Section 10.2 The Pythagorean Theorem

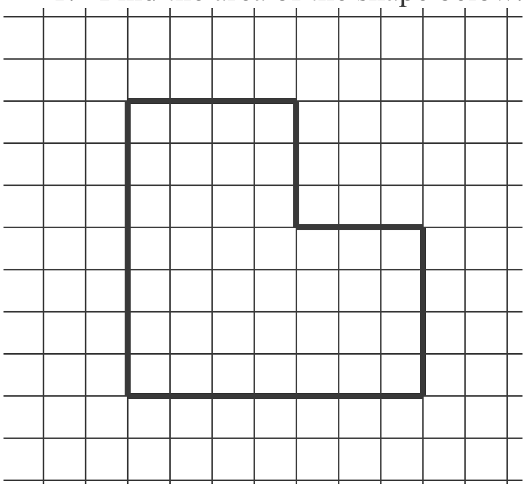
Section Overview:
In this section students begin to formalize many of the ideas learned in Chapter 7. They will transition from using the area of a square to find the length of a segment to generalizing the relationship between the side lengths of a right triangle, i.e. the Pythagorean Theorem, to find the length of a segment. They begin this transition by finding the areas of the squares adjacent to a given right triangle. Using these concrete examples, students describe the relationship between the sides of a right triangle. From here, students work to explain a proof by picture and subsequently a paragraph proof of the Pythagorean Theorem, starting first with a right triangle of side lengths 3, 4, and 5. Students then use a similar process to explain a proof of the Pythagorean Theorem for any right triangle with side lengths a, b, and *c* where a and b are the legs of the right triangle and *c* is the hypotenuse. Students arrive at the Pythagorean Theorem: $a^2 + b^2 = c^2$ where *a* and *b* are the legs of the right triangle and *c* is the hypotenuse. Throughout the section, students are connecting the Pythagorean Theorem to work done in Chapter 7. Next, students use the Pythagorean Theorem to solve for the missing side length of a right triangle given the measurements of the other two sides, relying on skills learned in Chapters 7 and 8. This is followed by explaining a proof of the converse of the Pythagorean Theorem: For a triangle with side lengths *a*, *b*, and c if $a^2 + b^2 = c^2$, then the triangle is a right triangle. Using this theorem, students determine whether three given side lengths form a right triangle. Throughout this section emphasis is placed on creating good arguments and explanation. Students are not formally proving the Pythagorean Theorem and its converse but explaining why the theorems are true by learning how to provide sufficient explanations and arguments. In addition students are providing evidence and warrants for claims that they make. At the end of the section is an optional exploration on Pythagorean triples.

Concepts and Skills to Master:
By the end of this section, students should be able to:

1. Know that in a right triangle $a^2 + b^2 = c^2$, where *a* and *b* are the legs of the right triangle and *c* is the hypotenuse.
2. Understand and explain a proof of the Pythagorean Theorem.
3. Use the Pythagorean Theorem to solve for the missing side length of a right triangle given the measurements of the other two sides.
4. Understand and explain a proof of the converse of the Pythagorean Theorem. That is, for a triangle with side lengths *a*, *b*, and *c* if $a^2 + b^2 = c^2$, then the triangle is a right triangle.
5. Use the converse of the Pythagorean Theorem to determine whether three given measurements are possible side lengths of a right triangle.

10.2a Class Activity: A Proof of the Pythagorean Theorem

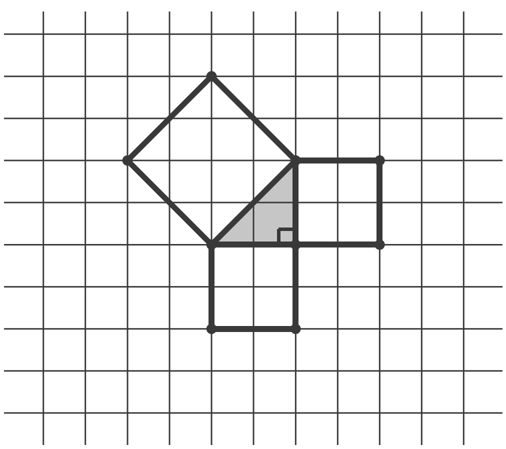
1. Find the area of the shape below. Each square on the grid has a side length of 1 unit.



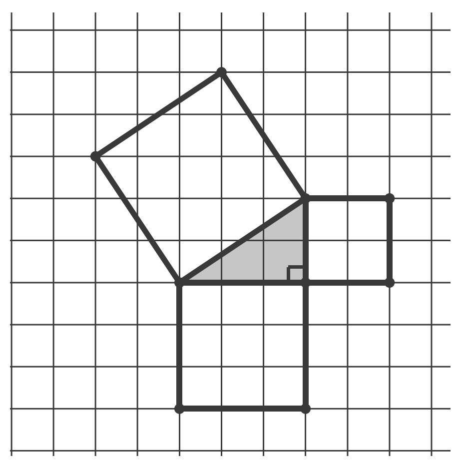
In numbers 2 and 3, a right triangle is shown in gray. The shorter sides of a right triangle are referred to as **legs**. The longer side of the right triangle (the side opposite of the right angle) is called the **hypotenuse**.

Directions: Squares have been drawn adjacent to the sides of the right triangle. Find the area of each of the squares. Assuming each square on the grid has a side length of 1 unit. Write the areas inside each of the squares.

2.

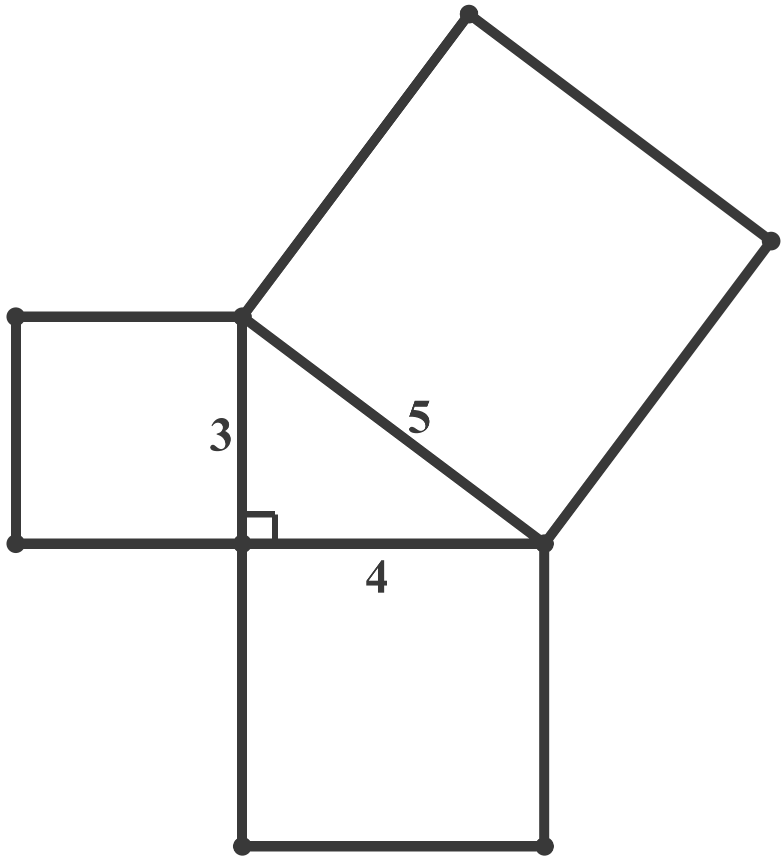


3.



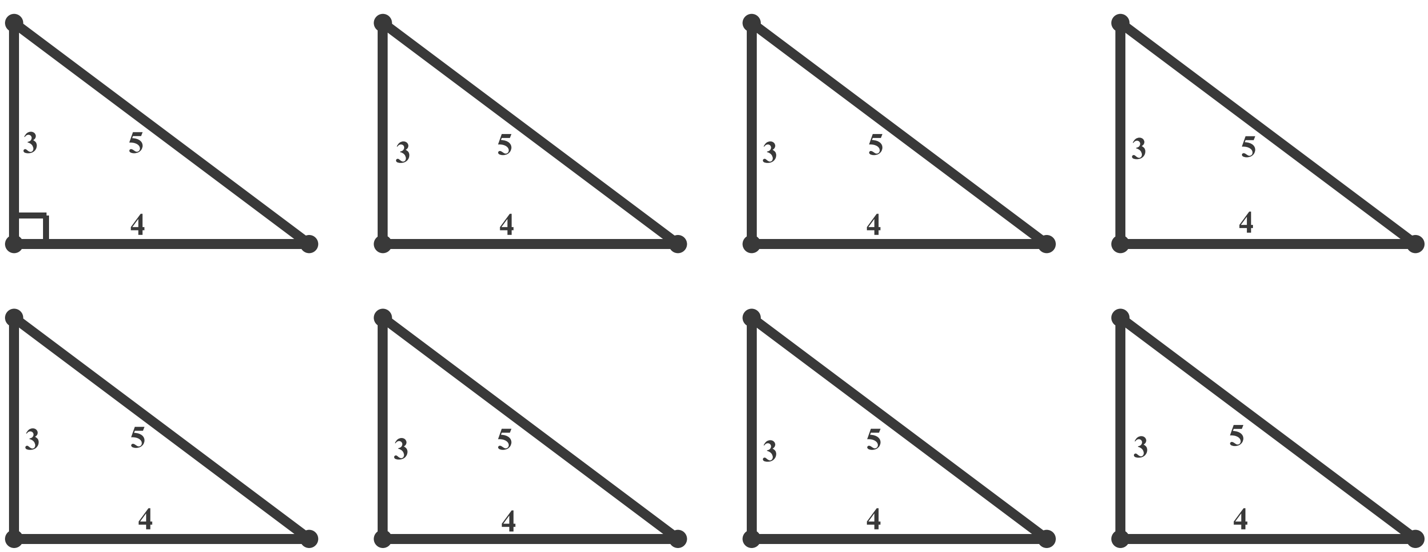
4. What do you notice about the relationship between the areas of the squares formed adjacent to the legs of a right triangle?

5. Below is a right triangle with side lengths 3, 4, and 5. Squares have been drawn adjacent to the sides of the right triangle.



a. Find the area of each of the squares. Write the area inside each of the squares. Then, cut out the three squares very carefully.

b. Below are 8 copies of the original right triangle. Cut out the 8 triangles very carefully.

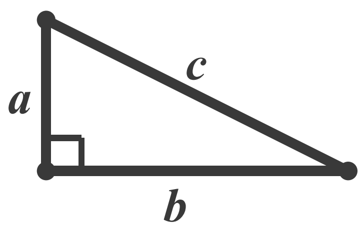


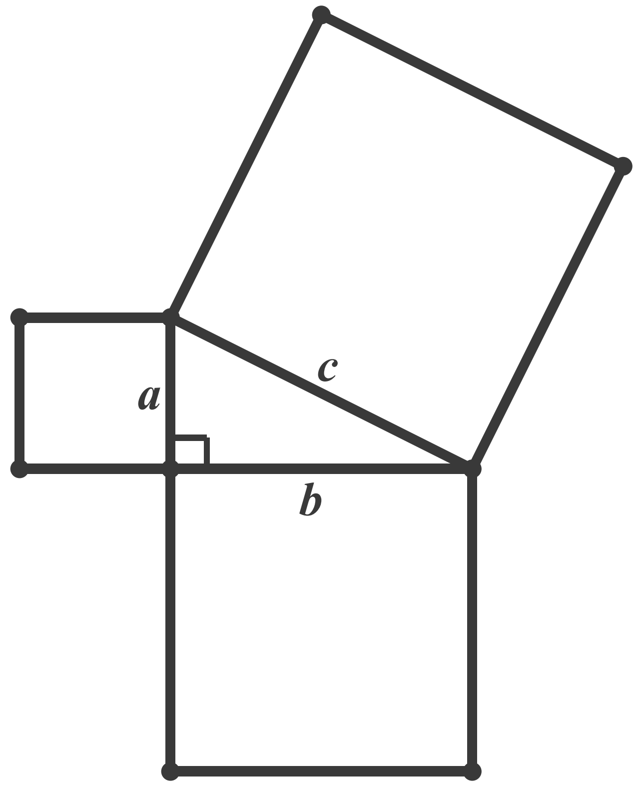
c. Below are two **congruent squares**. Since the squares are congruent, we know that their sides have the same length and subsequently they have the same area. Use your square with an area of 25 and four of the triangles from the previous page to cover one of the squares. Use your squares with areas 9 and 16 and four of the triangles from the previous page to cover the other square. Tape the pieces into place.

d. Use the large squares in part c) to explain the relationship you discovered in #2 – 4 between the squares formed adjacent to the sides of a right triangle.

