Section 8.3: Volume of Cylinders, Cones, and Spheres

Section Overview:
Throughout this section, students are solving real-world and mathematical problems involving volumes of cylinders, cones, and spheres. Students begin by deriving the volume of a cylinder, relying on their knowledge from previous grades that the volume of a right three-dimensional object can be found by taking the area of its base and multiplying it by the height. Students then use the formula for the volume of a cylinder to arrive at the formulas for the volumes of a cone and sphere. Using concrete models of these three-dimensional objects, students physically compare the volume of a cone to the volume of a cylinder. Students then manipulate the formula for the volume of a cylinder to reflect these differences, arriving at the formula for the volume of a cone. They use a similar process to derive the formula for the volume of a sphere. Once students understand where these formulas come from, they apply them to solve real-world problems, knowing when and how to use the formulas.

Concepts and Skills to be mastered:
By the end of this section students should be able to:

1. Find the volume of a cylinder, cone, and sphere given a radius and height.
2. Find a missing measurement (height, radius, or diameter) for a cylinder, cone, or sphere given the volume.
3. Use the formulas for the volumes of cylinders, cones, and spheres to solve a variety of real-world problems.

8.3a Class Activity: Wet or Dry (*This activity is optional)*

We have been discussing exponents throughout this chapter. You have learned how to simplify expressions with exponents in them and have looked at how expressing numbers in scientific notation can better help us deal with numbers that are really big and really small. Exponents are also used to find the volume of a three-dimensional object.

1. Describe what volume is. Compare it to finding perimeter or area.

To help us better understand how important it is to know how to find the volume of a three-dimensional object do the following activity.

2. Choose two different sizes of cylindrical cans to use for this activity. Measure the diameter and the height of each can **in centimeters.**

   Can 1: Diameter _____ Height _____

   Can 2: Diameter _____ Height _____

3. As a group determine the volume of each can. Show your work below or explain how you found the volume of your cans. Make sure that your units are correct. Once you have found the volume in cubic centimeters change your answer to millimeters. (Hint: One cubic centimeter is the same as one milliliter.)

   Can 1 **Can 2**

Select one of your cans and bring it up to the teacher with your calculation for the volume of the can. Also, select one member of the team to test your calculations.

4. Which can did your team choose and why did you choose this can?

5. How close were your calculations to the actual volume of the can?

6. What would you do differently if you could recalculate the volume of your can?

8.3b Class Activity: Volume of Cylinders

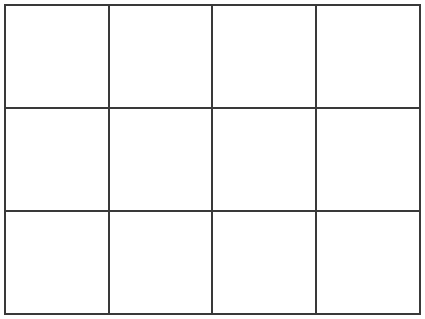
1. Gunner just started his summer job doing swimming pool maintenance. He has a variety of things to do for each pool. For each item below fill in the missing measurement in the space provided for each pool.

   a. He needs to build a fence around each of the swimming pools below. If each unit represents one meter determine how much fencing he needs for each pool. Write your answer below each pool in the appropriate spot.

   b. Gunner now has to cover each pool. Determine how much material he will need to cover each pool. Write your answer below each pool in the appropriate spot.

   c. After Gunner has put up a fence and knows how much material he needs to cover the pools he needs to fill the pools back up with water. Determine how much water he would need to fill each pool to a depth of one meter. Write your answer below each pool in the appropriate spot.

   d. Now determine of much water he would need to fill each pool to a depth of 2 meters. Continue filling in the chart to 10 meters deep for each pool.

