59. Continued

(g) The particle is on the positive side since $s(9)=\int_0^9 f(x)\,dx>0$ (the area below the x-axis is smaller than the area above the x-axis).

60. $f(x)=\frac{d}{dx}\left(\int_1^x f(t)\,dt\right)=\frac{d}{dx}\left(x^2-2x+1\right)=2x-2$

61. $f'(x)=\frac{d}{dx}\left(2+\int_0^x \frac{10}{1+t}dt\right)=\frac{10}{1+x}$

$f'(0)=10$

$f'(0)=2+\int_0^0 \frac{10}{1+t}dt=2$

$L(x)=2+10x$

62. $f(x)=\frac{d}{dx}\left(\int_0^x f(t)\,dt\right)$

$=\frac{d}{dx}(x\cos \pi x)$

$=x(-\pi \sin \pi x)+1\bullet\cos \pi x$

$=-\pi x \sin \pi x+\cos \pi x$

$f(4)=-4\pi \sin 4\pi+\cos 4\pi=1$

63. One arch of $\sin kx$ is from $x=0$ to $x=\frac{\pi}{k}$.

$\text{Area}=\int_0^{\pi/k}\sin kx\,dx=\left[-\frac{1}{k}\cos kx\right]_0^{\pi/k}=\frac{1}{k}-\left(-\frac{1}{k}\right)=\frac{2}{k}$

64. (a) $\int_{-3}^{2}\left(6-x-x^2\right)dx=\left[6x-\frac{1}{2}x^2-\frac{1}{3}x^3\right]_{-3}^{2}$

$=\frac{22}{3}-\left(-\frac{27}{2}\right)$

$=\frac{125}{6}$

(b) The vertex is at $x=\frac{-(-1)}{2(-1)}=-\frac{1}{2}$. (Recall that the vertex of a parabola $y=ax^2+bx+c$ is at $x=-\frac{b}{2a}$.)

$y\left(-\frac{1}{2}\right)=\frac{25}{4}$, so the height is $\frac{25}{4}$.

(c) The base is $2-(-3)=5$.

$\frac{2}{3}(\text{base})(\text{height})=\frac{2}{3}(5)\left(\frac{25}{4}\right)=\frac{125}{6}$

65. True. The Fundamental Theorem of Calculus guarantees that F is differentiable on I, so it must be continuous on I.

66. False. In fact, $\int_a^b e^{x^2}dx$ is a real number, so its derivative is always 0.

67. D.

68. D. See the Fundamental Theorem of Calculus.

69. E. $f(a)+f^{1}(a)(x-\pi)$

$f(\pi)=0$

$f(\pi)=-1$

$-1(x-\pi)=\pi-x$

70. E.

71. (a) $f(t)$ is an even function so $\int_{-x}^{0}\frac{\sin(t)}{t}dt=\int_0^x\frac{\sin(t)}{t}dt.$

$\text{Si}(-x)=\int_0^{-x}\frac{\sin(t)}{t}dt$

$=-\int_{-x}^{0}\frac{\sin(t)}{t}dt$

$=-\int_0^{x}\frac{\sin(t)}{t}dt=-\text{Si}(x)$

(b) $\text{Si}(0)=\int_0^0\frac{\sin t}{t}dt=0$

(c) $\text{Si}'(x)=f(t)=0$ when $t=\pi k$, k a nonzero integer.

(d)

[−20, 20] by [−3, 20, 203]

72. (a) $c(100)-c(1)=\int_1^{100}\left(\frac{dc}{dx}\right)dx$

$=\int_1^{100}\frac{1}{2\sqrt{x}}dx=\left[\sqrt{x}\right]_1^{100}$

$=10-1=9$ or \$9

(b) $c(400)-c(100)=\int_{100}^{400}\left(\frac{dc}{dx}\right)dx$

$=\int_{100}^{400}\frac{1}{2\sqrt{x}}dx=\left[\sqrt{x}\right]_{100}^{400}$

$=20-10=10$ or \$10

73. $\int_0^3\left(2-\frac{2}{(x+1)^2}\right)dx=\left[2x+2(x+1)^{-1}\right]_0^3$

$=\left[6+\frac{1}{2}\right]-2=\frac{9}{2}$

$=4.5$ thousand

The company should expect \$4500.

74. (a) $\frac{1}{30-0}\int_0^{30}\left(450-\frac{x^2}{2}\right)dx=\frac{1}{30}\left[450x-\frac{x^3}{6}\right]_0^{30}$

$=300$ drums

(b) (300 drums)(\$0.02 per drum) = \$6

75. (a) True, because $h'(x)=f(x)$ and therefore $h''(x)=f'(x)$.

(b) True because h and h' are both differentiable by part (a).

(c) True, because $h'(1)=f(1)=0$.

75. Continued

(d) True, becaue $h'(1) = f(1) = 0$ and $h''(1) = f'(1) < 0$.

(e) False, because $h''(1) = f'(1) < 0$.

(f) False, because $h''(1) = f'(1) \neq 0$

(g) True, because $h'(x) = f(x)$, and f is a decreasing function that includes the point (1,0).

76. Since $f(t)$ is odd, $\int_{-x}^{0} f(t)dt = -\int_{0}^{x} f(t)dt$ because the area between the curve and the x-axis from 0 to x is the opposite of the area between the curve and the x-axis from $-x$ to 0, but it is on the opposite side of the x-axis.

$$\int_0^{-x} f(t)\,dt = -\int_{-x}^{0} f(t)\,dt = -\left[-\int_0^x f(t)dt\right] = \int_0^x f(t)\,dt$$

Thus $\int_0^x f(t)\,dt$ is even.

77. Since $f(t)$ is even, $\int_{-x}^{0} f(t)\,dt = \int_0^x f(t)\,dt$ because the area between the curve and the x-axis from 0 to x is the same as the area between the curve and the x-axis from $-x$ to 0.

$$\int_0^{-x} f(t)\,dt = -\int_{-x}^{0} f(t)\,dt = -\int_0^x f(t)\,dt$$

Thus $\int_0^x f(t)\,dt$ is odd.

78. If f is an even continuous function, then $\int_0^x f(t)\,dt$ is odd, but $\frac{d}{dx}\int_0^x f(t)\,dt = f(x)$. Therefore, f is the derivative of the odd continuous function $\int_0^x f(t)\,dt$.

Similarly, if f is an odd continuous function, then f is the derivative of the even continuous function $\int_0^x f(t)\,dt$.