6. In the previous problems, we saw that for specific triangles **the sum of the areas of the squares along the legs of the right triangle equals the area of the square along the hypotenuse of the triangle** by looking at several examples. Now, we want to show that this relationship holds true for ***any*** right triangle.

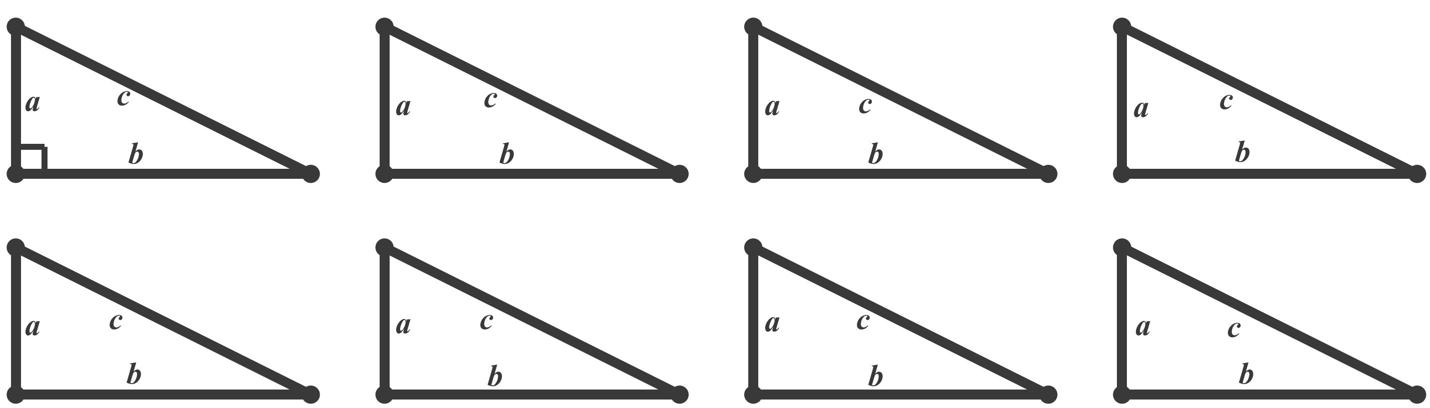
Suppose you have a right triangle with any side lengths a, b, and c where a and b are the legs of the triangle and c is the hypotenuse of the right triangle as shown below. The squares have been drawn along the sides of the right triangle. Our goal is to show that $a^2 + b^2 = c^2$ is always true.



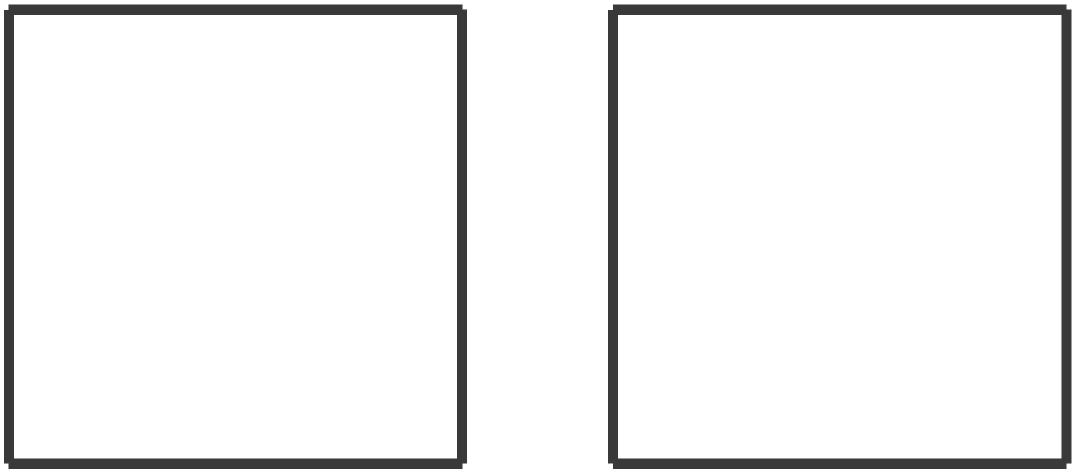


a. Find the area of each of the squares adjacent to the sides of the right triangle. Write the areas inside each square.

b. Cut out the squares formed on the sides of the triangle above as well as the 8 copies of the triangle with side lengths a, b, and c below.



c. Arrange the 3 squares and 8 triangles to cover the 2 squares shown below.



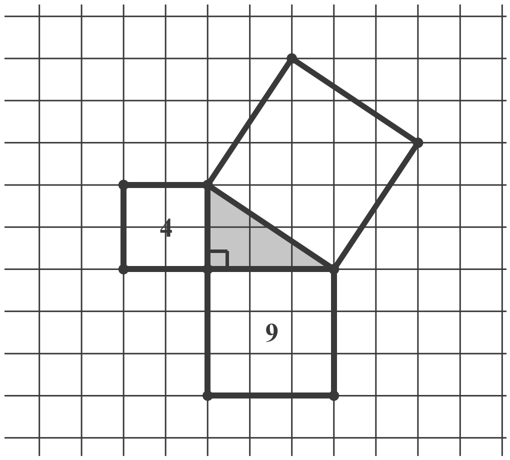
d. Using the picture above, show that **the sum of the areas of the squares adjacent to the legs of the right triangle equals the area of the square adjacent to the hypotenuse of the triangle for *any*** right triangle.

e. Conventionally, the leg lengths of a right triangle are denoted using the variables a and b and the hypotenuse of a right triangle is denoted using the variable c. State the relationship between the side lengths of a right triangle using the words **legs** and **hypotenuse**.

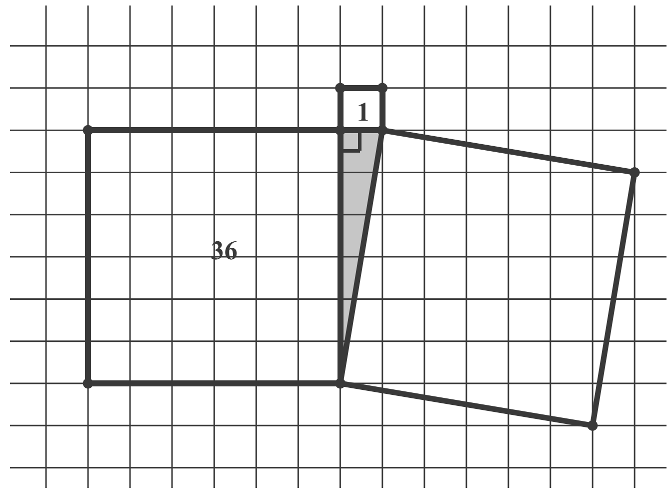
f. Write an **equation** that shows the relationship between the side lengths of a right triangle using a and b for the lengths of the legs and c for the length of the hypotenuse.

Directions: In each of the problems below, a right triangle is shown in gray. The squares along each of the three sides of the triangles have been drawn. The area of two of the squares is given. Determine the area of the third square. Write your answer in the square. Also find the side length of each square, write the sides lengths below each picture.

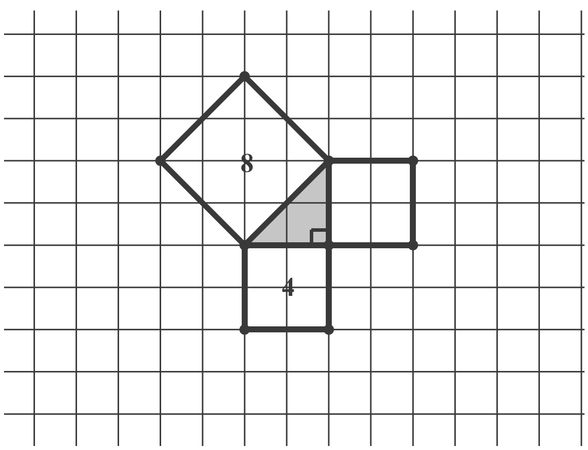
1.



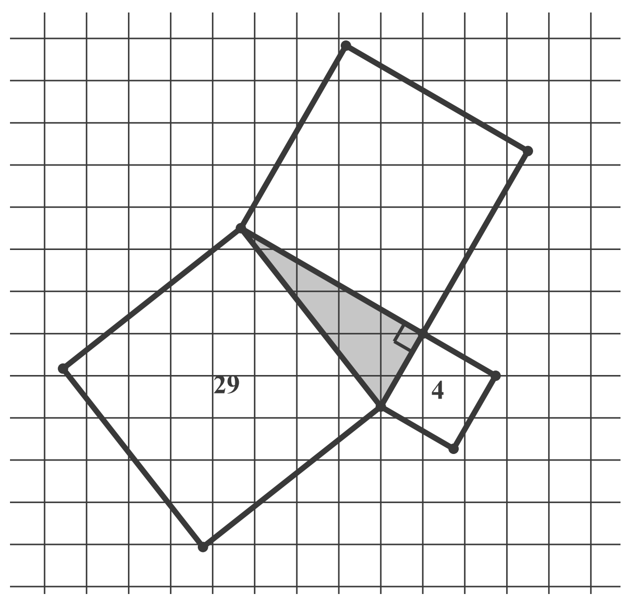
2.



3.



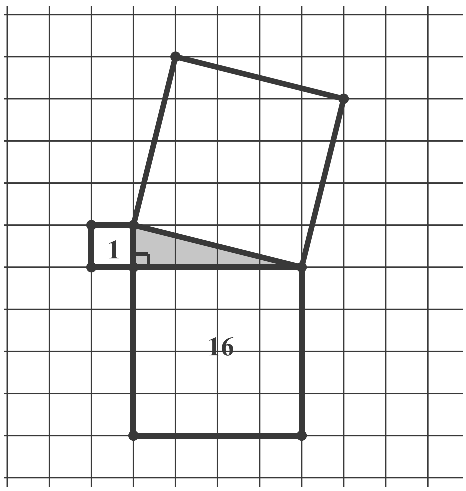
4.



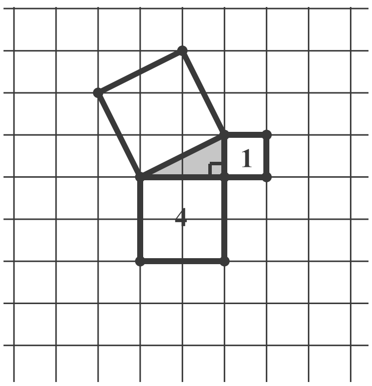
10.2a Homework: A Proof of the Pythagorean Theorem

Directions: In each of the problems below, a right triangle is shown in gray. The squares along each of the three sides of the triangles have been drawn. The area of two of the squares is given. Determine the area of the third square. Write your answer in the square.

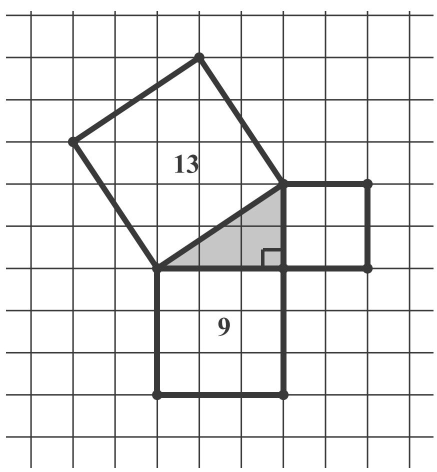
1.