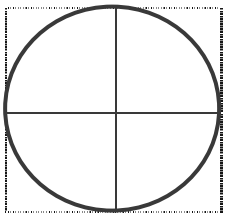
Pool #1	**Pool #2**
	
Perimeter:	Perimeter:
Area:	Area:
1 meter deep volume:	1 meter deep volume:
2 meter deep volume:	2 meter deep volume:
3 meter deep volume:	3 meter deep volume:
4 meter deep volume:	4 meter deep volume:
10 meter deep volume:	10 meter deep volume:

2. Describe how to find the volume of the pool for any given depth.

3. Explain how the formula $V=Bh$ helps you find the volume.

4. Gunner has one more pool to work on. Use what you know about the formula above to fill in the missing information for Pool #3. Recall that each unit represents 1 meter.

Pool #3



Perimeter:

Area:

1 meter deep volume:

2 meter deep volume:

3 meter deep volume:

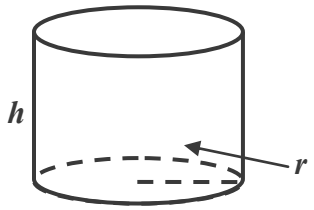
4 meter deep volume:

10 meter deep volume:

5. What type of three-dimensional object is Pool #4?

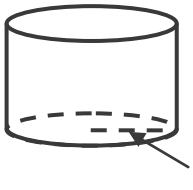
6. Use the picture given below to describe how to find the volume of a Cylinder. Be sure to describe each part of the formula and how it relates to the formula $V = Bh$.

A cylinder is a solid obtained by taking a circle in a plane (called the base) and drawing it out in a direction perpendicular to the base for a distance h (called the height).

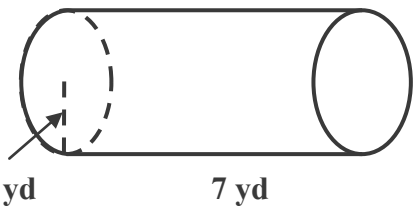


Directions: Find the volume for each cylinder described below. If needed draw and label a picture.

7.



8.



9. Cylinder with a Radius = 21 mm and Height = 19 mm.	10. Cylinder with a Diameter = 8.8 cm and Height = 9 cm.

Directions: Find the missing measurement for each cylinder described below.

11. The volume of a cylinder is 117.1 cubic feet, and its height is 15 ft. Find the diameter of the base of the cylinder.	12. The volume of a cylinder is 4,224.8 cubic millimeters, it has a diameter of 16.4 mm, find the height of the cylinder. Extension: Find the circumference of the base of the cylinder.

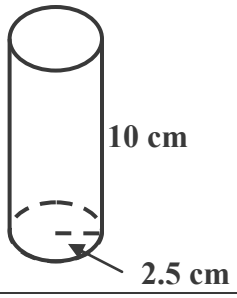
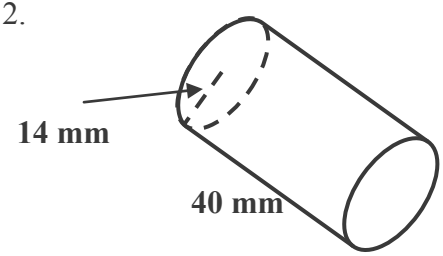
Directions: For each problem given below draw and label a picture that describes each cylinder. Then solve the problem.

13. An ice cream company wants to package a pint of ice cream in a circular cylinder that is 4 inches high. A pint is 16 fluid ounces and 1 fluid ounce is 1.8 cubic inches. What does the radius of the base circle have to be?

14. For a science project, Hassan put a can out to collect rainwater. The can was 11 inches tall and had a diameter of 8 inches. If it rained exactly 20 cubic inches each day, how many days did it take to fill the can?

8.3b Homework: Volume of Cylinders

Directions: Find the volume for each cylinder described below. If needed draw and label a picture.

1. 	2. 
3. Cylinder with a radius of 2 ft and a height of 7 ft.	4. Cylinder with a diameter of 2.7 m and a height of 30 m.

Find the missing measurement for each cylinder described below.