79. Solving $\text{NINT}\left(\frac{\sin t}{t}, t, 0, x\right) = 1$ graphically, the solution is $x \approx 1.0648397$. We now argue that there are no other solutions, using the functions Si(x) and $f(t)$ as defined in Exercise 56. Since $\frac{d}{dx}\text{Si}(x) = f(x) = \frac{\sin x}{x}$, Si($x$) is increasing on each interval $[2k\pi, (2k+1)\pi]$ and decreasing on each interval $[(2k+1)\pi, (2k+2)\pi]$, where K is a nonnegative integer. Thus, for $x > 0$, Si(x) has its local minima at $x = 2k\pi$, where k is a positive integer. Furthermore, each arch of $y = f(x)$ is smaller in height than the previous one, so $\int_{2k\pi}^{(2k+1)\pi} |f(x)|dx > \int_{(2k+1)\pi}^{(2k+2)\pi} |f(x)|dx$. This means that $\text{Si}(2k+2)\pi) - \text{Si}(2k\pi) = \int_{2k\pi}^{(2k+2)\pi} f(x)dx > 0$, so each successive minimum value is greater than the previous one. Since $f(2\pi) \approx \text{NINT}\left(\frac{\sin x}{x}, x, o, 2\pi\right) \approx 1.42$ and Si(x) is continuous for $x > 0$, this means Si(x) > 1.42 (and hence Si(x) $\neq 1$) for $x \geq 2\pi$. Now, Si(x) $= 1$ has exactly one solution in the interval $[0, \pi]$ because Si(x) is increasing on this interval and $x \approx 1.065$ is a solution. Furthermore, Si(x) $= 1$ has no solution on the interval $[\pi, 2\pi]$ because Si(x) is decreasing on this interval and Si(2π) $\approx 1.42 > 1$. Thus, Si(x) $= 1$ has exactly one solution in the interval $[0, \infty)$. Also, there is no solution in the interval $(-\infty, 0]$ because Si(x) is odd by Exercise 56 (or 62), which means that Si(x) ≤ 0 for $x \leq 0$ (since Si(x) ≥ 0 for $x \geq 0$).

Section 5.5 Trapezoidal Rule (pp. 306–315)

Exploration 1 Area Under a Parabolic Arc

1. Let $y = f(x) = Ax^2 + Bx + C$

Then $y_0 = f(-h) = Ah^2 - Bh + C$,

$y_1 = f(0) = A(0)^2 + B(0) + C = C$, and

$y_2 = f(h) = Ah^2 + Bh + C$.

2.
$$\begin{aligned} y_0 + 4y_1 + y_2 &= Ah^2 - Bh + C + 4C + Ah^2 + Bh + C \\ &= 2Ah^2 + 6C. \end{aligned}$$

3.
$$\begin{aligned} A_p &= \int_{-h}^{h} (Ax^2 + Bx + C)dx \\ &= \left[A\frac{x^3}{3} + B\frac{x^2}{2} + Cx\right]_{-h}^{h} \\ &= A\frac{h^3}{3} + B\frac{h^2}{2} + Ch - \left(-A\frac{h^3}{3} + B\frac{h^2}{2} - Ch\right) \\ &= 2A\frac{h^3}{3} + 2Ch \\ &= \frac{h}{3}(2Ah^2 + 6C) \end{aligned}$$

4. Substitute the expression in step 2 for the parenthetically enclosed expression in step 3:

$$\begin{aligned} A_p &= \frac{h}{3}(2Ah^2 + 6C) \\ &= \frac{h}{3}(y_0 + 4y_1 + y_2). \end{aligned}$$

Quick Review 5.5

1. $y' = -\sin x$

$y'' = -\cos x$

$y'' < 0$ on $[-1, 1]$, so the curve is concave down on $[-1, 1]$.

2. $y' = 4x^3 - 12$

$y'' = 12x^2$

$y'' > 0$ on $[8, 17]$, so the curve is concave up on $[8, 17]$.

3. $y' = 12x^2 - 6x$

$y'' = 24x - 6$

$y'' < 0$ on $[-8, 0]$, so the curve is concave down on $[-8, 0]$.

4. $y' = \frac{1}{2}\cos\frac{x}{2}$

$y'' = -\frac{1}{4}\sin\frac{x}{2}$

$y'' \le 0$ on $[48\pi, 50\pi]$, so the curve is concave down on $[48\pi, 50\pi]$.

5. $y' = 2e^{2x}$

$y'' = 4e^{2x}$

$y'' > 0$ on $[-5, 5]$, so the curve is concave up on $[-5, 5]$.

6. $y' = \frac{1}{x}$

$y'' = -\frac{1}{x^2}$

$y'' < 0$ on $[100, 200]$, so the curve is concave down on $[100, 200]$.

7. $y' = -\frac{1}{x^2}$

$y'' = \frac{2}{x^3}$

$y'' > 0$ on $[3, 6]$, so the curve is concave up on $[3, 6]$.

8. $y' = -\csc x \cot x$

$y'' = (-\csc x)(-\csc^2 x) + (\csc x \cot x)(\cot x)$

$= \csc^3 x + \csc x \cot^2 x$

$y'' > 0$ on $[0, \pi]$, so the curve is concave up on $[0, \pi]$.

9. $y' = -100x^9$

$y'' = -900x^8$

$y'' < 0$ on $[10, 10^{10}]$, so the curve is concave down on $[10, 10^{10}]$.

10. $y' = \cos x + \sin x$

$y'' = -\sin x + \cos x$

$y'' < 0$ on $[1, 2]$, so the curve is concave down.

Section 5.5 Exercises

1. (a) $f(x) = x, h = \frac{2-0}{4} = \frac{1}{2}$

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$f(x)$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2

$$T = \frac{1}{4}\left(0 + 2\left(\frac{1}{2}\right) + 2(1) + 2\left(\frac{3}{2}\right) + 2\right) = 2$$

(b) $f'(x) = 1, f''(x) = 0$

The approximation is exact.

(c) $\int_0^2 x\,dx = \left[\frac{1}{2}x^2\right]_0^2 = 2$

2. (a) $f(x) = x^2, h = \frac{2-0}{4} = \frac{1}{2}$

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$f(x)$	0	$\frac{1}{4}$	1	$\frac{9}{4}$	4

$$T = \frac{1}{4}\left(0 + 2\left(\frac{1}{4}\right) + 2(1) + 2\left(\frac{9}{4}\right) + 4\right) = 2.75$$

(b) $f'(x) = 2x, f''(x) = 2 > 0$ on $[0,2]$

The aproximation is an overestimate.

(c) $\int_0^2 x^2 dx = \left[\frac{1}{3}x^3\right]_0^2 = \frac{8}{3}$

3. (a) $f(x) = x^3, h = \frac{2-0}{4} = \frac{1}{2}$

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$f(x)$	0	$\frac{1}{8}$	1	$\frac{27}{8}$	8

$$T = \frac{1}{4}\left(0 + 2\left(\frac{1}{8}\right) + 2(1) + 2\left(\frac{27}{8}\right) + 8\right) = 4.25$$

(b) $f'(x) = 3x^2, f''(x) = 6x > 0$ on $[0, 2]$

The aproximation is an overestimate.

(c) $\int_0^2 x^3 dx = \left[\frac{1}{4}x^4\right]_0^2 = 4$

4. (a) $f(x) = \frac{1}{x}, h = \frac{2-1}{4} = \frac{1}{4}$

x	1	$\frac{5}{4}$	$\frac{3}{2}$	$\frac{7}{4}$	2
$f(x)$	1	$\frac{4}{5}$	$\frac{2}{3}$	$\frac{4}{7}$	$\frac{1}{2}$

$$T = \frac{1}{8}\left(1 + 2\left(\frac{4}{5}\right) + 2\left(\frac{2}{3}\right) + 2\left(\frac{4}{7}\right) + \frac{1}{2}\right) \approx 0.697$$

(b) $f'(x) = -\frac{1}{x^2}, f''(x) = \frac{2}{x^3} > 0$ on $[1, 2]$

The approximation is an overestimate.