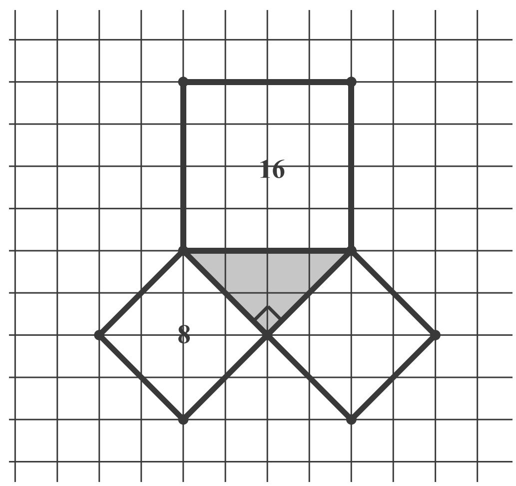
2.



3.

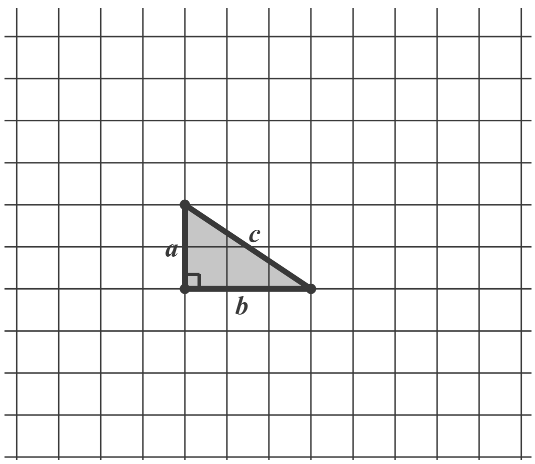


4.



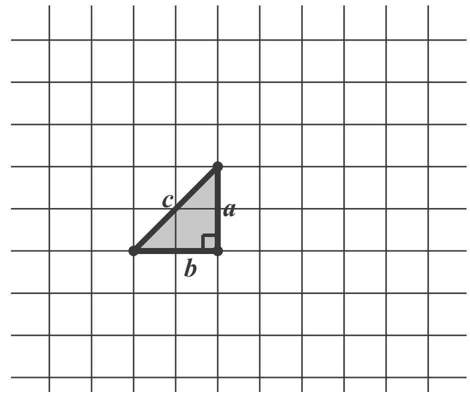
Directions: For each of the following problems, the gray triangle is a right triangle. Draw the squares adjacent to each of the three sides of the triangles. Find the area of each square and write the area in each square. Then, find the side lengths *a*, *b*, *c* of each triangle.

5.



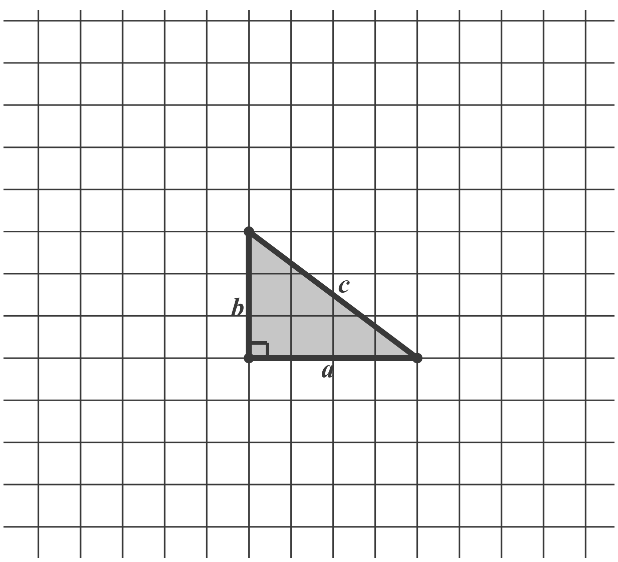
a = ________ *b* = ________ *c* = _______

6.



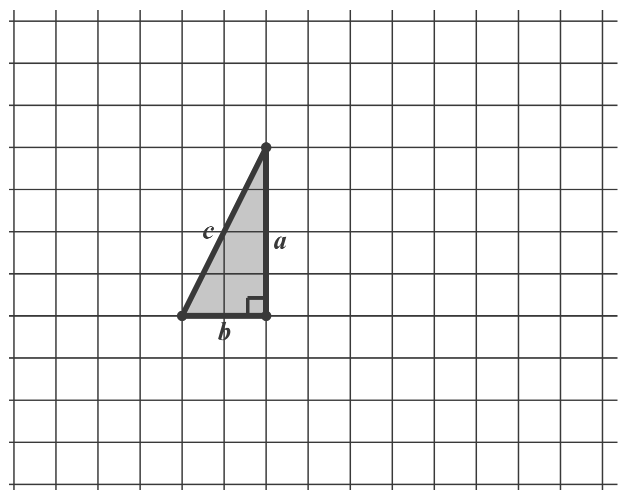
a = ________ *b* = ________ *c* = _________

7.



a = ________ *b* = ________ *c* = ________

8.



a = ________ *b* = ________ *c* = ________

10.2b Class Activity: The Pythagorean Theorem and Tilted Squares

1. On the grids below, construct the following and clearly label each object:
    a. Square $ABCD$ that has an area of 40 square units
    b. Square $PQRS$ that has an area of 10 square units
    c. $\overline{EF}$ that has a length of $\sqrt{8}$ units
    d. $\overline{LM}$ that has a length of $\sqrt{17}$ units

2. Draw as many different squares as you can with an area of 25 square units on the grids below. In this problem, different means that the squares are not tilted the same way.

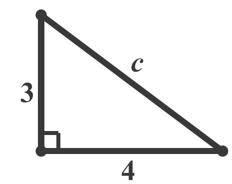
10.2b Homework: The Pythagorean Theorem and Tilted Squares

1. On the grids below, construct the following and clearly label each object:
   a. Square $ABCD$ that has an area of 5 square units
   b. Square $PQRS$ that has an area of 29 square units
   c. $\overline{EF}$ that has a length of $\sqrt{18}$ units
   d. $\overline{LM}$ that has a length of $\sqrt{13}$ units

10.2c Class Activity: The Pythagorean Theorem and Unknown Side Lengths

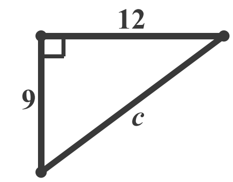
Directions: Find the length of the hypotenuse of each right triangle shown below using the Pythagorean Theorem. Leave your answer in simplest radical form.

1.



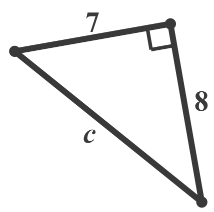
c = ______

2.



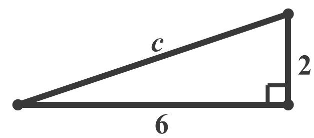
c = ______

3.



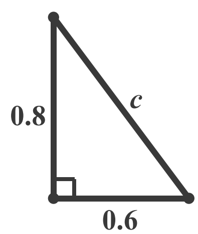
c = ______

4.



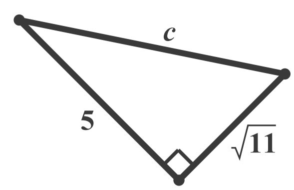
c = ______

5.



c = ______

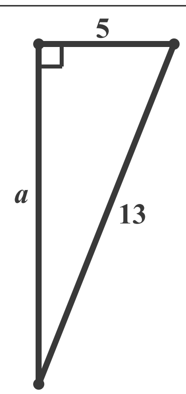
6.



c = ______

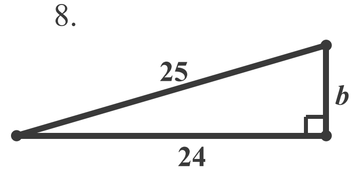
Directions: Find the length of the leg of each right triangle shown below using the Pythagorean Theorem. Leave your answer in simplest radical form.

7.



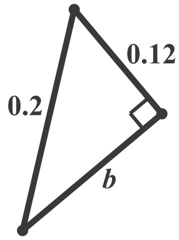
a = ______

8.



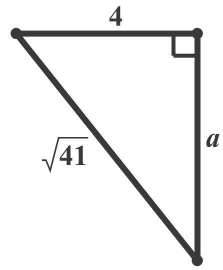
b = ______

9.



b = ______

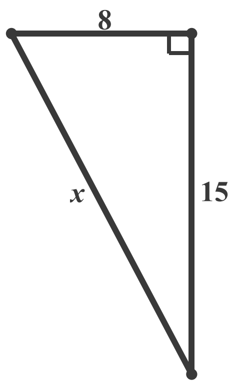
10.



a = ______

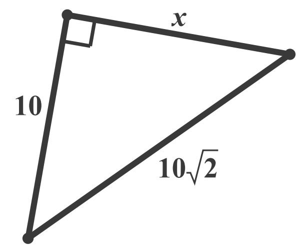
Directions: Find the value of x using the Pythagorean Theorem. Leave your answer in simplest radical form.

11.



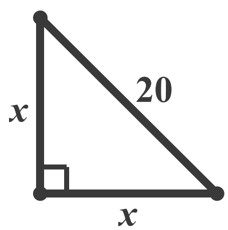
x = ______

12.



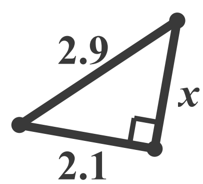
x = ______

13.



x = ______

14.



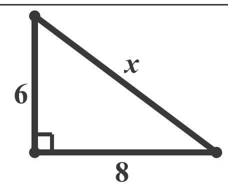
x = ______

10.2c Homework: The Pythagorean Theorem and Unknown Side Lengths

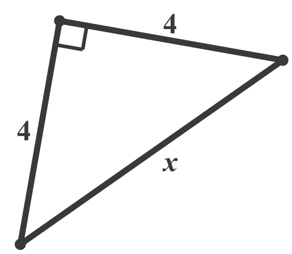
Directions: Two side lengths of a right triangle have been given. Solve for the missing side length if a and b are leg lengths and c is the length of the hypotenuse. Leave your answer in simplest radical form.

1. $a = 16$, $b = 30$, $c = ?$
2. $a = 2$, $b = 2$, $c = ?$
3. $a = 40$, $b = ?$, $c = 50$
4. $a = ?$, $b = 4\sqrt{3}$, $c = 8$

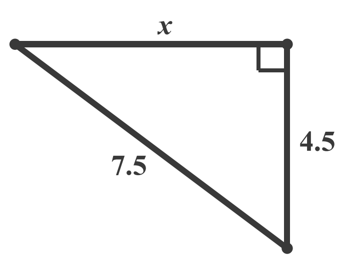
Directions: Find the value of x using the Pythagorean Theorem. Leave your answer in simplest radical form.

5.


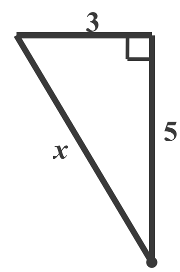
$x =$ ________

6.


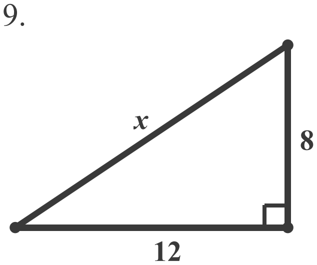
$x =$ _______

7.


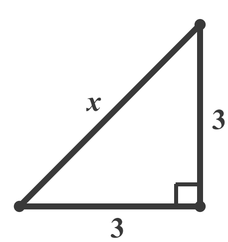
$x =$ ________

8.