5. The volume of a cylinder is 63.6 cubic inches, and its height is 9 inches. Find the diameter of the base of the cylinder. Extension: Find the circumference of the base of the cylinder.	6. The volume of a cylinder is 8,685.9 cubic ft, it has a diameter of 19.2 ft, find the height of the cylinder.

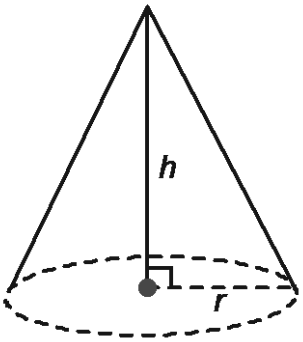
Directions: For each problem given below draw and label a picture that describes each cylinder. Then solve the problem.

7. What is the volume of Keisha's thermos if it has a radius of 2.5 in at the opening and 10 in for a height?

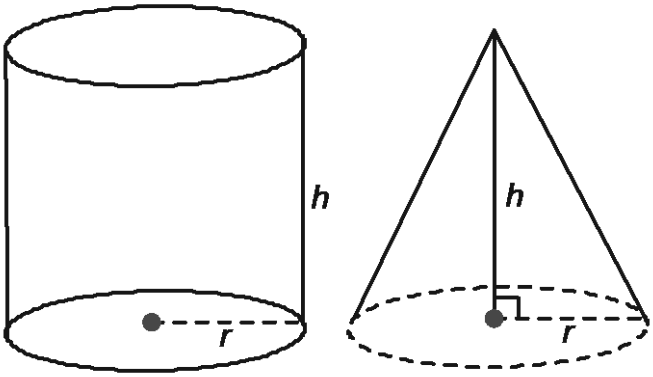
8. Mr. Riley bought 2 cans of paint to paint his garage. Each can had a radius of 5.5 inches and a height of 8 inches. How many cubic inches of paint did he buy in all?

8.3c Class Activity: Volume of Cones

Recall from seventh grade, that a cone is a three-dimensional figure with a circular base. A curved surface connects the base and the vertex.



The cylinder and cone given below have the same height and their bases are congruent.



1. Predict how the volume of the cone compares to the volume of the cylinder.

2. If you fill the cone with water or other filling material, predict how many cones of water will fit into the cylinder.

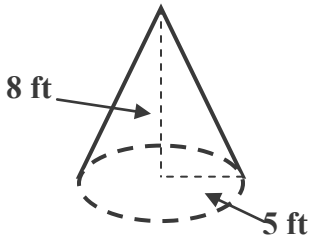
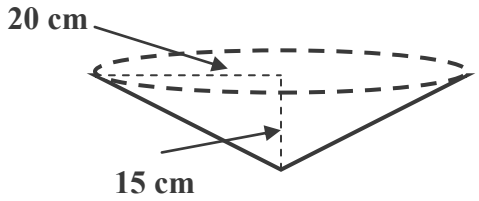
3. Now try it. How many cones fit into the cylinder?

4. About what fraction of the cylinder is filled by the volume of one cone?

5. Manipulate the equation for the volume of the cylinder to show the volume of the cone.

6. Explain in your own words how the volume of a cone compares to the volume of a cylinder. Describe the parts of the formula for the volume of a cone. Write this formula below the cone in the picture above.

Directions: Find the volume for each cone described below. If needed draw and label a picture.

7.  The volume of the cone is	8.  The volume of the cone is
9. A cone with a radius of 8.4 feet and a height of 5.5 feet.	10. A cone with a diameter of 9 meters and a height of 4.2 meters.

Directions: Find the missing measurement for each cylinder described below. Round your answer to the nearest tenth.

11. The volume of a cone is 122.8 cubic inches, and its height is 4.5 inches. Find the diameter of the base of the cone.	12. The volume of a cone is 188.5 cubic ft, it has a diameter of 12 ft, find the height of the cylinder.