(c) $\int_1^2 \frac{1}{x}dx = \left[\ln|x|\right]_1^2 = \ln 2 \approx 0.693$

5. (a) $f(x)=\sqrt{x}, h=\frac{4-0}{4}=1$

x	0	1	2	3	4
$f(x)$	0	1	$\sqrt{2}$	$\sqrt{3}$	2

$T=\frac{1}{2}\left(0+2(1)+2\sqrt{2}+2\sqrt{3}+2\right)\approx 5.146$

(b) $f'(x)=-\frac{1}{2}x^{-1/2}, f''(x)=-\frac{1}{4}x^{-3/2}<0$ on $[0, 4]$

The approximation is an underestimate.

(c) $\int_0^4 \sqrt{x}dx=\left[\frac{2}{3}x^{3/2}\right]_0^4=\frac{16}{3}\approx 5.333$

6. (a) $f(x)=\sin x, h=\frac{\pi-0}{4}=\frac{\pi}{4}$

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
$f(x)$	0	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{2}}{2}$	0

$T=\frac{\pi}{8}\left(0+2\left(\frac{\sqrt{2}}{2}\right)+2(1)+2\left(\frac{\sqrt{2}}{2}\right)+0\right)\approx 1.896$

(b) $f'(x)=\cos x, f''(x)=-\sin x<0$ on $[0, \pi]$

The approximation is an underestimate.

(c) $\int_0^{\pi} \sin x\, dx=\left[-\cos x\right]_0^{\pi}=2$

7. $T=\frac{h}{2}\left(y_0+2y_1+2y_2+\cdots+2y_{n-1}+y_n\right)$

$\int_0^6 f(x)dx\approx\frac{1}{2}(12+2(10)+2(9)+2(11)+2(13)+2(16)+18)=74$

8. $T=\frac{h}{2}\left(y_0+2y_1+2y_2+\cdots+2y_{n-1}+y_n\right)$

$\int_2^8 f(x)\,dx\approx\frac{1}{2}(16+2(19)+2(17)+2(14)+2(13)+2(16)+20)=97$

9. $\frac{5}{2}(6.0+2(8.2)+2(9.1)+\cdots+2(12.7)+13.0)(30)$

$=15.990 \text{ ft}^3$

10. (a) $\frac{200}{2}(0+2(520)+2(800)+2(1000)+\cdots+2(860)$

$+0)(20)=26{,}360{,}000 \text{ ft}^3$

(b) You plan to start with 26,360 fish. You intend to have $(0.75)(26{,}360)=19{,}770$ fish to be caught. Since $\frac{19{,}770}{20}=988.5$, the town can sell at most 988 licenses.

11. Sum the trapezoids and multiply by $\frac{1}{3600}$ to change seconds to hours

$\frac{1}{2}(2.0(0+30)+(3.2-2.0)(30+40)+(4.5-3.2)(40+50)$
$+(5.8-4.5)(50+60)+(7.7-5.8)(60+70)$
$+(9.5-7.7)(70+80)+(11.6-9.5)(80+90)$
$+(14.9-11.6)(90+100)+(17.8-14.9)(100+110)$
$+(21.7-17.8)(110+120)+(26.3-21.7)(120+130))$

$\frac{1}{3600}\approx 0.633 \text{ mi}\approx 3340$ feet.

12. Sum the trapezoids and multiply by $\frac{1}{3600}$ to change seconds to hours.

$\frac{1}{3600}\left(\frac{1}{2}\right)(0+2(3)+2(7)+2(12)+2(17)+2(25)+2(33)$
$+2(41)+48)=0.045 \text{ mi}\approx 238$ feet.

13. (a) $\int_0^2 x\,dx=\left(\frac{1/2}{3}\right)\left(0+4\left(\frac{1}{2}\right)+2(1)+4\left(\frac{3}{2}\right)+2\right)=2$

(b) $\int_0^2 x\,dx=\frac{x^2}{2}\Big|_0^2=\frac{2^2}{2}+\frac{0^2}{2}=2$

14. (a) $\int_0^2 x^2dx=\left(\frac{1/2}{3}\right)\left(0^2+4\left(\frac{1}{2}\right)^2+2(1)^2+4\left(\frac{3}{2}\right)^2+2^2\right)$

$=\frac{8}{3}$

(b) $\int_0^2 x^2dx=\frac{x^3}{3}\Big|_0^2=\frac{2^3}{3}+\frac{0^3}{3}=\frac{8}{3}$

15. (a) $\int_0^2 x^3dx=\left(\frac{1/2}{3}\right)\left(0^3+4\left(\frac{1}{2}\right)^3+2(1)^3+4\left(\frac{3}{2}\right)^3+2^3\right)=4$

(b) $\int_0^2 x^3dx=\frac{x^4}{4}\Big|_0^2=\frac{2^4}{4}-\frac{0^4}{4}=4$

16. (a) $\int_1^2\frac{1}{x}dx=\left(\frac{1/4}{3}\right)\left(\frac{1}{1}+4\left(\frac{1}{1.25}\right)+2\left(\frac{1}{1.5}\right)+4\left(\frac{1}{1.75}\right)+\frac{1}{2}\right)$

≈ 0.69325

(b) $\int_1^2\frac{1}{x}dx=\ln x\Big|_1^2=\ln 2-\ln 1\approx 0.69315$

17. (a) $\int_0^4\sqrt{x}\,dx=\left(\frac{1}{3}\right)(\sqrt{0}+4(\sqrt{1})+2(\sqrt{2})+4(\sqrt{3})+(\sqrt{4}))$

≈ 5.2522

(b) $\int_0^4\sqrt{x}dx=\frac{2}{3}x^{3/2}\Big|_0^4=\frac{2}{3}(4)^{3/2}-\frac{2}{3}(0)^{3/2}=\frac{16}{3}$

18. (a) $\int_0^{\pi} \sin x\,dx = \left(\frac{\pi/4}{3}\right)\left(\sin(0)+4\left(\sin\left(\frac{\pi}{4}\right)\right)\right.$
$\left. +2\left(\sin\left(\frac{\pi}{2}\right)\right)+4\left(\sin\left(\frac{3\pi}{4}\right)\right)+\sin\pi\right) \approx 2.00456$

(b) $\int_0^{\pi} \sin x\,dx = -\cos x\big|_0^{\pi} = -\cos\pi - (-\cos 0) = 2$

19. (a) $f(x) = x^3 - 2x,\ h = \frac{3-(-1)}{4} = 1$

x	−1	0	1	2	3
$f(x)$	1	0	−1	4	21

$S = \frac{1}{3}(1+4(0)+2(-1)+4(4)+21) = 12$

(b) $\int_{-1}^{3}(x^3-2x)dx = \left[\frac{1}{4}x^4 - x^2\right]_{-1}^{3}$
$= \left(\frac{81}{4}-9\right)-\left(\frac{1}{4}-1\right)$
$= 12$

$|E_s| = 0$

(c) For $f(x) = x^3-2x$, $M_{f^{(4)}} = 0$ since $f^{(4)} = 0$.

(d) Simpson's Rule for cubic polynomials will always give exact values since $f^{(4)} = 0$ for all cubic polynomials.

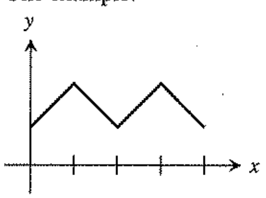
20. The average of the 13 discrete temperatures gives equal weight to the low values at the end.