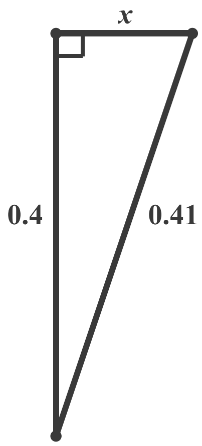
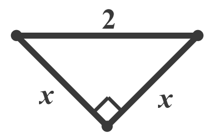
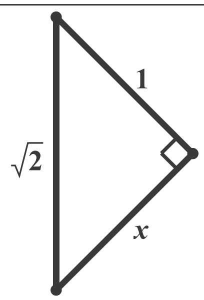
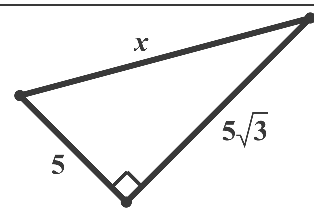
$x =$ ________

9.


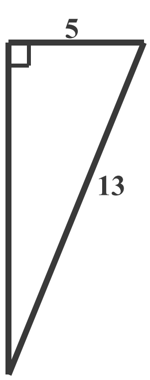
$x =$ ________

10.


$x =$ ________

11.  $x =$ _______	12.  $x =$ ________
13.  $x =$ ________	14.  $x =$ ________

15. **Find, Fix, and Justify:** Megan was asked to solve for the unknown side length in the triangle below. Her work is shown below. She made a mistake when solving. Explain the mistake she made and then solve the problem correctly.



<u>Megan's Solution:</u>

$a^2 + b^2 = c^2$
$5^2 + 13^2 = c^2$
$25 + 169 = c^2$
$194 = c^2$
$\sqrt{194} = c$

Explain Mistake:

<u>Correct Solution:</u>

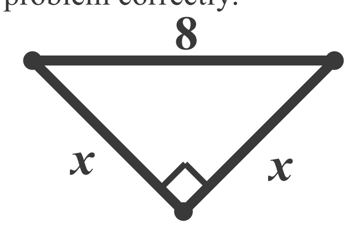
16. **Find, Fix, and Justify:** Raphael was asked to solve for the length of the hypotenuse in a right traingle with legs that have side lengths of 4 and 5. His work is shown below. He made a mistake when solving. Explain the mistake and then solve the problem correctly.

<u>**Raphael's Solution:**</u>
$a^2 + b^2 = c^2$
$4^2 + 5^2 = c^2$
$16 + 25 = c^2$
$41 = c$

<u>**Correct Solution:**</u>

Explain Mistake:

17. **Find, Fix, and Justify:** Nataani was asked to solve for the unknown side length in the triangle below. His work is shown below. He made a mistake when solving. Explain the mistake and then solve the problem correctly.

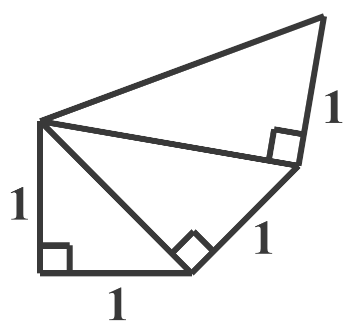


<u>**Nataani's Solution:**</u>
$a^2 + b^2 = c^2$
$x^2 + x^2 = 8$
$2x^2 = 8$
$x^2 = 4$
$x = 2$

<u>**Correct Solution:**</u>

Explain Mistake:

Extra for Experts: Use the picture below to answer questions a) and b).



a. Find all the missing side lengths and label the picture with the answers.
b. Using the picture above, devise a strategy for constructing a segment with a length of $\sqrt{5}$. Explain your strategy below.

10.2d Class Activity: The Converse of the Pythagorean Theorem

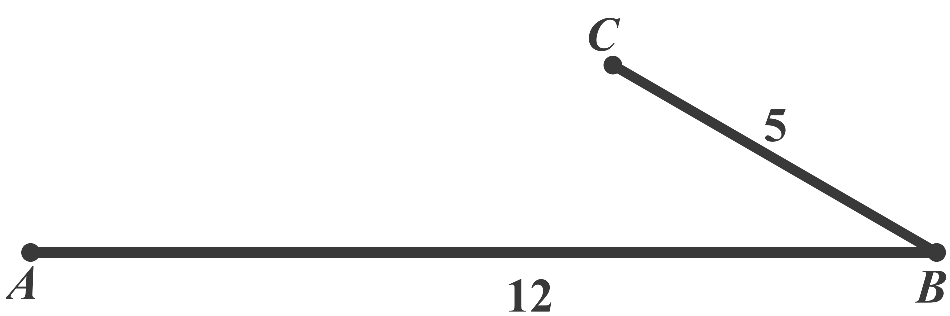
1. Mr. Riley's 8th grade class has been studying the Pythagorean Theorem. One day, he asked his class to find numbers a, b, and c where $a^2 + b^2 = c^2$, and draw triangles with those side lengths.

   Oscar determined that the numbers 5, 12, and 13 satisfy the Pythagorean Theorem as shown below:

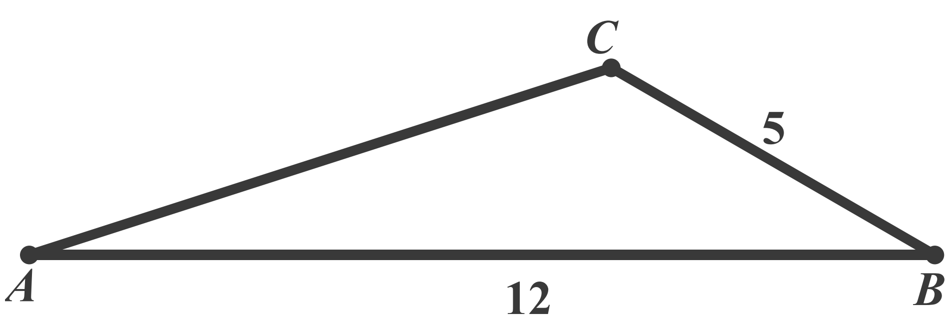
   $a^2 + b^2 = c^2$
   $5^2 + 12^2 = 13^2$
   $25 + 144 = 169$
   $169 = 169$

   Mr. Riley then said, "OK, so you have found three numbers that satisfy the Pythagorean Theorem. Now, show me that the triangle formed with these side lengths is a right triangle."

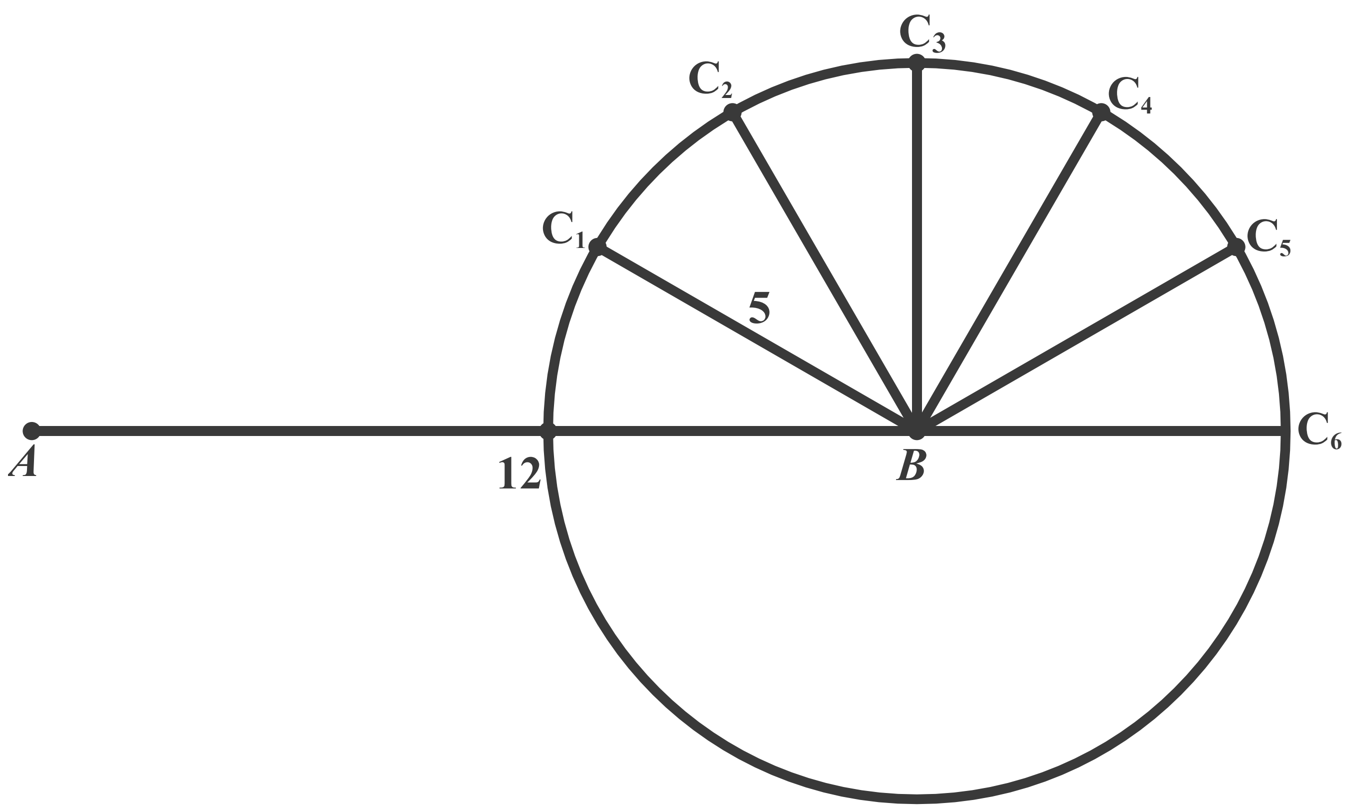
   a. Oscar continued working on the problem. He constructed a segment with a length of 12 cm and labeled the segment AB. From the endpoint B, he constructed a segment with a length of 5 cm and labeled the segment BC as shown in the picture below. Using a ruler, verify the lengths of the segments below.



   b. Then, he thought to himself, "I need to make the third side length AC equal to 13 because I know the triple 5, 12, 13 satisfies the Pythagorean Theorem." He connected A and C as shown below. He measured the length of AC and determined it did not measure 13 cm. Using a ruler, verify that AC does not measure 13 cm.



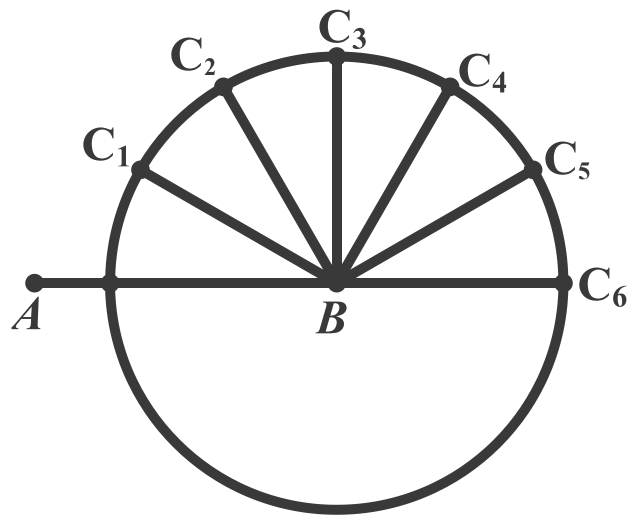
c. Then, he thought to himself, "What if I rotate $\overline{BC}$ around point B until AC measures 13 cm?" He began to rotate $\overline{BC}$ clockwise about B in increments as shown below. Help Oscar to find the location of C on the circle below that will give him a triangle with side lengths 5, 12, and 13.



d. What type of triangle is formed when AC equals 13 cm?

2. Lucy also found a set of numbers that satisfy the Pythagorean Theorem: 3, 4 and 5. Verify in the space below that Lucy's numbers satisfy the Pythagorean Theorem.

3. Using a process similar to Oscar's, Lucy set out to prove that a triangle with side lengths 3, 4 and 5 is in fact a right triangle. In the picture below $AB = 4$ cm and $CB = 3$ cm. Help Lucy determine the location of C that will create a triangle with side lengths 3 cm, 4 cm, and 5 cm.



