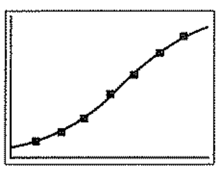
40. (a) Since $\dfrac{dy}{dx} = (x^2)(-e^{-x}) + (e^{-x})(2x) + (2x - x^2)e^{-x}$,

$dy = (2x - x^2)e^{-x}dx$.

(b) $dy = [2(1) - (1)^2](e^{-1})(0.01)$
$= 0.01e^{-1}$
≈ 0.00368

41. (a) With some rounding, $y = \dfrac{1633001.59}{1+17.471e^{-0.06378t}}$

(b)



[0, 80] by [0, 1600000]

(c) $y = \dfrac{1633001.59}{1+17.471e^{-0.06378(80)}} + 829{,}210 = 2{,}305{,}337$

(d) Using the Second Derivative, we find the maximum rate of growth about 1885. We find a point of inflection here, which shows the begining of a decline in the rate of growth.

(e) $y = \dfrac{1633001.59}{1+17.471e^{-0.06378(\infty)}} \approx 2{,}462{,}000$, which is the approxiate maximum population.

(f) There are many possible causes. Advances in transportation began drawing the population southward after 1920, and Tennessee was well-situated grographically to become a crossroads of river, railroad, and automobile routes. By the year 2000 there had been numerous other demographic changes. It should be pointed out that the census years in the data (1850–1910) include the years of the Civil War and Reconstruction, so the regression is based on unusual data.

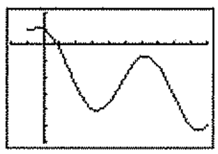
42. $f(x) = 2\cos x - \sqrt{1+x}$

$f'(x) = -2\sin x - \dfrac{1}{2\sqrt{1+x}}$

$x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

$= x_n - \dfrac{2\cos x_n - \sqrt{1+x_n}}{-2\sin x_n - \dfrac{1}{2\sqrt{1+x_n}}}$

The graph of $y = f(x)$ shows that $f(x) = 0$ has one solution, near $x = 1$.



[–2, 10] by [–6, 2]

$x_1 = 1$
$x_2 \approx 0.8361848$
$x_3 \approx 0.8283814$
$x_4 \approx 0.8283608$
$x_5 \approx 0.8283608$

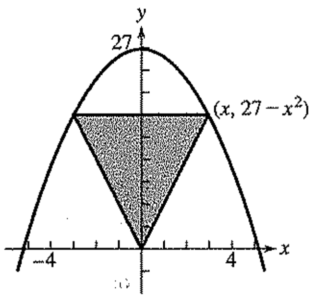
Solution: $x \approx 0.828361$

43. Let t represent time in seconds, where the rocket lifts off at $t = 0$. Since $a(t) = v'(t) = 20$, m/sec^2 and $v(0) = 0$ m/sec, we have $v(t) = 20t$, and so $v(60) = 1200$ m/sec. The speed after 1 minute (60 seconds) will be 1200 m/sec.

44. Let t represent time in seconds, where the rock is blasted upward at $t = 0$. Since $a(t) = v'(t) = -3.72$ m/sec^2 and $v(0) = 93$ m/sec, we have $v(t) = -3.72t + 93$. Since $s'(t) = -3.72t + 93$ and $s(0) = 0$, we have $s(t) = -1.86t^2 + 93t$. Solving $v(t) = 0$, we find that the rock attains its maximum height at $t = 25$ sec and its height at that time is $s(25) = 1162.5$ m.

45. Note that $s = 100 - 2r$ and the sector area is given by $A = \pi r^2\left(\frac{s}{2\pi r}\right) = \frac{1}{2}rs = \frac{1}{2}r(100-2r) = 50r - r^2$. To find the domain of $A(r) = 50r - r^2$, note that $r > 0$ and $0 < s < 2\pi r$, which gives $12.1 \approx \frac{50}{\pi+1} < r < 50$. Since $A'(r) = 50 - 2r$, the critical point occurs at $r = 25$. This value is in the domain and corresponds to the maximum area because $A''(r) = -2$, which is negative for all r. The greatest area is attained when $r = 25$ ft and $s = 50$ ft.

46.