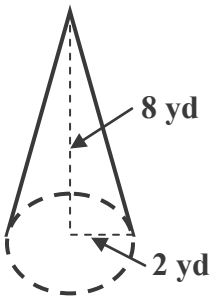
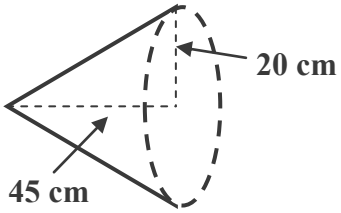
For each problem given below draw and label and picture that describes each cylinder. Then solve the problem.

13. Salt and sand mixtures are often used on icy roads. When the mixture is dumped from a truck into the staging area, it forms a cone-shaped mound with a diameter of 10 feet and a height of 6 feet. What is the volume of the salt-sand mixture?

14. A glass in the shape of a cone has a diameter of 8 cm. If the glass has a volume of 200 ml (or 200 cubic centimeters), what is the greatest depth that a liquid can be poured into the glass? Explain.

8.3c Homework: Volume of Cones

Directions: Find the volume for each cone described below. If needed draw and label a picture.

1. 	2. 
3. A cone with a radius of 40 feet and a height of 100 feet.	4. A cone with a diameter of 4.2 meters and a height of 5 meters.

Directions: Find the missing measurement for each cone described below.

5. The volume of a cone is 37.7 cubic inches, and its height is 4 inches. Find the diameter of the base of the cone.	6. The volume of a cone is 628.3 cubic ft, it has a diameter of 20 ft, find the height of the cone.

Directions: For each problem given below draw and label a picture that describes each cylinder. Then solve the problem.

7. The American Heritage Center at the University of Wyoming is a conical building. If the height is 77 feet, and the area of the base is about 38,000 square feet, find the volume of air that the heating and cooling systems would have to accommodate.

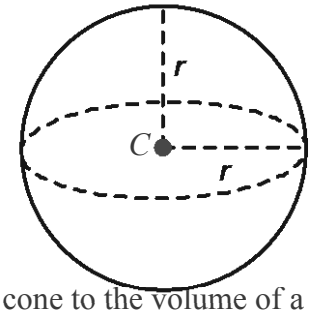
8. A stalactite, a geological formation, in the Endless Caverns in Virginia is cone-shaped. It is 4 feet long and has a diameter at its base of 1.5 feet.

   a. Assuming that the stalactite forms a perfect cone, find the volume of the stalactite.

   b. The stalactite is made of calcium carbonate, which weighs 131 pounds per cubic foot. What is the weight of the stalactite?

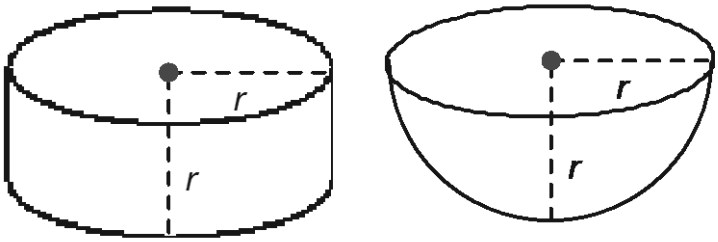
8.3d Class Activity Volume of Spheres

Recall that a sphere is a set of points in space that are a distance of r away from a point C, called the center of the sphere.



Just like you compared the volume of a cone to the volume of a cylinder to find the formula for the volume of a cone you are going to compare the volume of a sphere to the volume of a cylinder.

The cylinder and hemisphere given below have the same radius and the height of the cylinder is also the same as its radius.



1. Predict how the volume of the hemisphere compares to the volume of the cylinder. Which one holds more volume?

2. If you fill the hemisphere with water or other filling material, predict what fraction of the cylinder is filled by the volume of one hemisphere.