21. (a) $\frac{1}{2}(126+2 \cdot 65+2 \cdot 66+\cdots+2 \cdot 58+110) = 841$

$av(f) \approx \frac{1}{12} \cdot 841 \approx 70.08$

(b) We are approximating the area under the temperature graph. By doubling the endpoints, the error in the first and last trapezoids increases.

22. Sketch a graph of 4 line segments joined at sharp corners. One example:



23. $S_{50} \approx 3.13791,\ S_{100} \approx 3.14029$

24. $S_{50} \approx 1.08943,\ S_{100} \approx 1.08943$

25. $S_{50} = 1.37066,\ S_{100} = 1.37066$ using $a = 0.0001$ as lower limit
$S_{50} = 1.37076,\ S_{100} = 1.37076$ using $a = 0.000000001$ as lower limit

26. $S_{50} \approx 0.82812,\ S_{100} \approx 0.82812$

27. (a) $T_{10} \approx 1.983523538$
$T_{100} \approx 1.999835504$
$T_{1000} \approx 1.999998355$

(b)

n	$\lvert E_T \rvert = 2 - T_n$
10	$0.016476462 = 1.6476462 \times 10^{-2}$
100	1.64496×10^{-4}
1000	1.645×10^{-6}

(c) $\left|E_{T_{10n}}\right| \approx 10^{-2}\left|E_{T_n}\right|$

(d) $b-a = \pi,\ h^2 = \frac{\pi^2}{n^2},\ M = 1$

$\left|E_{T_n}\right| \le \frac{\pi}{12}\left(\frac{\pi^2}{n^2}\right) = \frac{\pi^3}{12n^2}$

$\left|E_{T_{10n}}\right| \le \frac{\pi^3}{12(10n)^2} = 10^{-2}\left|E_{T_n}\right|$

28. (a) $S_{10} \approx 2.000109517$
$S_{100} \approx 2.000000011$
$S_{1000} \approx 2.000000000$

(b)

n	$\lvert E_s \rvert = 2 - S_n$
10	1.09517×10^{-4}
100	1.1×10^{-8}
1000	0

(c) $\left|E_{S_{10n}}\right| = 10^{-4}\left|E_{S_n}\right|$

(d) $b-a = \pi, h^4 = \frac{\pi^4}{n^4}, M = 1$

$\left|E_{S_n}\right| \le \frac{\pi}{180}\left(\frac{\pi^4}{n^4}\right) = \frac{\pi^5}{180n^4}$

$\left|E_{S_{10n}}\right| \le \frac{\pi^5}{180(10n)^4} = 10^{-4}\left|E_{S_n}\right|$

29. $h = \frac{24 \text{ in.}}{6} = 4$ in.

Estimate the area to be

$\frac{4}{3}[0 + 4(18.75) + 2(24) + 4(26) + 2(24) + 4(18.75) + 0]$

$\approx 466.67\ in^2$

30. Note that the tank cross-section is represented by the shaded area, not the entire wing cross-section. Using Simpson's Rule, estimate the cross-section area to be

$\frac{1}{3}[y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + 4y_5 + y_6]$

$= \frac{1}{3}[1.5 + 4(1.6) + 2(1.8) + 4(1.9) + 2(2.0)$
$+ 4(2.1) + 2.1] = 11.2 \text{ ft}^2$

Length $\approx (5000 \text{ lb})\left(\frac{1}{42 \text{ lb}/\text{ft}^3}\right)\left(\frac{1}{11.2\text{ft}^2}\right) \approx 10.63 \text{ ft}$

31. False. The Trapezoidal Rule will over estimate the integral if it is concave up.

32. False. For example, the two approximations will be the same if f is constant on $[a, b]$.

33. A. LRAM < T < RRAM, so RRAM < 16.4.

34. B. $\int_{-2}^{4} \frac{e^x}{2} dx = \frac{1}{2}\left(2\frac{e^{-2}}{2} + 4\frac{e^0}{2} + 4\frac{e^2}{2} + 2\frac{e^4}{2}\right)$

$= e^4 + 2e^2 + 2e^0 + e^{-2}$

35. C. $\int_0^{\pi} \sin x\, dx = \frac{\pi/4}{2}\left(\sin 0 + 4\left(\sin\frac{\pi}{4}\right)\right.$

$\left.+ 2\left(\sin\frac{\pi}{2}\right) + 4\left(\sin\frac{3\pi}{4}\right) + \sin \pi\right)$

$= \frac{\pi/4}{2}\left(0 + 4\left(\frac{\sqrt{2}}{2}\right) + 2(1) + 4\left(\frac{\sqrt{2}}{2}\right) + 0\right)$

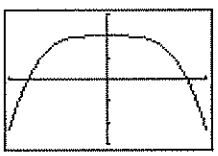
$= \frac{\pi}{4}\left(1 + \sqrt{2}\right)$

36. C.

37. (a) $f'(x) = 2x\cos(x^2)$

$f''(x) = 2x \cdot -2x\sin(x^2) + 2\cos(x^2)$
$= -4x^2\sin(x^2) + 2\cos(x^2)$

(b)



$[-1, 1]$ by $[-3, 3]$

(c) The graph shows that $-3 \le f''(x) \le 2$ so $|f''(x)| \le 3$ for $-1 \le x \le 1$.

(d) $|E_T| \le \frac{1-(-1)}{12}(h^2)(3) = \frac{h^2}{2}$

(e) For $0 < h \le 0.1, |E_T| \le \frac{h^2}{2} \le \frac{0.1^2}{2} = 0.005 < 0.01$

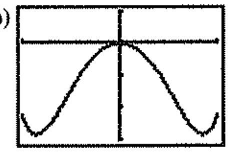
(f) $n \ge \frac{1-(-1)}{h} \ge \frac{2}{0.1} = 20$

38. (a) $f'''(x) = -4x^2 \cdot 2x\cos(x^2) - 8x\sin(x^2) - 4x\sin(x^2)$

$= -8x^3\cos(x^2) - 12x\sin(x^2)$

$f^{(4)}(x) = -8x^3 \cdot -2x\sin(x^2) - 24x^2\cos(x^2)$
$- 12x \cdot 2x\cos(x^2) - 12\sin(x^2)$
$= (16x^4 - 12)\sin(x^2) - 48x^2\cos(x^2)$

(b)



$[-1, 1]$ by $[-30, 10]$

(c) The graph shows that $-30 \le f^{(4)}(x) \le 10$ so $\left|f^{(4)}(x)\right| \le 30$ for $-1 \le x \le 1$.

(d) $|E_S| \le \frac{1-(-1)}{180}(h^4)(30) = \frac{h^4}{3}$

(e) For $0 < h \le 0.4, |E_S| \le \frac{h^4}{3} \le \frac{0.4^4}{3} \approx 0.00853 < 0.01$

(f) $n \ge \frac{1-(-1)}{h} \ge \frac{2}{0.4} = 5$

39. $T_n = \frac{h}{2}[y_0 + 2y_1 + 2y_2 + \cdots + 2y_{n-1} + y_n]$

$= \frac{h[y_0 + y_1 + \cdots + y_{n-1}] + h[y_1 + y_2 + \cdots + y_n]}{2}$

$= \frac{\text{LRAM}_n + \text{RRAM}_n}{2}$

40. $S_{2n} = \frac{h}{3}[y_0 + 4y_1 + 2y_2 + 4y_3 + \cdots + 2y_{2n-2}$
$+ 4y_{2n-1} + y_{2n}]$

$= \frac{1}{3}[h(y_0 + 2y_1 + 2y_2 + \cdots + 2y_{2n-1} + y_{2n})$
$+ (2h)(y_1 + y_3 + y_5 + \cdots + y_{2n-1})]$

$= \frac{2T_{2n} + \text{MRAM}_n}{3}$, where $h = \frac{b-a}{2n}$.