For $0 < x < \sqrt{27}$, the triangle with vertices at (0, 0) and $(\pm x,\ 27 - x^2)$ has an area given by

$A(x) = \frac{1}{2}(2x)(27 - x^2) = 27x - x^3$. Since

$A' = 27 - 3x^2 = 3(3-x)(3+x)$ and $A'' = -6x$, the critical point in the interval $(0, \sqrt{27})$ occurs at $x = 3$ and corresponds to the maximum area because $A''(x)$ is negative in this interval. The largest possible area is $A(3) = 54$ square units.

47. If the dimensions are x ft by x ft by h ft, then the total amount of steel used is $x^2 + 4xh$ ft^2. Therefore, $x^2 + 4xh = 108$ and so $h = \frac{108 - x^2}{4x}$. The volume is given by $V(x) = x^2 h = \frac{108x - x^3}{4} = 27x - 0.25x^3$. Then

$V'(x) = 27 - 0.75x^2 = 0.75(6+x)(6-x)$ and $V''(x) = -1.5x$. The critical point occurs at $x = 6$, and it corresponds to the maximum volume because $V''(x) < 0$ for $x > 0$. The corresponding height is $\frac{108 - 6^2}{4(6)} = 3$ ft. The base measures 6 ft by 6 ft, and the height is 3 ft.

48. If the dimensions are x ft by x ft by h ft, then we have $x^2 h = 32$ and so $h = \frac{32}{x^2}$. Neglecting the quarter-inch thickness of the steel, the area of the steel used is $A(x) = x^2 + 4xh = x^2 + \frac{128}{x}$. We can minimize the weight of the vat by minimizing this quantity. Now

$A'(x) = 2x - 128x^{-2} = \frac{2}{x^2}(x^3 - 4^3)$ and

$A''(x) = 2 + 256x^{-3}$. The critical point occurs at $x = 4$ and corresponds to the minimum possible area because $A''(x) > 0$ for $x > 0$. The corresponding height is $\frac{32}{4^2} = 2$ ft.

The base should measure 4 ft by 4 ft, and the height should be 2 ft.

49. We have $r^2 + \left(\frac{h}{2}\right)^2 = 3$, so $r^2 = 3 - \frac{h^2}{4}$. We wish to minimize the cylinder's volume

$V = \pi r^2 h = \pi\left(3 - \frac{h^2}{4}\right)h = 3\pi h - \frac{\pi h^3}{4}$ for $0 < h < 2\sqrt{3}$.

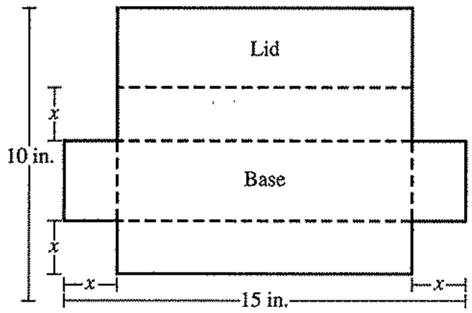
Since $\frac{dV}{dh} = 3\pi - \frac{3\pi h^2}{4} \frac{3\pi}{4}(2+h)(2-h)$ and $\frac{d^2V}{dh^2} = -\frac{3\pi h}{2}$, the critical point occurs at $h = 2$ and it corresponds to the maximum value because $\frac{d^2V}{dh^2} < 0$ for $h > 0$. The corresponding value of r is $\sqrt{3 - \frac{2^2}{4}} = \sqrt{2}$. The largest possible cylinder has height 2 and radius $\sqrt{2}$.

50. Note that, from similar cones, $\frac{r}{6} = \frac{12-h}{12}$, so $h = 12 - 2r$. The volume of the smaller cone is given by

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2(12 - 2r) = 4\pi r^2 - \frac{2\pi}{3}r^3$ for $0 < r < 6$.

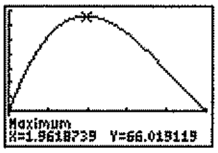
Then $\frac{dV}{dr} = 8\pi r - 2\pi r^2 = 2\pi r(4 - r)$, so the critical point occurs at $r = 4$. This critical point corresponds to the maximum volume because $\frac{dV}{dr} > 0$ for $0 < r < 4$ and $\frac{dV}{dr} < 0$ for $4 < r < 6$. The smaller cone has the largest possible value when $r = 4$ ft and $h = 4$ ft.