3. Now try it, what fraction of the cylinder is filled by the volume of one hemisphere?

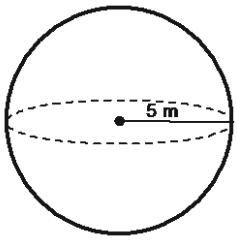
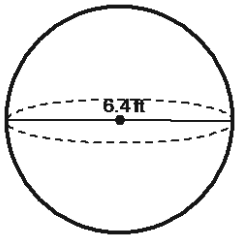
4. Write down the formula for the volume of the cylinder below the cylinder, be sure to write your height in terms of the radius or r.

5. Manipulate the equation for the volume of the cylinder to show the volume of the hemisphere.

6. In number 10 you found the volume for a hemisphere. Adjust this formula to find the volume of a sphere.

7. Explain in your own words how the volume of a sphere compares to the volume of a cylinder. Describe the parts of the formula for the volume of a sphere. Write this formula below the sphere in the picture on the previous page.

Directions: Find the volume for each sphere described below. If needed draw and label a picture.

8. 	9. 
10. A sphere with a radius of 1.3 yds.	11. A sphere with a diameter of 25 inches

Directions: Find the missing measurement for each sphere described below. Round your answer to the nearest tenth.

12. The volume of a sphere is 6882.3 in^3; find the diameter of the sphere.	13. The volume of a sphere is 1436.8 ft^3; find the radius of the sphere.

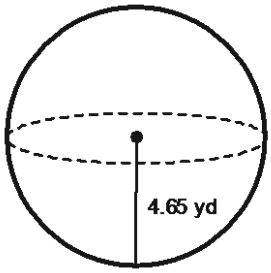
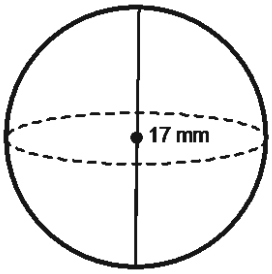
Directions: For each problem given below draw and label a picture that describes each sphere. Then solve the problem.

14. If a golf ball has a diameter of 4.3 centimeters and a tennis ball has a diameter of 6.9 centimeters, find the difference between the volumes of the two balls.

15. Kauri pours the water out of a cylindrical flower vase with a height of 5 inches and a radius of 4 inches into a spherical flower vase. The spherical vase has a radius of 4 inches. Will the water overflow? If so, by how much? If not, how much space is left in the spherical vase?

8.3d Homework: Volume of Spheres

Directions: Find the volume for each sphere described below. If needed draw and label a picture.

1. 	2. 
3. A sphere with a radius of 10 yards.	4. A sphere with a diameter of 60 inches

Directions: Find the missing measurement for each sphere described below.

5. The volume of a sphere is 113.1 cm^3; find the diameter of the sphere.	6. The volume of a sphere is 4,188.8 cubic feet; find the radius of the sphere.

Directions: For each problem given below draw and label a picture that describes each sphere. Then solve the problem.

7. The diameter of the moon is 3,476 kilometers. Approximate the volume of the moon.

8. Find the volume of the empty space in a cylindrical tube of three tennis balls. The diameter of each ball is 2.5 inches. The cylinder is 2.5 inches in diameter and is 7.5 inches tall.

8.3e Class Activity: Volume of Cylinders, Cones, and Spheres 

Task 1: Silos

A silo is a storage bin that is a cylinder with a hemisphere on top. A farmer has a silo with a base radius of 30 feet and a storage height of 100 feet. The "storage height" is the part which can be filled with grain - it is just the cylinder. A cubic foot of grain weighs 62 lbs.

a. Draw and label a picture of the silo

b. How many pounds of grain can the farmer store in the silo?

c. How high (including the hemispherical top) is the silo?

d. One thousand square feet of wheat produces 250 pounds of grain. The farmer's wheat field is 3,500 feet by 20,000 feet. Is the silo large enough to hold the grain? By how much? Explain your answer.

e. If the farmer decides to fill the silo all the way to the top of the hemisphere how many cubic feet of grain can he store?