Quick Quiz Sections 5.4 and 5.5

1. C. $\int_1^7 f(x)\, dx = \frac{1}{2}((4-1)(10+30) + (6-4)(30+40)$
$+ (7-6)(40+20)) = 160$

2. D. $\int \sin x^3 dx = \left(\frac{-(\sin^2 x)}{3} - \frac{2}{3}\right)\cos x$

$\left(\frac{-(\sin^2(8))}{3} - \frac{2}{3}\right)\cos 8 - \left(\frac{-(\sin^2(1))}{3} - \frac{2}{3}\right)\cos 1 = 0.632$

3. C. $df(x) = \dfrac{d}{dx}\int_{-2}^{x^2-3x} e^{t^2}\,dt$

$$\frac{df(x)}{dx} = (2x-3)e^{(x^2-3x)^2} = 0$$
$$2x-3=0$$
$$x=\frac{3}{2}.$$

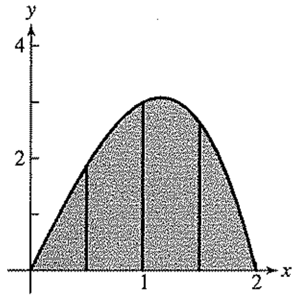
4. (a) $\dfrac{2-0}{2(4)}(\sin\ 0 + 2\ \sin(0.5^2) + 2\ \sin(1.0^2)$
$+\ 2\ \sin(1.5^2) + 2\ \sin(2^2)) = 0.744$

(b) F increases on $[0,\sqrt{\pi}]$ and $[\sqrt{2\pi},3]$ because $\sin(t^2) > 0$

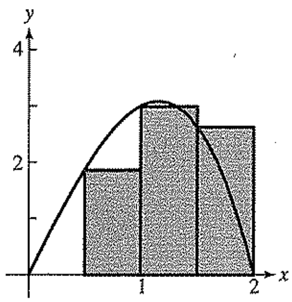
(c) $f(t) = k = \dfrac{d}{dx}\int_0^3 \sin(t^2)\ dt = 3K - 0K = 3K$

Chapter 5 Review (315–319)

1.

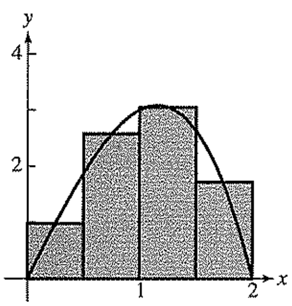


2.



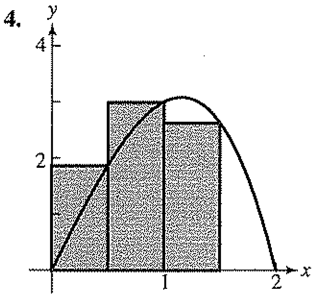
$$\text{LRAM}_4 : \frac{1}{2}\left(0+\frac{15}{8}+3+\frac{21}{8}\right) = \frac{15}{4} = 3.75$$

3.



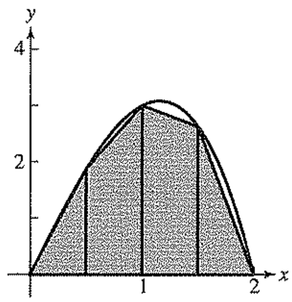
$$\text{MRAM}_4 : \frac{1}{2}\left(\frac{63}{64}+\frac{165}{64}+\frac{195}{64}+\frac{105}{64}\right) = 4.125$$

4.



$$\text{RRAM}_4 : \frac{1}{2}\left(\frac{15}{8}+3+\frac{21}{8}+0\right) = \frac{15}{4} = 3.75$$

5.



$$T_4 = \frac{1}{2}\left(\text{LRAM}_4 + \text{RRAM}_4\right) = \frac{1}{2}\left(\frac{15}{4}+\frac{15}{4}\right) = 3.75$$

6. $\int_0^2 (4x - x^3)\ dx = \left[2x^2 - \frac{1}{4}x^4\right]_0^2 = 8 - 4 = 4$

7.

n	LRAM_n	MRAM_n	RRAM_n
10	1.78204	1.60321	1.46204
20	1.69262	1.60785	1.53262
30	1.66419	1.60873	1.55752
50	1.64195	1.60918	1.57795
100	1.62557	1.60937	1.59357
1000	1.61104	1.60944	1.60784

8. $\int_1^5 \frac{1}{x}\ dx = \left[\ln|x|\right]_1^5 = \ln 5 - \ln 1 = \ln 5 \approx 1.60944$

9. (a) $\int_5^2 f(x)\ dx = -\int_2^5 f(x)\ dx = -3$

The statement is true.

(b) $\int_{-2}^5 [f(x)+g(x)]\ dx$

$= \int_{-2}^5 f(x)\,dx + \int_{-2}^5 g(x)\,dx$
$= \int_{-2}^2 f(x)\,dx + \int_2^5 f(x)\,dx + \int_{-2}^5 g(x)\,dx$
$= 4+3+2 = 9$

The statement is true.

9. Continued

(c) If $f(x) \le g(x)$ on [–2, 5], then $\int_{-2}^{5} f(x)\,dx \le \int_{-2}^{5} g(x)\,dx$, but this is not true since $\int_{-2}^{5} f(x)\,dx = \int_{-2}^{2} f(x) + \int_{2}^{5} f(x) = 4 + 3 = 7$ and $\int_{-2}^{5} g(x)\,dx = 2$. The statement is false.

10. (a) Volume of one cylinder: $\pi r^2 h = \pi \sin^2(m_i)\,\Delta x$

Total volume: $V = \lim_{n\to\infty} \sum_{i=1}^{n} \pi \sin^2(m_i)\,\Delta x$

(b) Use $\pi \sin^2 x$ on $[0, \pi]$.

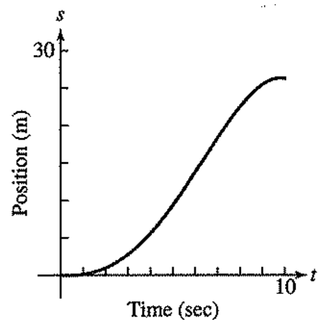
NINT $(\pi \sin^2 x, x, 0, \pi) \approx 4.9348$

11. (a) Approximations may vary. Using Simpson's Rule, the area under the curve is approximately

$\frac{1}{3}[0 + 4(0.5) + 2(1) + 4(2) + 2(3.5) + 4(4.5) + 2(4.75) + 4(4.5) + 2(3.5) + 4(2) + 0] = 26.5$

The body traveled about 26.5 m.

(b)



The curve is always increasing because the velocity is always positive, and the graph is steepest when the velocity is highest, at $t = 6$.