4. What type of triangle is formed when AC equals 5 cm?

5. Based on the problems above, what type of triangle is formed with side lengths that satisfy the Pythagorean Theorem? Write down the Converse of the Pythagorean Theorem.

6. Do the side lengths given below satisfy the Pythagorean Theorem? Remember to distinguish between legs (shorter sides) and the hypotenuse (longest side) and enter them into the equation correctly.

a. 11, 60, 61	b. 2, 4, 6
c. 14, 50, 48	d. 1, 3, $\sqrt{10}$;
e. 2, 4, and $2\sqrt{5}$.	f. 5, 6, 8

7. Mr. Garcia then asks the class, "What if the tick marks in Lucy's picture are each 2 cm instead of 1 cm? What are the measures of the side lengths that form the right triangle? Do they satisfy the Pythagorean Theorem?"

8. What if the tick marks in Lucy's picture are each 3 cm? 0.1 cm? 10 cm? What are the measures of the side lengths that form the right triangles given these different scales and do they satisfy the Pythagorean Theorem?

10.2d Homework: The Converse of the Pythagorean Theorem

Directions: Determine whether the three side lengths form a right triangle. Write yes or no on the line provided.

1. 9, 12, 15 ______
2. 18, 36, 45 ______
3. 12, 37, 35 _______
4. 8, 15, 16 ______
5. $\sqrt{6}$, $\sqrt{10}$, 4 ______
6. 6.4, 12, 12.2 ______
7. 8.6, 14.7, 11.9 ______
8. 8, $8\sqrt{3}$, 16 ______
9. 8, 8, $8\sqrt{2}$ ______
10. 7, 9, 11.4 _____

10.2e Class Activity: Exploration with Pythagorean Triples Extension

While we have seen several different sets of numbers that form a right triangle, there are special sets of numbers that form right triangles called Pythagorean triples. A **Pythagorean triple** is a set of nonzero **whole numbers** *a*, *b*, and *c* that can be put together to form the side lengths of a right triangle. 3, 4, 5 and 5, 12, 13 are examples of Pythagorean triples. We have seen many other sets of numbers that form a right triangle such as 0.09, 0.4, 0.41 that are not Pythagorean triples because their side lengths are not whole numbers.

a. The chart below shows some sets of numbers *a*, *b*, and *c* that are Pythagorean triples. Verify that the sets satisfy the equation $a^2 + b^2 = c^2$.

b. Can you find additional Pythagorean triples? Explain the method you used.

a	*b*	*c*	a^2	b^2	c^2
3	4	5			
5	12	13			
7	24	25			

c. The chart above starts with values for *a* that are odd numbers. Why didn't the chart start with a value of 1 for *a*.

d. Can you find Pythagorean triples where *a* is even? What is the smallest Pythagorean triple you can find with *a* being an even number?

e. Design a method to confirm that these numbers actually form right triangles. Write a short paragraph describing the method you used, and the results you obtained.

10.2f Self-Assessment: Section 10.2

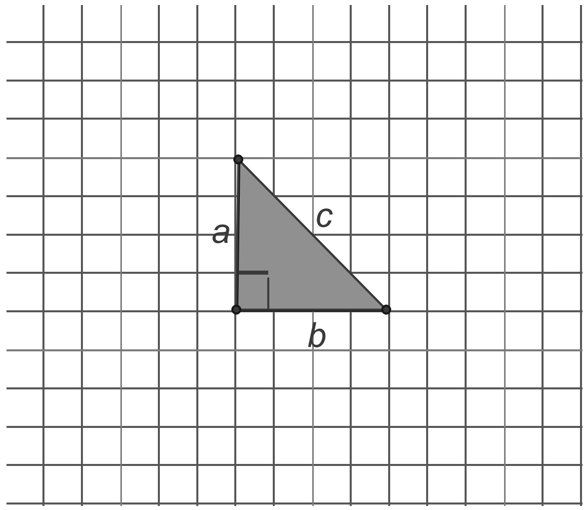
Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems are provided on the next page that match each skill/concept.

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Understanding 3	Substantial Understanding 4
1. Know that in a right triangle $a^2 + b^2 = c^2$, where a and b are the legs of the right triangle and c is the hypotenuse. *See sample problem #1*				
2. Understand and explain a proof of the Pythagorean Theorem. *See sample problem #2*				
3. Use the Pythagorean Theorem to solve for the missing side length of a right triangle given the measurements of the other two sides. *See sample problem #3*				
4. Understand and explain a proof of the converse of the Pythagorean Theorem. That is, for a triangle with side lengths a, b, and c if $a^2 + b^2 = c^2$, then the triangle is a right triangle. *See sample problem #4*				
5. Use the converse of the Pythagorean Theorem to determine whether three given measurements are possible side lengths of a right triangle. *See sample problem #5*				

10.2f Sample Problems: Section 10.2

Sample Problem #1

In the picture below the gray triangle is a right triangle. Draw the squares along each of the three sides of the triangles. Find the area of each square and write the area in each square. Then, find the side lengths *a*, *b*, *c* of the triangle.



Sample Problem #2

Below is a geometric explanation for a proof of the Pythagorean Theorem: Given a right triangle with side lengths *a* and *b* and a hypotenuse of *c*, then $a^2 + b^2 = c^2$. The figures for the proof are given in order. Choose the explanation that provides a sound argument accompanied with reasoning and warrants to support the claims given for each figure. Write the letter that matches each explanation in the space provided.

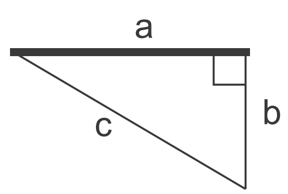


Figure 1_____

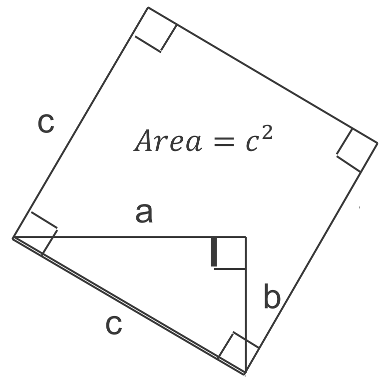


Figure 2_____

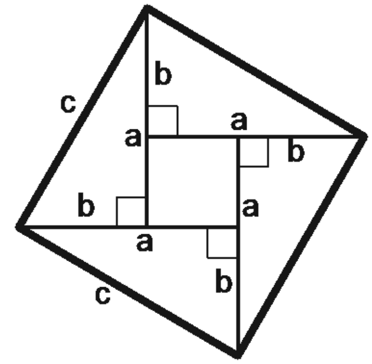


Figure 3_____

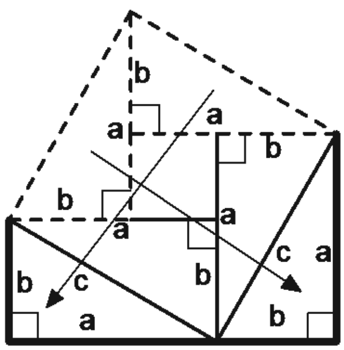


Figure 4_____

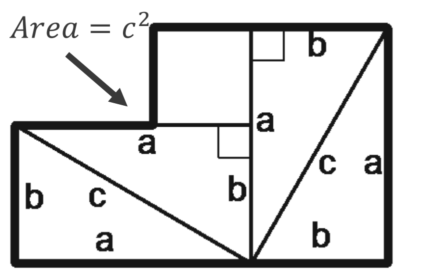


Figure 5_____

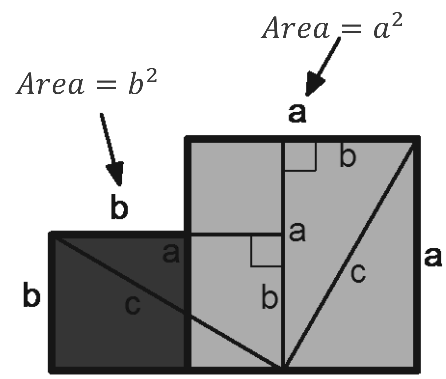


Figure 6 ________

A. Inside of the square draw 4 congruent right triangles with side length *a* and *b* and a hypotenuse of *c*.

B. Draw a square with off of this triangle with a side length of *c*. The area of square this square is c^2. This is because the area of a square is the side length squared

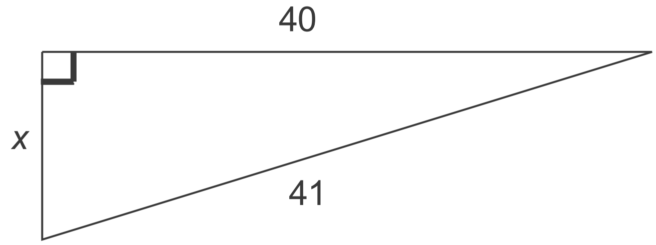
C. You can view the area of this figure as the composition of two squares with sides length *a* and *b*. The area of the darker square is b^2 and the area of the lighter shaded square is a^2. Thus the area of the whole figure is a^2+b^2. As stated above this is the same as the area of the original square with side length *c*. Thus $a^2 + b^2 = c^2$.

D. Rearrange the square by translating the top two triangles to the bottom of the figure.

E. The area of this figure is the same as the area of the original square because we have not added or removed any of the pieces.

F. Begin with right triangle with a horizontal side length of *a* and a vertical side length of *b* and a hypotenuse of *c*.

Sample Problem #3
Find the value of x using the Pythagorean Theorem. Leave your answer in simplest radical form.



Sample Problem #4

The Converse of the Pythagorean Theorem states that given a triangle with side lengths *a*, *b*, and *c*, if $a^2 + b^2 = c^2$, then the triangle is a right triangle. Explain the proof of the Converse of the Pythagorean Theorem that your teacher provides for you.

Sample Problem #5

Determine whether the three side lengths form a right triangle. Show your work to verify your answer.

5.5, 12.5, 13.5

Section 10.3 Applications of the Pythagorean Theorem

Section Overview:
In this section, students apply the Pythagorean Theorem to solve real-world problems in two- and three-dimensions. Then, students use the Pythagorean Theorem to find the distance between two points. After the students gain an understanding of the process being used to find the distance between two points in a coordinate system, students have the opportunity to derive the distance formula from the Pythagorean Theorem and the process being used. Rather than memorizing the distance formula, the emphasis is placed on the process used to find the distance between two points in a coordinate system and the connection between the Pythagorean Theorem and the distance formula.

Concepts and Skills to Master:
By the end of this section, students should be able to:

1. Use the Pythagorean Theorem to solve problems in real-world contexts, including three-dimensional contexts.
2. Find the distance between two points in a coordinate system.

10.3a Class Activity: Applications of the Pythagorean Theorem

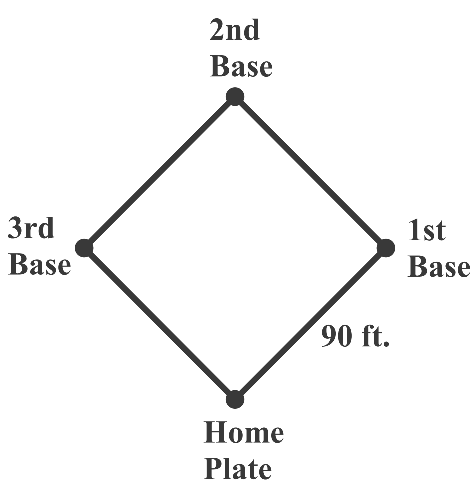
Directions: For each problem, first draw a picture if one is not provided and then solve the problem.

1. What is the length of the diagonal of a rectangle of side lengths 1 inch and 4 inches?

2. A square has a diagonal with a length of $2\sqrt{2}$ inches. What is the side length of the square?

3. Two ships leave a dock. The first ship travels 6 miles east and then 8 miles north and anchors for the night. The second ship travels 5 miles west and then 12 miles south and anchors for the night. How far are each of the ships from the dock when they anchor for the night?

4. A baseball diamond is in the shape of a square. The distance between each of the consecutive bases is 90 feet. What is the distance from Home Plate to 2^{nd} Base?