51.



(a) $V(x) = x(15-2x)(5-x)$

(b, c) Domain: $0 < x < 5$



The maximum volume is approximately 66.019 and it occurs when $x \approx 1.962$ in.

(d) Note that $V(x) = 2x^3 - 25x^2 + 75x$,

so $V'(x) = 6x^2 - 50x + 75$.

Solving $V'(x) = 0$, we have

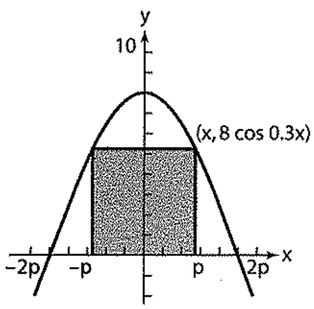
$$x = \frac{50 \pm \sqrt{(-50)^2 - 4(6)(75)}}{2(6)} = \frac{50 \pm \sqrt{700}}{12}$$

$$= \frac{50 \pm 10\sqrt{7}}{12} = \frac{25 \pm 5\sqrt{7}}{6}.$$

These solutions are approximately $x \approx 1.962$ and $x = 6.371$, so the critical point in the appropriate domain occurs at

$$x = \frac{25 - 5\sqrt{7}}{6}.$$

52.



For $0 < x < \frac{5\pi}{3}$, the area of the rectangle is given by $A(x) = (2x)(8\cos 0.3x) = 16x\cos 0.3x$.

Then $A'(x) = 16x(-0.3\sin 0.3x) + 16(\cos 0.3x)(1)$

$= 16(\cos 0.3x - 0.3x\sin 0.3x)$

Solving $A'(x) = 0$ graphically, we find that the critical point occurs at $x \approx 2.868$ and the corresponding area is approximately 29.925 square units.

53. The cost (in thousands of dollars) is given by

$$C(x) = 40x + 30(20 - y) = 40x + 600 - 30\sqrt{x^2 - 144}.$$

Then $C'(x) = 40 - \frac{30}{2\sqrt{x^2-144}}(2x) = 40 - \frac{30x}{\sqrt{x^2-144}}.$

Solving $C'(x) = 0$, we have:

$$\frac{30x}{\sqrt{x^2-144}} = 40$$

$$3x = 4\sqrt{x^2 - 144}$$

$$9x^2 = 16x^2 - 2304$$

$$2304 = 7x^2$$

Choose the positive solution:

$$x = +\frac{48}{\sqrt{7}} \approx 18.142 \text{ mi}$$

$$y = \sqrt{x^2 - 12^2} = \frac{36}{\sqrt{7}} \approx 13.607 \text{ mi}$$

54. The length of the track is given by $2x + 2\pi r$, so we have $2x + 2\pi r = 400$ and therefore $x = 200 - \pi r$. Then the area of the rectangle is

$A(r) = 2rx$

$= 2r(200 - \pi r)$

$= 400r - 2\pi r^2$, for $0 < r < \frac{200}{\pi}$.

Therefore, $A'(r) = 400 - 4\pi r$ and $A''(r) = -4\pi$, so the critical point occurs at $r = \frac{100}{\pi}$ m and this point corresponds to the maximum rectangle area because $A''(r) < 0$ for all r.

The corresponding value of x is

$$x = 200 - \pi\left(\frac{100}{\pi}\right) = 100 \text{ m}.$$

The rectangle will have the largest possible area when $x = 100$ m and $r = \frac{100}{\pi}$ m.

55. Assume the profit is k dollars per hundred grade B tires and $2k$ dollars per hundred grade A tires.

Then the profit is given by

$P(x) = 2kx + k \cdot \frac{40 - 10x}{5 - x}$

$= 2k \cdot \frac{(20 - 5x) + x(5 - x)}{5 - x}$

$= 2k \cdot \frac{20 - x^2}{5 - x}$

$P'(x) = 2k \cdot \frac{(5-x)(-2x) - (20 - x^2)(-1)}{(5-x)^2}$

$= 2k \cdot \frac{x^2 - 10x + 20}{(5-x)^2}$

55. Continued

The solutions of $P'(x)=0$ are

$$x=\frac{10\pm\sqrt{(-10)^2-4(1)(20)}}{2(1)}=5\pm\sqrt{5},$$ so the solution in the appropriate domain is $x=5-\sqrt{5}\approx 2.76$.