12. (a) $\int_{0}^{10} x^3\,dx$

(b) $\int_{0}^{10} x \sin x\,dx$

(c) $\int_{0}^{10} x(3x-2)^2\,dx$

(d) $\int_{0}^{10} \frac{1}{1+x^2}\,dx$

(e) $\int_{0}^{10} \pi\left(9 - \sin^2 \frac{\pi x}{10}\right)dx$

13. The graph is above the x-axis for $0 \le x < 4$ and below the x-axis for $4 < x \le 6$

Total area $= \int_{0}^{4} (4-x)\,dx - \int_{4}^{6} (4-x)\,dx$

$= \left[4x - \frac{1}{2}x^2\right]_0^4 - \left[4x - \frac{1}{2}x^2\right]_4^6$

$= [8-0] - [6-8] = 10$

14. The graph is above the x-axis for $0 \le x < \frac{\pi}{2}$ and below the x-axis for $\frac{\pi}{2} < x \le \pi$

Total area $= \int_{0}^{\pi/2} \cos x\,dx - \int_{\pi/2}^{\pi} \cos x\,dx$

$= \left[\sin x\right]_0^{\pi/2} - \left[\sin x\right]_{\pi/2}^{\pi}$

$= (1-0) - (0-1) = 2$

15. $\int_{-2}^{2} 5\,dx = \left[5x\right]_{-2}^{2} = 10 - (-10) = 20$

16. $\int_{2}^{5} 4x\,dx = \left[2x^2\right]_2^5 = 50 - 8 = 42$

17. $\int_{0}^{\pi/4} \cos x\,dx = \left[\sin x\right]_0^{\pi/4} = \frac{\sqrt{2}}{2} - 0 = \frac{\sqrt{2}}{2}$

18. $\int_{-1}^{1} (3x^2 - 4x + 7)\,dx = \left[x^3 - 2x^2 + 7x\right]_{-1}^{1}$

$= 6 - (-10) = 16$

19. $\int_{0}^{1} (8s^3 - 12s^2 + 5)\,ds = \left[2s^4 - 4s^3 + 5s\right]_0^1 = 3 - 0 = 3$

20. $\int_{1}^{2} \frac{4}{x^2}\,dx = \left[-\frac{4}{x}\right]_1^2 = -2 - (-4) = 2$

21. $\int_{1}^{27} y^{-4/3}\,dy = \left[-3y^{-1/3}\right]_1^{27} = -1 - (-3) = 2$

22. $\int_{1}^{4} \frac{dt}{t\sqrt{t}} = \int_{1}^{4} t^{-3/2}\,dt = \left[-2t^{-1/2}\right]_1^4 = -1 - (-2) = 1$

23. $\int_{0}^{\pi/3} \sec^2\theta\,d\theta = \left[\tan\theta\right]_0^{\pi/3} = \sqrt{3} - 0 = \sqrt{3}$

24. $\int_{1}^{e} \frac{1}{x}\,dx = \left[\ln|x|\right]_1^e = 1 - 0 = 1$

25. $\int_{0}^{1} \frac{36}{(2x+1)^3}\,dx = \int_{0}^{1} 36(2x+1)^{-3}\,dx$

$= \left[-9(2x+1)^{-2}\right]_0^1$

$= -1 - (-9) = 8$

26. $\int_1^2 \left(x+\frac{1}{x^2}\right) dx = \int_1^2 (x+x^{-2})\,dx$

$= \left[\frac{1}{2}x^2 - x^{-1}\right]_1^2$

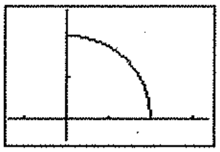
$= \frac{3}{2} - \left(-\frac{1}{2}\right) = 2$

27. $\int_{-\pi/3}^{0} \sec x \tan x\,dx = \left[\sec x\right]_{-\pi/3}^{0} = 1-2=-1$

28. $\int_{-1}^{1} 2x\sin(1-x^2)\,dx = \left[\cos(1-x^2)\right]_{-1}^{1} = 1-1=0$

29. $\int_0^2 \frac{2}{y+1}\,dy = \left[2\ln|y+1|\right]_0^2 = 2\ln 3 - 0 = 2\ln 3$

30. Graph $y=\sqrt{4-x^2}$ on $[0, 2]$.

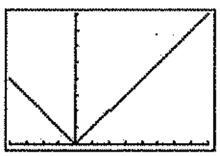


$[-1.35, 3.35]$ by $[-0.5, 2.6]$

The region under the curve is a quarter of a circle of radius 2.

$\int_0^2 \sqrt{4-x^2}\,dx = \frac{1}{4}\pi(2)^2 = \pi$

31. Graph $y=|x|dx$ on $[-4,8]$.

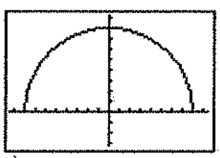


$[-4, 8]$ by $[0, 8]$

The region under the curve consists of two triangles.

$\int_{-4}^{8} |x|\,dx = \frac{1}{2}(4)(4) + \frac{1}{2}(8)(8) = 40$

32. Graph $y=\sqrt{64-x^2}$ on $[-8,8]$.



$[-9.4, 9.4]$ by $[-3.2, 9.2]$

The region under the curve $y=\sqrt{64-x^2}$ is half a circle of radius 8.

$\int_{-8}^{8} 2\sqrt{64-x^2}\,dx = 2\int_{-8}^{8}\sqrt{64-x^2}\,dx = 2\left[\frac{1}{2}\pi(8)^2\right] = 64\pi$

33. (a) Note that each interval is 1 day = 24 hours

Upper estimate:

$24(0.020+0.021+0.023+0.025+0.028+0.031+0.035) = 4.392$ L

Lower estimate:

$24(0.019+0.020+0.021+0.023+0.025+0.028+0.031) = 4.008$ L

(b) $\frac{24}{2}[0.019+2(0.020)+2(0.021)+\cdots+2(0.031) + 0.035] = 4.2$ L

34. (a) Upper estimate:

$3(5.30+5.25+5.04+\cdots+1.11) = 103.05$ ft

Lower estimate:

$3(5.25+5.04+4.71+\cdots+0) = 87.15$ ft

(b) $\frac{3}{2}[5.30+2(5.25)+2(5.04)+\cdots+2(1.11)+0] = 95.1$ ft

35. One possible answer:

The *dx* is important because it corresponds to the actual physical quantity Δx in a Riemann sum. Without the Δx, our integral approximations would be way off.