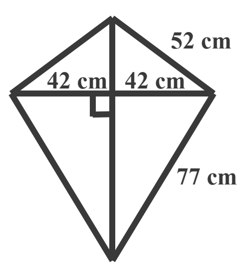


5. Ray is a contractor that needs to access his client's roof in order to assess whether the roof needs to be replaced. He sees that he can access a portion of the roof that is 15 feet from the ground. He has a ladder that is 20 feet long.

   a. How far from the base of the house should Ray place the ladder so that it just hits the top of the roof? Round your answer to the nearest tenth of a foot.

   b. How far should he place the ladder from the base of the house if he wants it to sit 3 feet higher than the top of the roof? Round your answer to the nearest tenth of a foot.

6. The dimensions of a kite sail are shown below. The support rod that runs from the top of the kite to the bottom of the kite has been broken and needs to be replaced. What length of rod is needed to replace the broken piece? Round your answer to the nearest tenth.

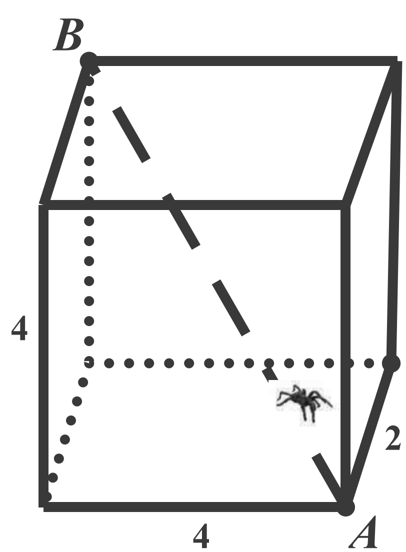


7. A new restaurant is putting in a wheelchair ramp. The landing that people enter the restaurant from is 2 feet higher than street level. Safety standards require that for every 1 foot of rise on a wheelchair ramp there must be a run of 12 feet. How long will the ramp have to be to meet safety standards? Round your answer to the nearest tenth.

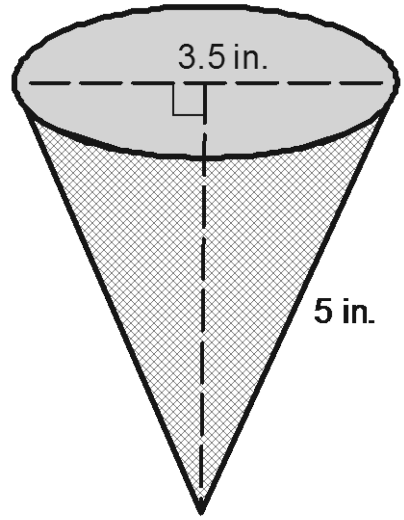
8. Melanie is having a rectangular-shaped patio built in her backyard. It is very important to Melanie that the corners of the patio are right angles. The contractor built a patio with a width of 10 feet and a length of 15 feet. The diagonal measures 20 feet. Does the patio have the right angles that Melanie requested?

9. Fred is safety conscious. He knows that to be safe, the distance between the foot of the ladder and the wall should be $\frac{1}{4}$ the height of the wall. Fred needs to get on the roof of the school building which is 20 ft. tall. How long should the ladder be if he wants it to rest on the edge of the roof and meet safety standards? Round your answer to the nearest tenth.

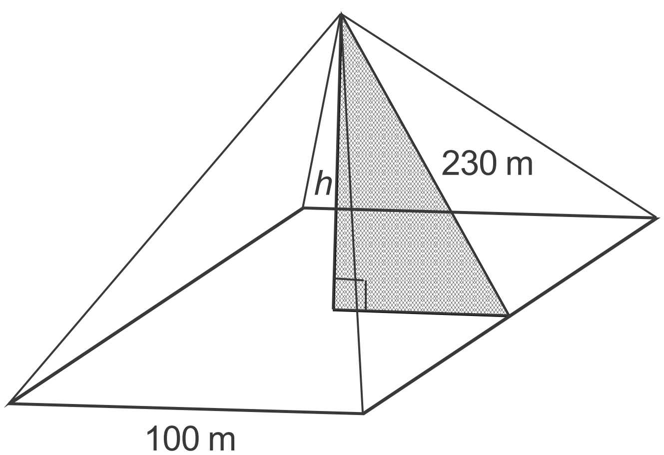
10. A spider has taken up residence in a small cardboard box which measures 2 inches by 4 inches by 4 inches. What is the length, in inches, of a straight spider web that will carry the spider from the lower right front corner of the box to the upper left back corner of the box?



11. Sunny made a paper cone to hold candy for favors for a baby shower. After making the cones she measures the slant height of the cone and the diameter of the base of the cone. Her measurements are shown in the picture below. Find the volume of the cone.



12. In the movie Despicable Me, an inflatable model of The Great Pyramid of Giza in Egypt was created by Vector to trick people into thinking that the actual pyramid had not been stolen. When inflated, the false Great Pyramid had a square base of side length 100 m. and the height of one of the side triangles was 230 meters. This is also called the slant height of the pyramid. What is the volume of gas that was used to fully inflate the fake Pyramid? (Hint: Recall the formula for the volume of a pyramid is $\frac{1}{3}Bh$ where B is the area of the base and h is the height of the pyramid (the distance from the base to the apex).



10.3a Homework: Applications of the Pythagorean Theorem

1. What is the length of the diagonal of a square with a side length of 4 cm?

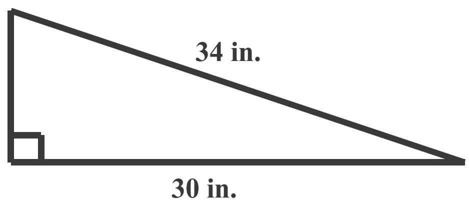
2. One side length of a rectangle is 2 inches. The diagonal of the rectangle has a length of $2\sqrt{5}$ inches. What is the length of the other side of the rectangle?

3. A football field is 360 feet long and 160 feet wide. What is the length of the diagonal of a football field assuming the field is in the shape of a rectangle?

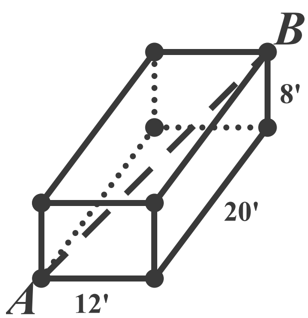
4. The length of an Olympic-size swimming pool is 55 meters. The width of the pool is 25 meters. What is the length of the diagonal of the pool assuming the pool is in the shape of a rectangle?

5. You are locked out of your house. You can see that there is a window on the second floor that is open so you plan to go and ask your neighbor for a ladder long enough to reach the window. The window is 20 feet off the ground. There is a vegetable garden on the ground below the window that extends 10 ft. from the side of the house that you can't put the ladder in. What size ladder should you ask your neighbor for?

6. Kanye just purchased a skateboarding ramp. The ramp is 34 inches long and the length of the base of the ramp is 30 inches as shown below. What is the height of the ramp?

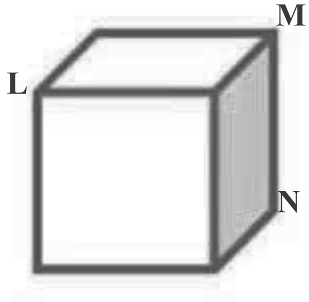


7. A rectangular-shaped room has a width of 12 feet, a length of 20 feet, and a height of 8 feet. What is the approximate distance from one corner on the floor (Point A in the figure) to the opposite corner on the ceiling (Point B in the figure)?



8. A large pile of sand has been dumped into a conical pile in a warehouse. The slant height of the pile is 20 feet. The diameter of the base of the sand pile is 32 ft. Find the volume of the pile of sand.

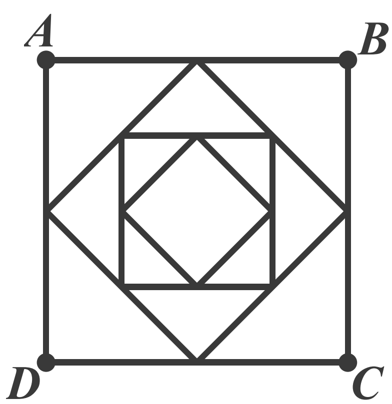
9. The cube below is a unit cube. A unit cube is a cube of side length 1.



a. What is the length of $\overline{LM}$? Leave your answer in simplest radical form.

b. What is the length of $\overline{LN}$? Leave your answer in simplest radical form.

Extra for Experts: Square *ABCD* has side lengths equal to 4 inches. Connecting the midpoints of each side forms the next square inside *ABCD*. This pattern of connecting the midpoints to form a new square is repeated.

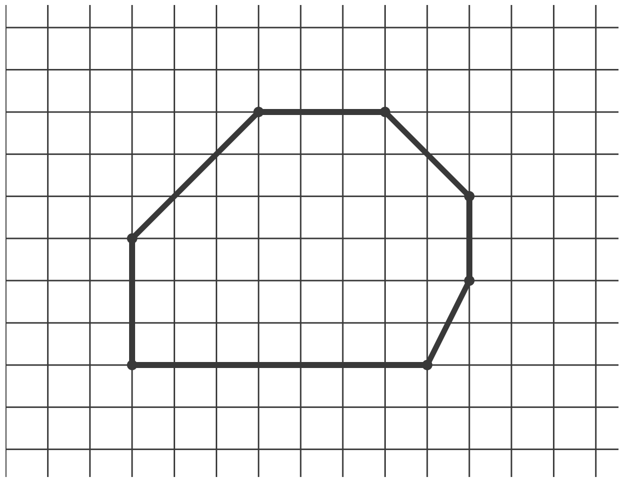


a. What is the side length of the inner-most square?

b. What is the area of the inner-most square?

c. What is the ratio of the area of each square to the area of the next square created?

Extra for Experts: The following is a scale drawing of a patio that Mr. Davis plans to build in his backyard. Each box in the scale drawing represents 1 unit.



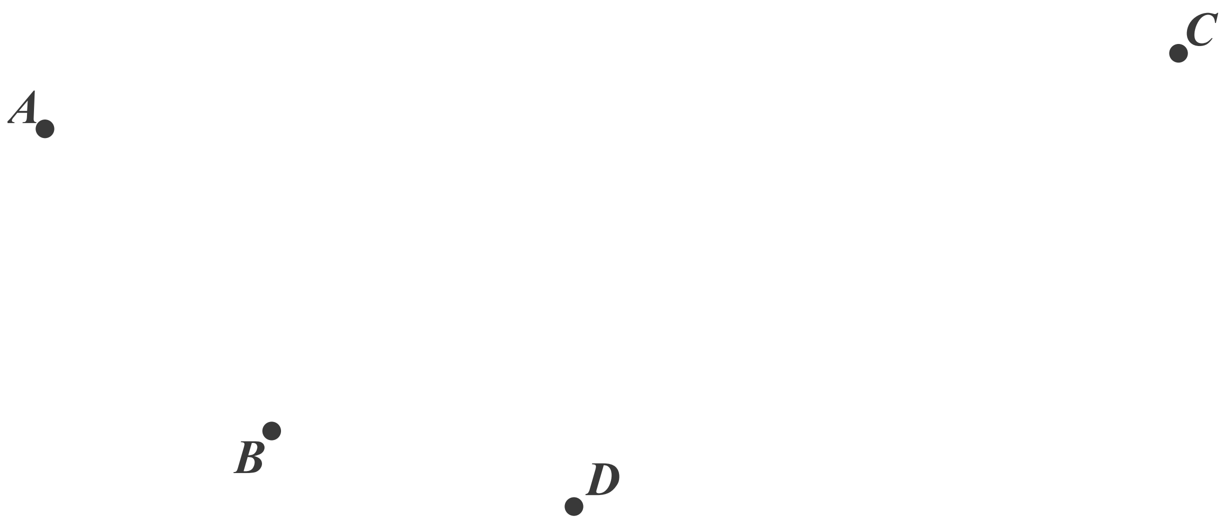
a. Find the exact value of the perimeter of the scale drawing of the patio. Show all work and thinking.

b. Find the area of the scale drawing of the patio. Show all work and thinking.