Check the profit for the critical point and endpoints:

Critical point:	$x\approx 2.76$	$P(x)\approx 11.06k$
End points:	$x=0$	$P(x)=8k$
	$x=4$	$P(x)=8k$

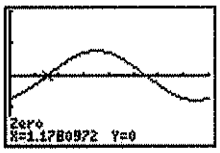
The highest profit is obtained when $x\approx 2.76$ and $y\approx 5.53$, which corresponds to 276 grade A tires and 553 grade B tires.

56. (a) The distance between the particles is $|f(t)|$ where

$$f(t)=-\cos t+\cos\left(t+\frac{\pi}{4}\right).$$ Then

$$f'(t)=\sin t-\sin\left(t+\frac{\pi}{4}\right)$$

Solving $f'(t)=0$ graphically, we obtain $t\approx 1.178$, $t\approx 4.230$, and so on.



$[0, 2\pi]$ by $[-2, 2]$

Alternatively, $f'(t)=0$ may be solved analytically as follows.

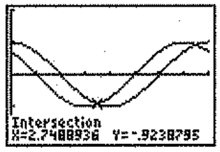
$$f'(t)=\sin\left[\left(t+\frac{\pi}{8}\right)-\frac{\pi}{8}\right]-\sin\left[\left(t+\frac{\pi}{8}\right)+\frac{\pi}{8}\right]$$
$$=\left[\sin\left(t+\frac{\pi}{8}\right)\cos\frac{\pi}{8}-\cos\left(t+\frac{\pi}{8}\right)\sin\frac{\pi}{8}\right]$$
$$-\left[\sin\left(t+\frac{\pi}{8}\right)\cos\frac{\pi}{8}+\cos\left(t+\frac{\pi}{8}\right)\sin\frac{\pi}{8}\right]$$
$$=-2\sin\frac{\pi}{8}\cos\left(t+\frac{\pi}{8}\right),$$

so the critical points occcur when

$\cos\left(t+\frac{\pi}{8}\right)=0$, or $t=\frac{3\pi}{8}+k\pi$. At each of these values,

$f(t)=\pm 2\cos\frac{3\pi}{8}\approx\pm 0.765$ units, so the maximum distance between the particles is 0.765 units.

(b) Solving $\cos t=\cos\left(t+\frac{\pi}{4}\right)$ graphically, we obtain $t\approx 2.749$, $t\approx 5.890$, and so on.



$[0, 2\pi]$ by $[-2, 2]$

Alternatively, this problem may be solved analytically as follows.

$$\cos t=\cos\left(t+\frac{\pi}{4}\right)$$
$$\cos\left[\left(t+\frac{\pi}{8}\right)-\frac{\pi}{8}\right]=\cos\left[\left(t+\frac{\pi}{8}\right)+\frac{\pi}{8}\right]$$
$$\cos\left(t+\frac{\pi}{8}\right)\cos\frac{\pi}{8}+\sin\left(t+\frac{\pi}{8}\right)\sin\frac{\pi}{8}=\cos\left(t+\frac{\pi}{8}\right)\cos\frac{\pi}{8}-\sin\left(t+\frac{\pi}{8}\right)\sin\frac{\pi}{8}$$
$$2\sin\left(t+\frac{\pi}{8}\right)\sin\frac{\pi}{8}=0$$
$$\sin\left(t+\frac{\pi}{8}\right)=0$$
$$t=\frac{7\pi}{8}+k\pi$$

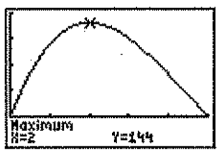
The particles collide when $t=\frac{7\pi}{8}\approx 2.749$ (plus multiples of π if they keep going.)

57. The dimensions will be x in. by $10-2x$ in. by $16-2x$ in., so $V(x)=x(10-2x)(16-2x)=4x^3-52x^2+160x$ for $0<x<5$.

Then $V'(x)=12x^2-104x+160=4(x-2)(3x-20)$, so the critical point in the correct domain is $x=2$.

This critical point corresponds to the maximum possible volume because $V'(x)>0$ for $0<x<2$ and $V'(x)<0$ for $2<x<5$. The box of largest volume has a height of 2 in. and a base measuring 6 in. by 12 in., and its volume is 144 in^3.