36. $\int_{-4}^{4} f(x)\,dx = \int_{-4}^{0} f(x)\,dx + \int_0^4 f(x)\,dx$

$= \int_{-4}^{0} (x-2)\,dx + \int_0^4 x^2\,dx$

$= \left[\frac{1}{2}x^2 - 2x\right]_{-4}^{0} + \left[\frac{1}{3}x^3\right]_0^4$

$= [0-16] + \left[\frac{64}{3} - 0\right] = \frac{16}{3}$

37. Let $f(x) = \sqrt{1+\sin^2 x}$

$\max f = \sqrt{2}$ since $\max \sin^2 x = 1$

$\min f = 1$ since $\min \sin^2 x = 0$

$(\min f)(1-0) \le \int_0^1 \sqrt{1+\sin^2 x}\,dx \le (\max f)(1-0)$

$0 < 1 \le \int_0^1 \sqrt{1+\sin^2 x}\,dx \le \sqrt{2}$

38. (a) $av(y) = \frac{1}{4-0}\int_0^4 \sqrt{x}\,dx = \frac{1}{4}\left[\frac{2}{3}x^{3/2}\right]_0^4 = \frac{1}{4}\left(\frac{16}{3} - 0\right) = \frac{4}{3}$

(b) $av(y) = \frac{1}{a-0}\int_0^a a\sqrt{x}\,dx = \frac{1}{a}\left[\frac{2}{3}ax^{3/2}\right]_0^a = \frac{2}{3}a^{3/2}$

39. $\frac{dy}{dx} = \sqrt{2+\cos^3 x}$

40. $\frac{dy}{dx} = \sqrt{2+\cos^3(7x^2)} \cdot \frac{d}{dx}(7x^2) = 14x\sqrt{2+\cos^3(7x^2)}$

41. $\frac{dy}{dx} = \frac{d}{dx}\left(-\int_1^x \frac{6}{3+t^4}\,dt\right) = -\frac{6}{3+x^4}$

42. $\frac{dy}{dx} = \frac{d}{dx}\left(\int_0^{2x} \frac{1}{t^2+1}dt - \int_0^x \frac{1}{t^2+1}dt\right)$

$= \frac{1}{(2x)^2+1} \cdot 2 - \frac{1}{x^2+1}$

$= \frac{2}{4x^2+1} - \frac{1}{x^2+1}$

43. $c(x) = \int_{25}^x \frac{2}{\sqrt{t}}dt + 50$

$= \left[4t^{1/2}\right]_{25}^x + 50$

$= 4\sqrt{x} - 20 + 50$

$= 4\sqrt{x} + 30$

$c(2500) = 4\sqrt{2500} + 30 = 230$

The total cost for printing 2500 newsletters is $230.

44. $av(I) = \frac{1}{14}\int_0^{14}(600 + 600t)\, dt$

$= \frac{1}{14}[600t + 300t^2]_0^{14} = 4800$

Rich's average daily inventory is 4800 cases.

$c(t) = 0.04I(t) = 24 + 24t$

$av(c) = \frac{1}{14}\int_0^{14}(24 + 24t)\, dt = \frac{1}{14}\left[24t + 12t^2\right]_0^{14} = 192$

Rich's average daily holding cost is $192.

We could also say (0.04)4800 = 192.

45. $\int_0^x (t^3 - 2t + 3)\, dt = \left[\frac{1}{4}t^4 - t^2 + 3t\right]_0^x$

$= \frac{1}{4}x^4 - x^2 + 3x$

$\frac{1}{4}x^4 - x^2 + 3x = 4$

$\frac{1}{4}x^4 - x^2 + 3x - 4 = 0$

$x^4 - 4x^2 + 12x - 16 = 0$

Using a graphing calculator, $x \approx -3.09131$ or $x \approx 1.63052$.

46. **(a)** True, because $g'(x) = f(x)$.

(b) True, because g is differentiable.

(c) True, because $g'(1) = f(1) = 0$.

(d) False, because $g''(1) = f'(1) > 0$.

(e) True, because $g'(1) = f(1) = 0$ and $g''(1) = f'(1) > 0$.

(f) False, because $g''(1) = f'(1) \neq 0$.

(g) True, because $g'(x) = f(x)$, and f is an increasing function which includes the point (1, 0).

47. $\int_0^1 \sqrt{1+x^4}\, dx = F(1) - F(0)$

48. $y(x) = \int_5^x \frac{\sin t}{t}dt + 3$

49. $y' = 2x + \frac{1}{x}$

$y'' = 2 - \frac{1}{x^2}$

Thus, it satisfies condition i.

$y(1) = 1 + \int_1^1 \frac{1}{t}dt + 1 = 1 + 0 + 1 = 2$

$y'(1) = 2 + \frac{1}{1} = 2 + 1 = 3$

Thus, it satisfies condition ii.

50. Graph (b).

$y = \int_1^x 2t\, dt + 4 = \left[t^2\right]_1^x + 4 = (x^2 - 1) + 4 = x^2 + 3$

51. **(a)** Each interval is $5\text{ min} = \frac{1}{12}$ h.

$\frac{1}{24}[2.5 + 2(2.4) + 2(2.3) + \cdots + 2(2.4) + 2.3]$

$= \frac{29}{12} \approx 2.42$ gal

(b) $(60\text{ mi/h})\left(\frac{12}{29}\text{h/gal}\right) \approx 24.83$ mi/gal

52. **(a)** Using the freefall equation $s = \frac{1}{2}gt^2$ from Section 3.4, the distance A falls in 4 seconds is $\frac{1}{2}(32)(4^2) = 256$ ft. When her parachute opens, her altitude is 6400 – 256 = 6144 ft.

(b) The distance B falls in 13 seconds is $\frac{1}{2}(32)(13^2) = 2704$ ft. When her parachute opens, her altitude is 7000 – 2704 = 4296 ft.

(c) Let t represent the number of seconds after A jumps. For $t \geq 4$ sec, A's position is given by $S_A(t) = 6144 - 16(t-4) = 6208 - 16t$, so A lands at $t = \frac{6208}{16} = 388$ sec. For $t \geq 45 + 13 = 58$ sec, B's position is given by $S_B(t) = 4296 - 16(t-58) = 5224 - 16t$, so B lands at $t = \frac{5224}{16} = 326.5$ sec. B lands first.

53. **(a)** Area of the trapezoid $= \frac{1}{2}(2h)(y_1+y_3) = h(y_1+y_3)$

Area of the rectangle $= (2h)y_2 = 2hy_2$

$h(y_1+y_3)+2(2hy_2) = h(y_1+4y_2+y_3)$

(b) Let $h = \frac{b-a}{2n}$.

$$S_{2n} = \frac{h}{3}[y_0+4y_1+2y_2+4y_3+2y_4+\cdots+2y_{2n-2} + 4y_{2n-1}+y_{2n}]$$
$$= \frac{1}{3}[h(y_0+4y_1+y_2)+h(y_2+4y_3+y_4)+\cdots + h(y_{2n-2}+4y_{2n-1}+y_{2n})]$$

Since each expression of the form $h(y_{2i-2}+4y_{2i-1}+y_{2i})$ is equal to twice the area of the ith of n rectangles plus the area of the ith of n trapezoids, $S_{2n} = \frac{2 \cdot \text{MRAM}_n + T_n}{3}$.

54. **(a)** $g(1) = \int_1^1 f(t)\,dt = 0$

(b) $g(3) = \int_1^3 f(t)\,dt = -\frac{1}{2}(2)(1) = -1$

(c) $g(-1) = \int_1^{-1} f(t)\,dt = -\int_{-1}^1 f(t)\,dt = -\frac{1}{4}\pi(2)^2 = -\pi$

(d) $g'(x) = f(x)$; Since $f(x) > 0$ for $-3 < x < 1$ and $f(x) < 0$ for $1 < x < 3$, $g(x)$ has a relative maximum at $x = 1$.

(e) $g'(-1) = f(-1) = 2$

The equation of the tangent line is $y-(-\pi) = 2(x+1)$ or $y = 2x+2-\pi$

(f) $g''(x) = f'(x)$, $f'(x) = 0$ at $x = -1$ and $f'(x)$ is not defined at $x = 2$. The inflection points are at $x = -1$ and $x = 2$. Note that $g''(x) = f'(x)$ is undefined at $x = 1$ as well, but since $g''(x) = f'(x)$ is negative on both sides of $x = 1$, $x = 1$ is not an inflection point.