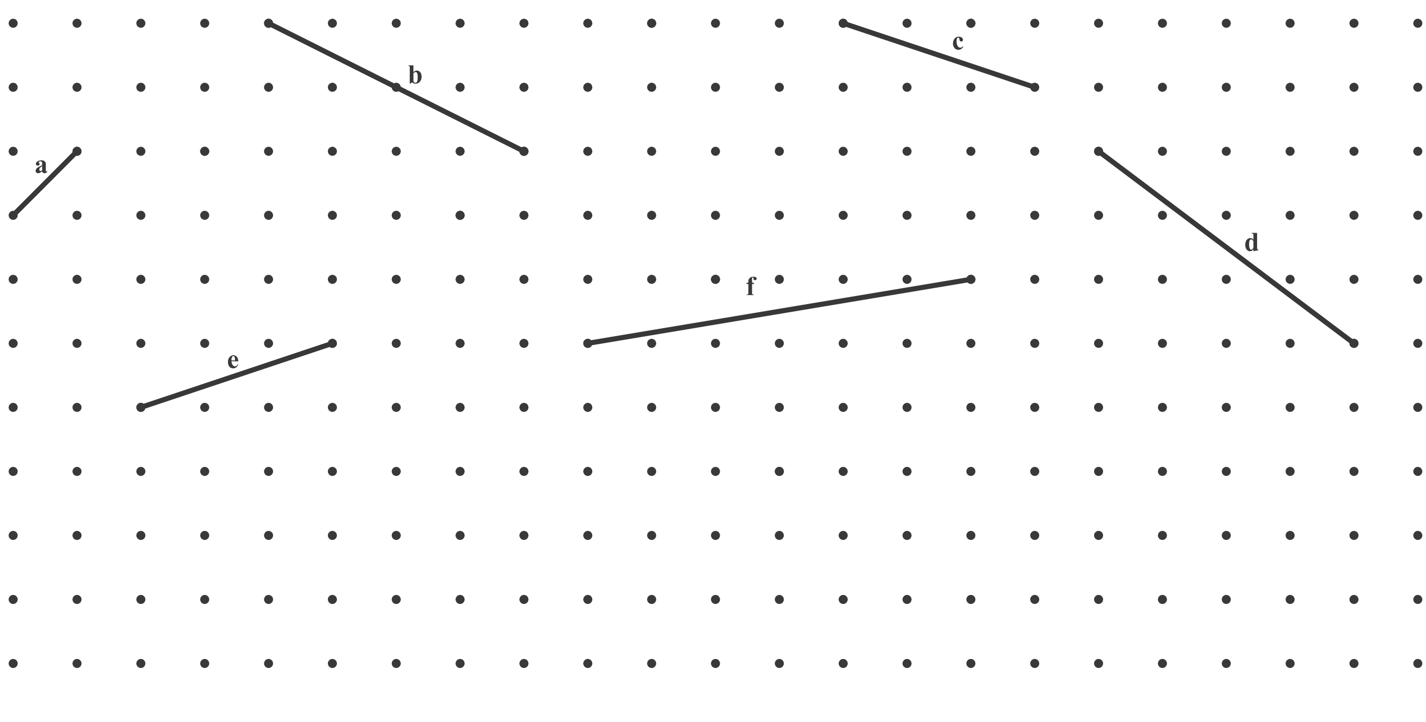
c. If the scale on the drawing above is 1 unit = 3 feet, what is the actual measure of the perimeter of the patio? The area? Show all work.

10.3b Class Activity: Finding Distance Between Two Points

1. Using a centimeter ruler, find the distance between the following sets of points shown below. Then draw the slope triangle of each segment, measure the lengths of the rise and run, and verify that the Pythagorean Theorem holds true.
   a. *A* to *B*
   b. *B* to *C*
   c. *C* to *D*

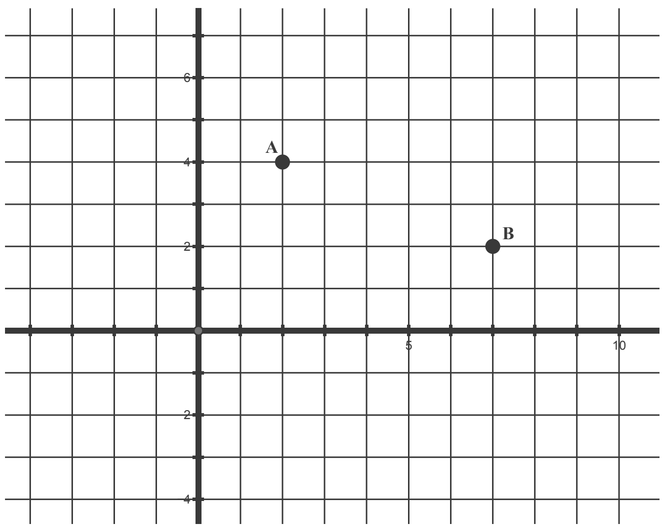


2. Find the lengths of the segments below. Assume that each horizontal and vertical segment connecting the dots has a length of 1 unit.

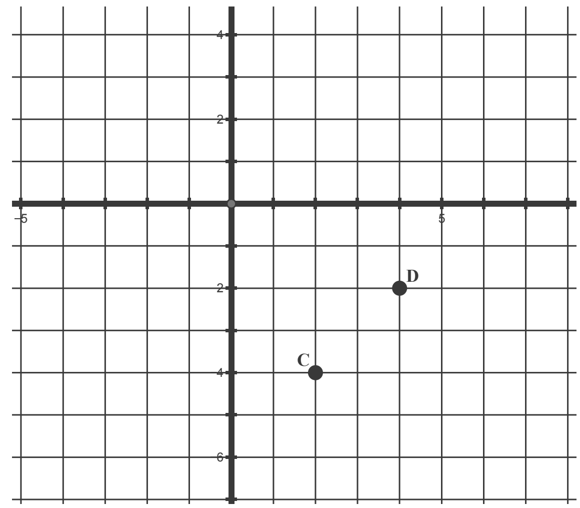


Directions: Label the coordinates of each point. Then, find the distance between the two points shown on each grid below.

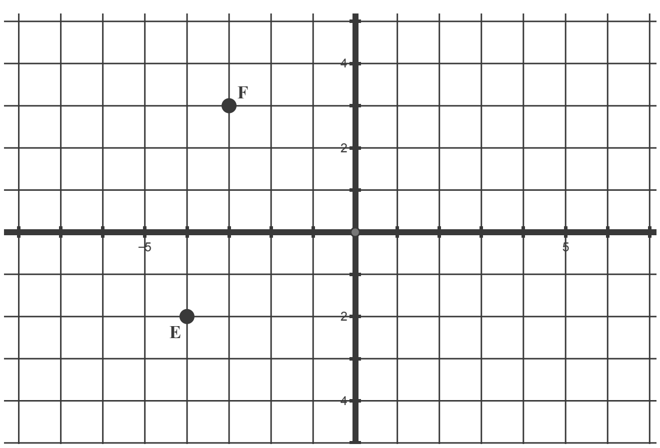
3.



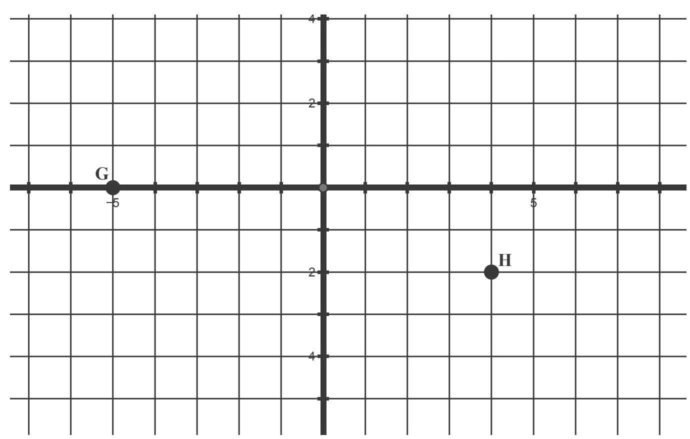
4.



5.

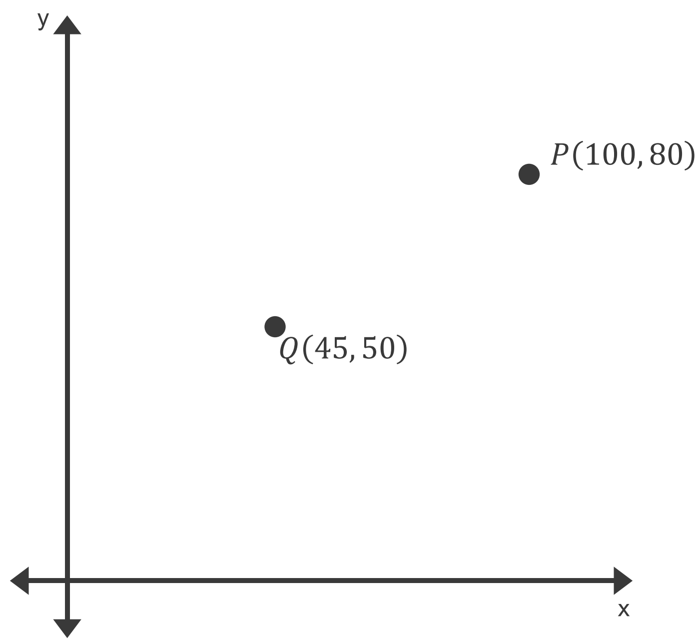


6.



<u>The Coordinate Distance Formula</u>

7. Find the distance between the two points given on the graph below.



8. Find the distance between the two points given below. Leave your answers in simplest radical form.

a. $A: (3, 5) \quad B: (6, 9)$

b. $R: (-1, 4) \quad S: (3, 8)$

c. $C: (0, 5) \quad D: (2, -3)$

d. $S: (-3, -5) \quad T: (2, -7)$

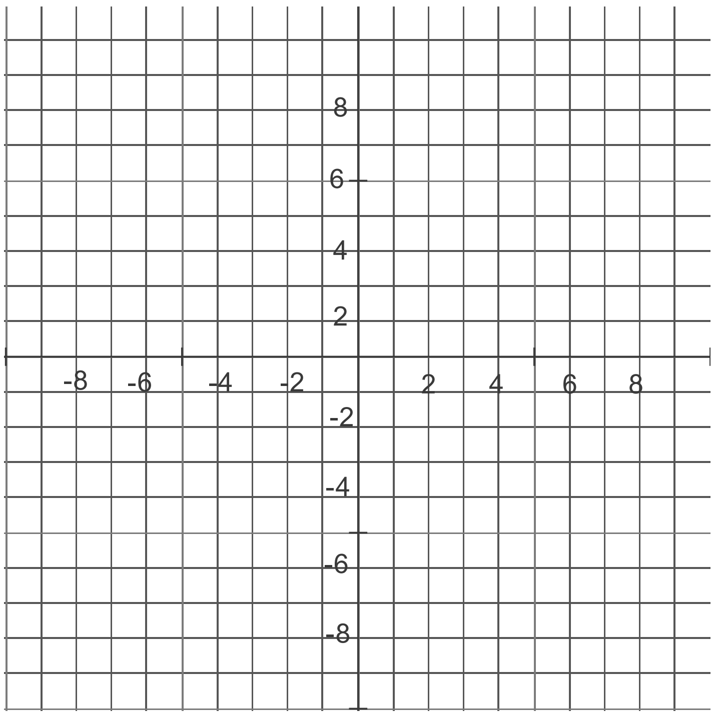
9. A triangle has vertices at the points (2,3) and (4,8), and (6,3) on the coordinate plane.
   a. Find the perimeter of the triangle. Use the grid below if needed.

   b. Find the area of the triangle.