Graphical support:



$[0, 5]$ by $[-40, 160]$

58. Step 1:
r = radius of circle
A = area of circle

Step 2:

At the instant in question, $\frac{dr}{dt} = -\frac{2}{\pi}$ m/sec and $r = 10$ m.

Step 3:

We want to find $\frac{dA}{dt}$.

Step 4:
$A = \pi r^2$
Step 5:
$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$
Step 6:
$\frac{dA}{dt} = 2\pi(10)\left(-\frac{2}{\pi}\right) = -40$

The area is changing at the rate of $-40\ \text{m}^2/\text{sec}$.

59. Step 1:
x = x-coordinate of particle
y = y-coordinate of particle
D = distance from origin to particle

Step 2:
At the instant in question, $x = 5$ m, $y = 12$ m,
$\frac{dx}{dt} = -1$ m/sec, and $\frac{dy}{dt} = -5$ m/sec.

Step 3:

We want to find $\frac{dD}{dt}$.

Step 4:
$D = \sqrt{x^2 + y^2}$

Step 5:

$$\frac{dD}{dt} = \frac{1}{2\sqrt{x^2+y^2}}\left(2x\frac{dx}{dt} + 2y\frac{dy}{dt}\right) = \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{\sqrt{x^2+y^2}}$$

Step 6:
$$\frac{dD}{dt} = \frac{(5)(-1) + (12)(-5)}{\sqrt{5^2 + 12^2}} = -5 \text{ m/sec}$$

Since $\frac{dD}{dt}$ is negative, the particle is *approaching* the origin at the *positive* rate of 5 m/sec.

60. Step 1:

x = edge of length of cube
V = volume of cube

Step 2:
At the instant in question,
$\frac{dV}{dt} = 1200\ \text{cm}^3/\text{min}$ and $x = 20$ cm.

Step 3:

We want to find $\frac{dx}{dt}$.

Step 4:
$V = x^3$

Step 5:
$\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$

Step 6:
$1200 = 3(20)^2 \frac{dx}{dt}$

$\frac{dx}{dt} = 1$ cm/min
The edge length is increasing at the rate of 1 cm/min.

61. Step 1:
x = x-coordinate of point
y = y-coordinate of point
D = distance from origin to point

Step 2:

At the instant in question, $x = 3$ and $\frac{dD}{dt} = 11$ units per sec.

Step 3:

We want to find $\frac{dx}{dt}$.

Step 4:

Since $D^2 = x^2 + y^2$ and $y = x^{3/2}$, we have
$D = \sqrt{x^2 + x^3}$ for $x \ge 0$.

Step 5:

$$\frac{dD}{dt} = \frac{1}{2\sqrt{x^2+x^3}}(2x + 3x^2)\frac{dx}{dt}$$
$$= \frac{2x + 3x^2}{2x\sqrt{1+x}}\frac{dx}{dt} = \frac{3x+2}{2\sqrt{1+x}}\frac{dx}{dt}$$

61. Continued

Step 6:

$$11=\frac{3(3)+2}{2\sqrt{4}}\frac{dx}{dt}$$

$$\frac{dx}{dt}=4 \text{ units per sec}$$

62. **(a)** Since $\frac{h}{r}=\frac{10}{4}$, we may write $h=\frac{5r}{2}$ or $r=\frac{2h}{5}$.

(b) Step 1:

h = depth of water in tank
r = radius of surface of water
V = volume of water in tank

Step 2:
At the instant in question,

$$\frac{dV}{dt}=-5 \text{ ft}^3/\text{min and } h=6 \text{ ft.}$$

Step 3:

We want to find $-\frac{dh}{dt}$.

Step 4:

$$V=\frac{1}{3}\pi r^2 h=\frac{4}{75}\pi h^3$$

Step 5:

$$\frac{dV}{dt}=\frac{4}{25}\pi h^2\frac{dh}{dt}$$

Step 6:

$$-5=\frac{4}{25}\pi(6)^2\frac{dh}{dt}$$

$$\frac{dh}{dt}=-\frac{125}{144\pi}\approx -0.276 \text{ ft/min}$$

Since $\frac{dh}{dt}$ is negative, the water level is *dropping* at the positive rate of ≈ 0.276 ft/min.