(g) Note that the absolute maximum is $g(1) = 0$ and the absolute minimum is

$$g(-3) = \int_1^{-3} f(t)\,dt = -\int_{-3}^1 f(t)\,dt = -\frac{1}{2}\pi(2)^2 = -2\pi.$$

The range of g is $[-2\pi, 0]$.

55. **(a)** $\text{NINT}(e^{-x^2/2}, x, -10, 10) \approx 2.506628275$

$\text{NINT}(e^{-x^2/2}, x, -20, 20) \approx 2.506628275$

(b) The area is $\sqrt{2\pi}$.

56. First estimate the surface area of the swamp.

$$\frac{20}{2}[146+2(122)+2(76)+2(54)+2(40)+2(30) + 13] = 8030 \text{ ft}^2$$

$$(5 \text{ ft})(8030 \text{ ft}^2) \cdot \frac{1 \text{ yd}^3}{27 \text{ ft}^3} \approx 1500 \text{ yd}^3$$

57. **(a)** $V^2 = (v_{max})^2 \sin^2(120\pi t)$

Using NINT:

$$av(V^2) = \frac{1}{1}\int_0^1 (V_{max})^2 \sin^2(120\pi t)\,dt$$
$$= (V_{max})^2 \int_0^1 \sin^2(120\pi t)\,dt = (V_{max})^2 \frac{1}{2} = \frac{(V_{max})^2}{2}$$
$$V_{rms} = \sqrt{\frac{(V_{max})^2}{2}} = \frac{V_{max}}{\sqrt{2}}$$

(b) $V_{max} = 240\sqrt{2} \approx 339.41$ volts

58. **(a)** $\int_0^{24} R(t)dt \approx \left(\frac{4}{2}\right)(9.6+2(10.3+10.9+11.1 + 10.9+10.5)+9.6) \approx 253.2$, which is the total number of gallons of water that flowed through the pipe during the 24 hour period.

(b) Yes, because $R(0) = R(24)$, the Mean Value Theorem guarantees that there is a number c between 0 and 24 such that $R'(c) = 0$.

(c) $Q(t) = 0.01(950+25(4)-(4)^2) = 10.58$ gal/hr

59. $f'(1) = a(1)^2 + b(1) = -6$

$f''(x) = 2ax+b$

$f''(1) = 2a(1)+b = 6$

$2a+b = 6$

$-(a+b = -6)$

$a = 12$

$b = -18$

$f'(x) = 12x^2-18x$

$f(x) = 4x^3-9x^2+c$

$\int_1^2 f(x)dx = \int_1^2 4x^3-9x^2+c\,dx = 14$

$(x^4-3x^3+cx)\Big|_1^2 = 14$

$c = 20$

$f(x) = 4x^3-9x^2+20$

60. **(a)** $g(4) = \frac{1}{2}(1(3+1)+2(1+(-1))) = 2$

$g(-2) = \frac{1}{2}(-3(3+0)) = -\frac{9}{2}$

(b) $g(2) = f(2) = 1$

(c) The minimum value is $g(-2) = -\frac{9}{2}$

(d) g has a point of inflection at $x = 1$. It is the only place where the slope goes from positive to negative.

Chapter 6
Differential Equations and Mathematical Modeling

Section 6.1 Slope Fields and Euler's Method (pp. 321–330)

Exploration 1 Seeing the Slopes

1. Since $\frac{dy}{dx}=0$ represents a line with a slope of 0, we should expect to see intervals with no change in y. We see this at odd multiples of $\pi/2$.
2. Since y is the dependent variable, I

t will have no effect on the value of $\frac{dy}{dx}=\cos x$.

3. The graph of $\frac{dy}{dx}$ will look the same at all values of y.
4. When $x=0$, $\frac{dy}{dx}=\cos x=1$. This can be seen on the graph near the origin. At that point, the change in y and change in x are the same.
5. When $x=\pi$, $\frac{dy}{dx}=\cos x=-1$. This can be seen in the graph at $x=\pi$. At this point, the change in y is negative of the change in x.
6. This is true because each point on the graph has a negative of itself.

Quick Review 6.1

1. Yes. $\frac{d}{dx}e^x=e^x$
2. Yes. $\frac{d}{dx}e^{4x}=4e^{4x}$
3. No. $\frac{d}{dx}(x^2e^x)=2xe^x+x^2e^x$
4. Yes. $\frac{d}{dx}e^{x^2}=2xe^{x^2}$
5. No. $\frac{d}{dx}(e^{x^2}+5)=2xe^{x^2}$
6. Yes. $\frac{d}{dx}\sqrt{2x}=\frac{1}{2\sqrt{2x}}(2)=\frac{1}{\sqrt{2x}}$
7. Yes. $\frac{d}{dx}\sec x=\sec x\tan x$
8. No. $\frac{d}{dx}x^{-1}=-x^{-2}$
9. $y=3x^2+4x+C$
   $2=3(1)^2+4(1)+C$
   $C=-5$
10. $y=2\sin x-3\cos x+C$
    $4=2\sin(0)-3\cos(0)+C$
    $C=-7$
11. $y=e^{2x}+\sec x+C$
    $5=e^{2(0)}+\sec(0)+C$
    $C=3$
12. $y=\tan^{-1}x+\ln(2x-1)+C$
    $\pi=\tan^{-1}(1)+\ln(2(1)-1)+C$
    $C=\frac{3\pi}{4}$

Section 6.1 Exercises

1. $\int dy=\int(5x^4-\sec^2 x)\,dx$
   $y=x^5-\tan x+C$
2. $\int dy=\int(\sec x\tan x-e^x)\,dx$
   $y=\sec x-e^x+C$
3. $\int dy=\int(\sin x-e^{-x}+8x^3)\,dx$
   $y=-\cos x+e^{-x}+2x^4+C$
4. $\int dy=\int\left(\frac{1}{x}-\frac{1}{x^2}\right)dx=\ln x+\frac{1}{x}+C$
5. $\int dy=\int\left(5^x\ln 5+\frac{1}{x^2+1}\right)dx=5^x+\tan^{-1}x+C$
6. $\int dy=\int\left(\frac{1}{\sqrt{1-x^2}}-\frac{1}{\sqrt{x}}\right)dx=\sin^{-1}x-2\sqrt{x}+C$
7. $\int dy=\int(3t\cos(t^3))\,dt=\sin(t^3)+C$
8. $\int dy=\int\cos t\, e^{\sin t}dt$
   $=e^{\sin t}+C$
9. $\int dy=\int(\sec^2(x^5)(5x^4))\,dx$
   $=\tan x^5+C$
10. $\int dy=\int 4(\sin u)^3\cos u\,du$
    $=(\sin u)^4+C$
11. $\int dy=\int 3\sin x\,dx=-3\cos x+C$
    $2=-3\cos(0)+C,\quad C=5$
    $y=-3\cos x+5$
12. $\int dy=\int 2e^x-\cos x\,dx=2e^x-\sin x+C$
    $3=2e^0-\sin(0)+C,\quad C=1$
    $y=2e^x-\sin x+1$
13. $\int du=\int(7x^6-3x^2+5)\,dx=x^7-x^3+5x+C$
    $1=1^7-1^3+5+C,\quad C=-4$
    $u=x^7-x^3+5x-4$