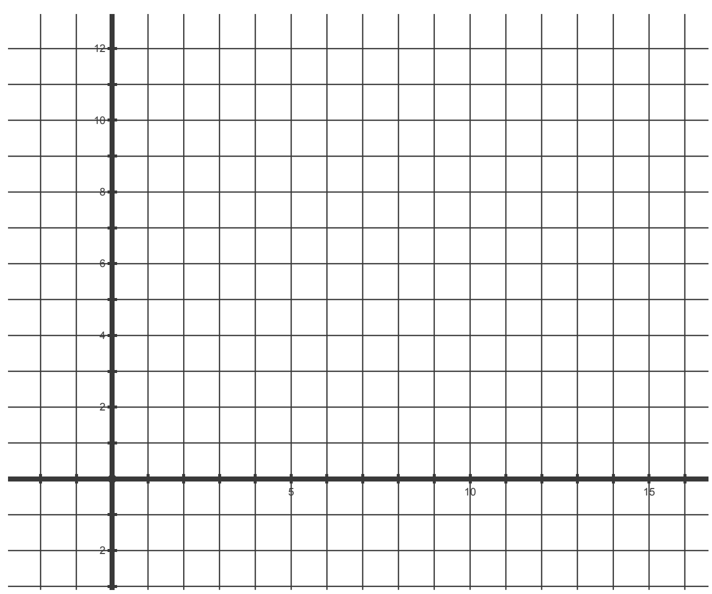
   c. If the triangle is dilated by a scale factor of 3 what will the new perimeter be?

   d. If the triangle is dilated by a scale factor of 3 what will the new area be?

   e. Plot the original triangle, label it triangle A. Then reflect the triangle over the *y*-axis, label the new triangle A'. Does this transformation change the perimeter of the triangle? Explain your answer.



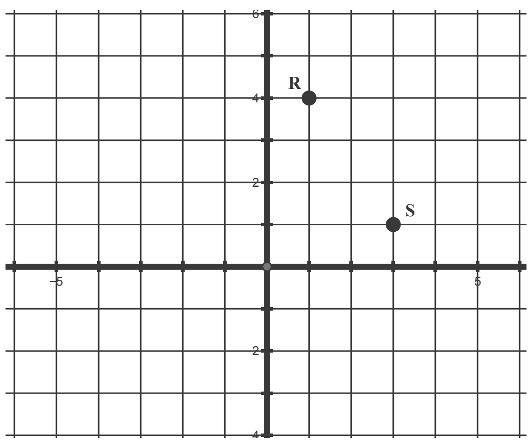
10. List **three** coordinate pairs that are 5 units away from the origin in the first quadrant. Describe how to find the points and justify your reasoning. The grid has been provided to help you.
(**Note:** Points on the axes are not in the first quadrant).



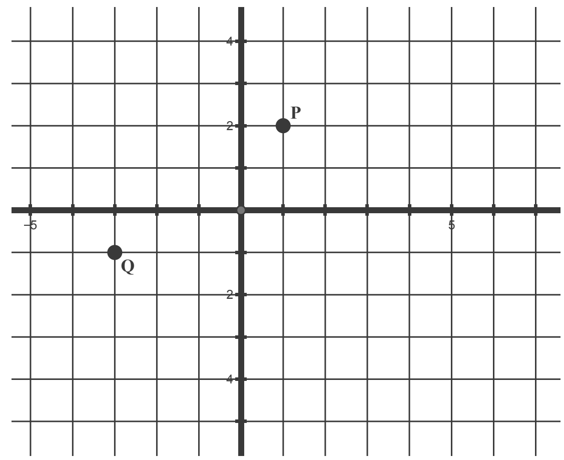
10.3b Homework: Finding Distance Between Two Points

Directions: Find the distance between the two points shown on each grid below. Leave your answers in simplest radical form.

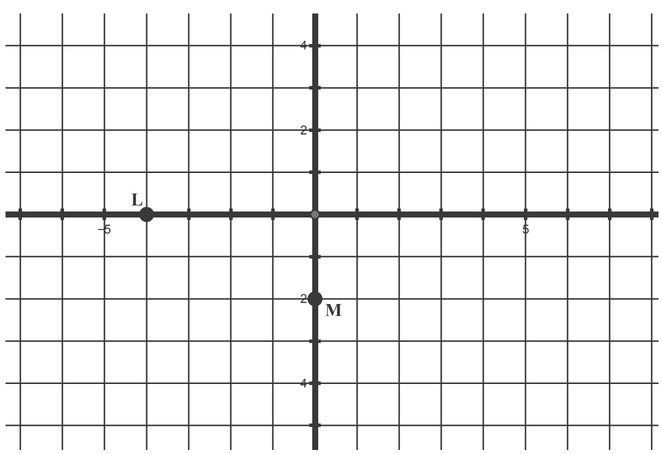
1.



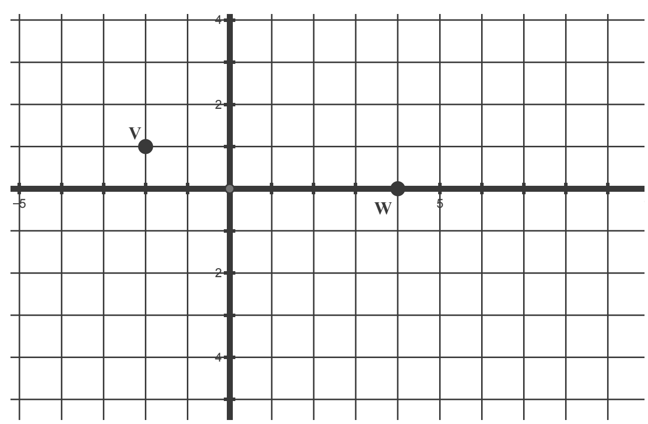
2.



3.



4.



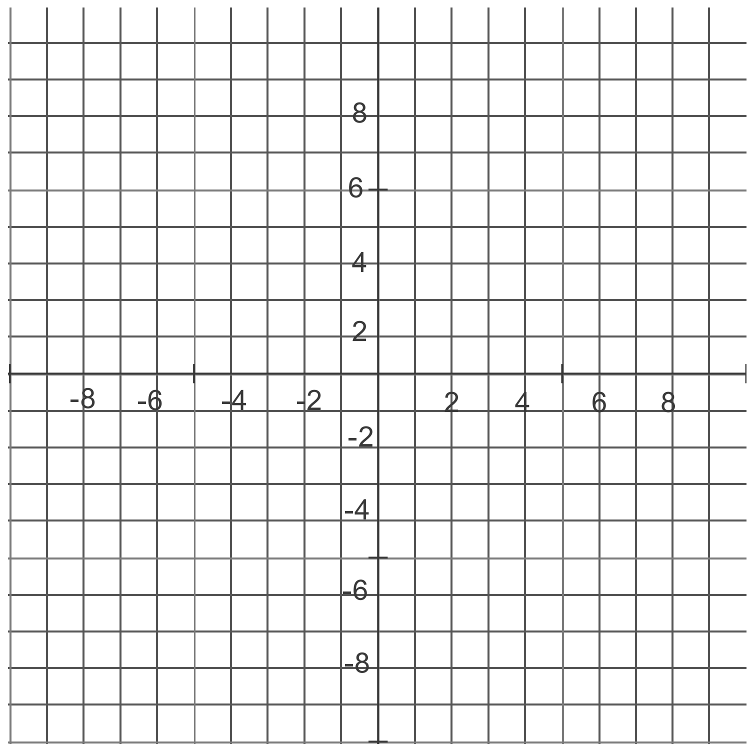
5. Find the distance between the two points given below. Leave your answers in simplest radical form.
   a. $A:(2,1)\quad B:(4,7)$

   b. $R:(2,-1)\quad S:(8,-7)$

   c. $C:(1,0)\quad D:(2,-3)$

   d. $S:(-2,-4)\quad T:(2,-5)$

6. Plot any letter of the alphabet that is made up of segments that are straight lines on the coordinate plane given below. For example you can plot the letter A, E, F, etc. but not the letter B,C, D, etc.



a. Find the total distance for the segments that make up this letter.

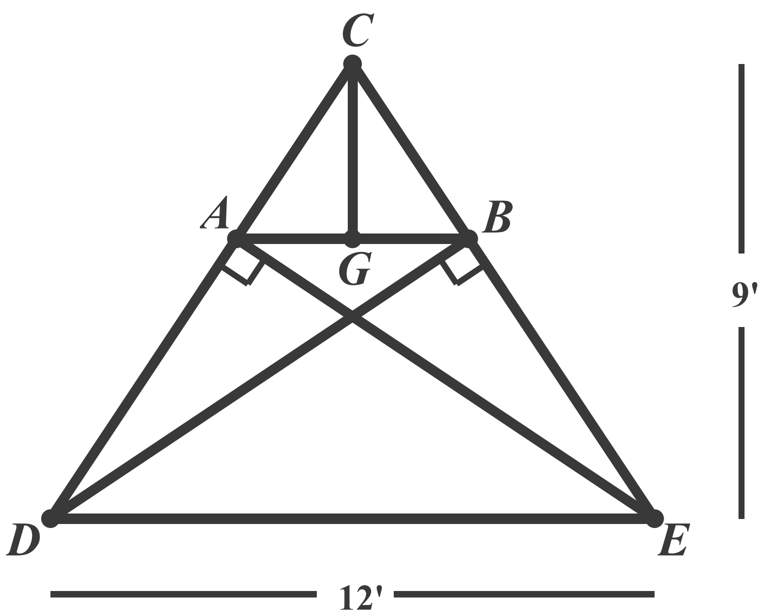
b. If you dilated this letter by a scale factor of 4 what is the total distance of the segments that make up your letter?

c. If you dilated this letter by a scale factor of $\frac{1}{5}$ what is the total distance of the segments that make up your letter?

d. Rotate your letter 180 degrees about the origin. Does this transformation change the size or shape of the letter? Explain your answer.

10.3c Extension: Construction

Mario is designing an A-frame for the lodge of a ski resort. Below is a scale drawing of his design.
Given: C lies over the center of the building
$\overline{AB} \parallel \overline{DE}$
$\angle DAE$ and $\angle EBD$ are right angles.



What are the lengths of all segments in the diagram?

10.3d Self-Assessment: Section 10.3

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept. Sample problems are provided on the next page that match each skill/concept.

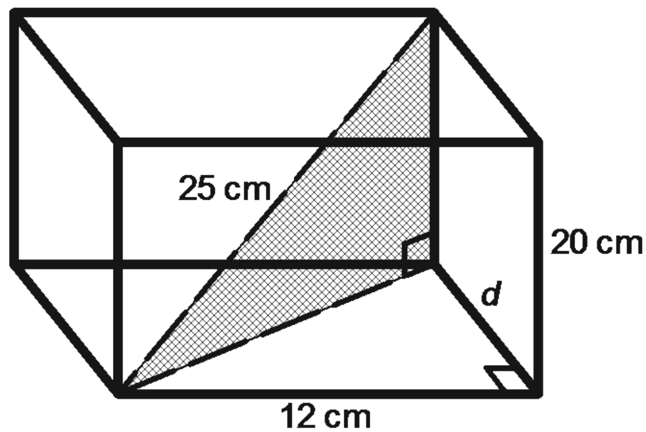
Skill/Concept	**Minimal Understanding 1**	**Partial Understanding 2**	**Sufficient Understanding 3**	**Substantial Understanding 4**
1. Use the Pythagorean Theorem to solve problems in real-world contexts, including three-dimensional contexts. *See sample problem #1*				
2. Find the distance between two points in a coordinate system. *See sample problem #2*				

10.3d Sample Problems: Section 10.3

Sample Problem #1

a. A park is 6 miles east of your home. The bakery is 4 miles north of the park. How far is your home from the bakery as the crow flies?

b. Find the volume of the rectangular prism given below.



Sample Problem #2

Find the distance between each set of points.

a. $A(-10,2)$ and $B(-7,6)$

b. $C(-2,-6)$ and $D(6,9)$

c. $E(3,5)$ and $F(7,9)$

d. $G(3,4)$ and $H(-2,-2)$