63. Step 1:

r = radius of outer layer of cable on the spool
θ = clockwise angle turned by spool
s = length of cable that has been unwound

Step 2:

At the instant in question, $\frac{ds}{dt}=6$ ft/sec and $r=1.2$ ft

Step 3:

We want to find $\frac{d\theta}{dt}$.

Step 4:

$s=r\theta$

Step 5:

Since r is essentially constant, $\frac{ds}{dt}=r\frac{d\theta}{dt}$

Step 6:

$$6=1.2\frac{d\theta}{dt}$$

$$\frac{d\theta}{dt}=5 \text{ radians/sec}$$

The spool is turning at the rate of 5 radians per second.

64. $a(t)=v'(t)=-g=-32$ ft/sec^2

Since $v(0)=32$ ft/sec, $v(t)=s'(t)=-32t+32$.

Since $s(0)=-17$ ft, $s(t)=-16t^2+32t-17$.
The shovelful of dirt reaches its maximum height when $v(t)=0$, at $t=1$ sec. Since $s(1)=-1$, the shovelful of dirt is still below ground level at this time. There was not enough speed to get the dirt out of the hole. Duck!

65. We have $V=\frac{1}{3}\pi r^2 h$, so $\frac{dV}{dr}=\frac{2}{3}\pi rh$ and $dV=\frac{2}{3}\pi rh\ dr$.
When the radius changes from a to $a+dr$, the volume change is approximately $dV=\frac{2}{3}\pi ah$ dr.

66. **(a)** Let x = edge of length of cube and S = surface area of cube. Then $S=6x^2$, which means $\frac{dS}{dx}=12x$ and $dS=12x\ dx$. We want $|dS|\le 0.02S$, which gives $|12x\ dx|\le 0.02(6x^2)$ or $|dx|\le 0.01x$. The edge should be measured with an error of no more than 1%.

(b) Let V = volume of cube. Then $V=x^3$, which means $\frac{dV}{dx}=3x^2$ and $dV=3x^2dx$. We have $|dx|\le 0.01x$, which means $|3x^2dx|\le 3x^2(0.01x)=0.03V$, so $|dV|\le 0.03V$. The volume calculation will be accurate to within approximately 3% of the correct volume.

67. Let C = circumference, r = radius, S = surface area, and V = volume.

(a) Since $C = 2\pi r$, we have $\frac{dC}{dr} = 2\pi$ and so $dC = 2\pi\ dr$.

Therefore, $\left|\frac{dC}{C}\right| = \left|\frac{2\pi dr}{2\pi r}\right| = \left|\frac{dr}{r}\right| < \frac{0.4\text{ cm}}{10\text{ cm}} = 0.04$ The calculated radius will be within approximately 4% of the correct radius.

(b) Since $S = 4\pi r^2$, we have $\frac{dS}{dr} = 8\pi r$ and so $dS = 8\pi r\ dr$. Therefore,

$\left|\frac{dS}{S}\right| = \left|\frac{8\pi r\ dr}{4\pi r^2}\right| = \left|\frac{2\ dr}{r}\right| \le 2(0.04) = 0.08.$ The calculated surface area will be within approximately 8% of the correct surface area.

(c) Since $V = \frac{4}{3}\pi r^3$, we have $\frac{dV}{dr} = 4\pi r^2$ and so $dV = 4\pi r^2\ dr$. Therefore

$$\left|\frac{dV}{V}\right| = \left|\frac{4\pi r^2\ dr}{\frac{4}{3}\pi r^3}\right| = \left|\frac{3\ dr}{r}\right| \le 3(0.04) = 0.12.$$

The calculated volume will be within approximately 12% of the correct volume.

68. By similar triangles, we have $\frac{a}{6} = \frac{a+20}{h}$, which gives $ah = 6a + 120$, or $h = 6 + 120a^{-1}$ The height of the lamp post is approximately $6 + 120(15)^{-1} = 14$ ft. The estimated error in measuring a was

$|da| \le 1$ in. $= \frac{1}{12}$ ft. Since $\frac{dh}{da} = -120a^{-2}$, we have

$|dh| = \left|-120a^{-2}\ da\right| \le 120(15)^{-2}\left(\frac{1}{12}\right) = \frac{2}{45}$ ft, so the

estimated possible error is $\pm\frac{2}{45}$ ft or $\pm\frac{8}{15}$ in.

69. $\frac{dy}{dx} = 2\sin x\cos x - 3$. Since $\sin x$ and $\cos x$ are both between 1and -1, the value of $2\sin x\cos x$ is never greater than 2. Therefore, $\frac{dy}{dx} \le 2 - 3 = -1$ for all values of x.

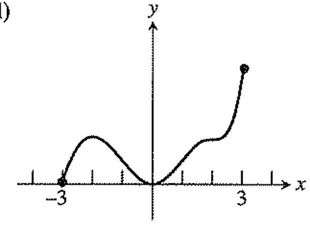
Since $\frac{dy}{dx}$ is always negative, the function decreases on every interval.

70. **(a)** f has a relative maximum at $x = -2$. This is where $f'(x) = 0$, causing f' to go from positive to negative.

(b) f has a relative minimum at $x = 0$. This is where $f'(x) = 0$, causing f' to go from negative to positive.

(c) The graph of f is concave up on $(-1, 1)$ and on $(2, 3)$. These are the intervals on which the derivatives of f are increasing.

(d)



71. **(a)** $A = \pi r^2$

$\frac{dA}{dt} = 2\pi r dr$

$\frac{dA}{dt} = 2\pi(2)\left(\frac{1}{3}\right) = \frac{4}{3}\pi\frac{\text{in.}^2}{\text{sec}}$

(b) $dA = dV$

$\frac{4}{3}\pi = \frac{1}{3}\pi r^2 dh$

$\frac{4}{3}\pi = \frac{1}{3}\pi(2)^2 dh$

$\frac{dh}{dt} = 1\frac{\text{in.}}{\text{sec}}$

(c) $\frac{dA}{dh} = \frac{\frac{4}{3}\pi}{1} = \frac{4}{3}\pi\frac{\text{in.}^2}{\text{in.}}$

72. **(a)** $2a + 4b = 60$

$b = 15 - 2a$

$V = \pi a^2 b = \pi a^2(15 - 2a)$

$\frac{dV}{da} = 30\pi a - \frac{3\pi a^2}{2}$

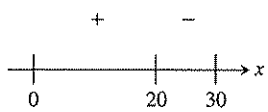
$30\pi a = \frac{3\pi a^2}{2}$

$a = 20$

$2(20) + 4b = 60$

$b = 5$

(b) The sign graph for the derivative $\frac{dV}{da} = \frac{3\pi a}{2}(20 - a)$ on the interval (0,30) is as follows:



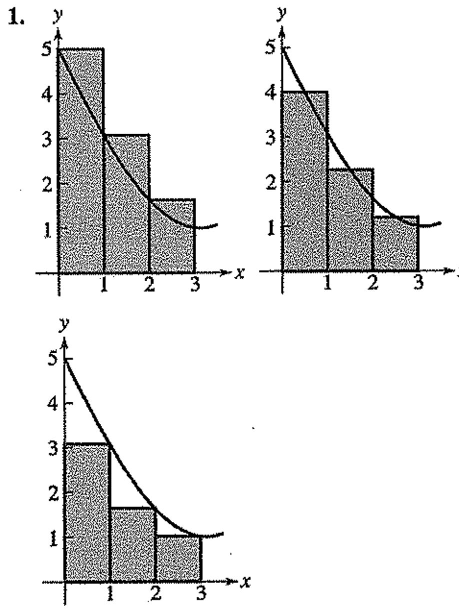
By the First Derivative Test, there is a maximum at $x = 20$.

Chapter 5
The Definite Integral

Section 5.1 Estimating with Finite Sums (pp. 263-273)

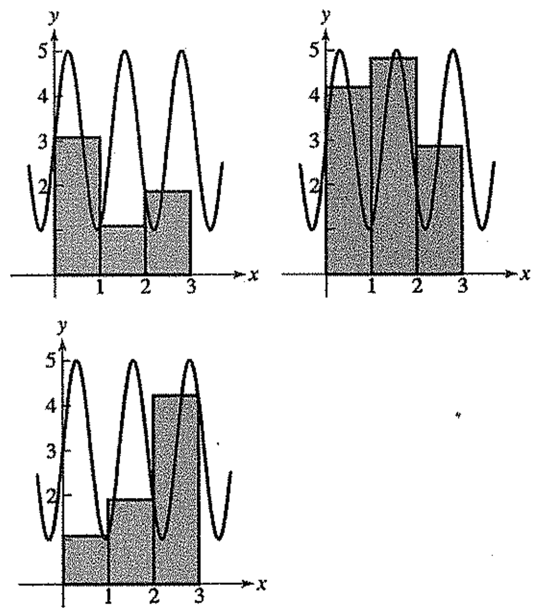
Exploration 1 Which RAM is the Biggest?

1.



LRAM > MRAM > RRAM

2.



MRAM > RRAM > LRAM

3. RRAM > MRAM > LRAM, because the heights of the rectangles increase as you move toward the right under an increasing function.

4. LRAM > MRAM > RRAM, because the heights of the rectangles decrease as you move toward the right under a decreasing function.

Quick Review 5.1

1. 80 mph • 5 hr = 400 mi

2. 48 mph • 3 hr = 144 mi

3. $10 \text{ ft/sec}^2 \cdot 10 \text{ sec} = 100 \text{ ft/sec}$

$100 \text{ ft/sec} \cdot \frac{1 \text{ mi}}{5280 \text{ ft}} \cdot \frac{3600 \text{ sec}}{1 \text{ h}} \approx 68.18 \text{ mph}$

4. $300{,}000 \text{ km / sec} \cdot \frac{3600 \text{ sec}}{1 \text{ hr}} \cdot \frac{24 \text{ hr}}{1 \text{ day}} \cdot \frac{365 \text{ days}}{1 \text{ yr}} \cdot 1 \text{ yr}$

$\approx 9.46 \times 10^{12} \text{ km}$

5. $(6 \text{ mph})(3 \text{ h}) + (5 \text{ mph})(2 \text{ h}) = 18 \text{ mi} + 10 \text{ mi} = 28 \text{ mi}$

6. $20 \text{ gal/min} \cdot 1 \text{ h} \cdot \frac{60 \text{ min}}{1 \text{ h}} = 1200 \text{ gal}$

7. $(-1°\text{C/h})(12 \text{ h}) + (1.5°\text{C})(6 \text{ h}) = -3°\text{C}$

8. $300 \text{ ft}^3/\text{sec} \cdot \frac{3600 \text{ sec}}{1 \text{ h}} \cdot \frac{24 \text{ h}}{1 \text{ day}} \cdot 1 \text{ day} = 25{,}920{,}000 \text{ ft}^3$

9. $350 \text{ people/mi}^2 \cdot 50 \text{ mi}^2 = 17{,}500 \text{ people}$

10. $70 \text{ times/sec} \cdot \frac{3600 \text{ sec}}{1 \text{ h}} \cdot 1 \text{ h} \cdot 0.7 = 176{,}400 \text{ times}$

Section 5.1 Exercises

1. Since $v(t) = 5$ is a strait line, compute the area under the curve.
$x = (t)\, v(t) = (4)(5) = 20$

2. Since $v(t) = 2t + 1$ creates a trapezoid with the x-axis, compute the area of the curve under the trapezoid.

$A = \frac{h}{2}(a+b)$
$a = t = 0 = v(0) = 2(0) + 1 = 1$
$b = t = 4 = v(4) = 2(4) + 1 = 9$
$h = 4$
$A = \frac{4}{2}(9+1) = 20$

3. Each rectangle has base 1. The height of each rectangle is Found by using the points $t = (0.5, 1.5, 2.5, 3.5)$ in the equation $v(t)\, t^2 + 1$. The area under the curve is approximately $1\left(\frac{5}{4} + \frac{13}{4} + \frac{29}{4} + \frac{53}{4}\right) = 25$, so the particle is close to $x = 25$.

4. Each rectangle has base 1. The height of each rectangle is found by using the points $y = (0.5, 1.5, 2.5, 3.5, 4.5)$ in the equation $v(t) = t^2 + 1$. The area under the curve is approximately $1\left(\frac{5}{4} + \frac{13}{4} + \frac{29}{4} + \frac{53}{4} + \frac{85}{4}\right) = 46.25$, so the particle is close to $x = 46.25$.