Task 2: Snow Cones

A snow cone consists of a cone filled with flavored shaved ice topped with hemisphere of flavored shaved ice. The cone is 4 inches long and the top has a diameter of 3 inches.

a. Draw and label a picture of the snowcone.

b. How much shaved ice, in cubic inches, is there altogether?

c. If 6 cubic inches of flavored ice is equal to 1 ounce, how many ounces of shaved ice is that?

d. If one ounce of flavored shaved ice is 50 calories, how many calories will you consume if you eat this snow cone?

Task 3: Pipes

Which will carry the most water? Explain your answer.

- Two pipes each 100 cm tall. One with a 3 cm radius and the other with a 4 cm radius
- One pipe that is also 100 cm tall with a 5 cm radius.

Task 4: Fruit

A cantaloupe a diameter of 23 centimeters and a Clementine orange has a diameter of 7 centimeters. Predict how many times bigger the cantaloupe is than the orange. Then calculate the volume of each fruit to determine how many times bigger the cantaloupe is than the orange.

8.3e Homework: Volume of Cylinders, Cones, and Spheres

Task 1: Containers

A cylindrical glass 7 cm in diameter and 10 *cm* tall is filled with water to a height of 9 cm. If a ball 5 cm in diameter is dropped into the class and sinks to the bottom, will the water in the glass overflow? If it does overflow, how much water will be lost? Explain and justify your response.

Task 2: Ice Cream

Izzi's Ice Cream Shoppe is about to advertise giant spherical scoops of ice cream 8 cm in diameter! Izzi wants to be sure there is enough ice cream and wonders how many scoops can be obtained from each cylindrical container of ice cream. The containers are 20 cm in diameter and 26 cm tall.

a. Draw and label a picture of the ice cream containers and the scoop of ice cream.

b. Determine the number of scoops of ice cream one container will give her?

c. Ingrid purchases one of these famous giant scoops of ice cream but does not get to it fast enough and the ice cream melts! The radius of the cone and the ice cream (sphere) is 4 cm and the height of the cone is 10 cm. Will all of the melted ice cream fit inside the cone?

d. If it does fit, how much more ice cream will fit in the cone? If it doesn't fit, how many cubic centimeters of ice cream does she need to eat before it melts in order to make it fit?

8.3f Class Activity: Banana Splits

Materials: graph paper, string, rulers, pen or pencil, banana, ice cream scoop
Use any of the materials on your table to approximate the volume of your banana and one scoop of ice cream. Be prepared to show and explain all your methods and your results.

1. What is your estimate for the volume of the peeled banana (include units)? _____

   Show how you found this volume.

2. What is your estimate for the volume of one scoop of ice cream (include units)? _____

   Show how you found this volume.

3. Comment on each of the other groups' methods and results. Compare their strategies and their results to yours.

4. How do you think you could have a more accurate approximation for the volume of the banana?

5. How do you think you could have a more accurate approximation for the volume of the scoop of ice cream?

6. If you make your banana split sundae with one banana, 3 scoops of ice cream, and 2 Tbsp chocolate syrup, what will be the total volume of your sundae?
(Hint: 1 Tbsp $\approx$ 14.8 cm^3 and 1 $in^3 \approx$ 16.4 cm^3)

8.3g Self-Assessment: Section 8.3

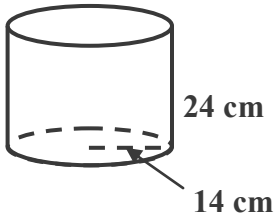
Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept.

Skill/Concept	Minimal Understanding 1	Partial Understanding 2	Sufficient Understanding 3	Substantial Understanding 4
1. Find the volume of a cylinder, cone, and sphere given a radius and height. *See sample problem #1*				
2. Find a missing measurement (height, radius, or diameter) for a cylinder, cone, or sphere given the volume. *See sample problem #2*				
3. Use the formulas for the volumes of cylinders, cones, and spheres to solve a variety of real-world problems. *See sample problem #3*				

Sample Problem #1

Find the volume for each object described below. Find the exact volume and the approximate volume rounded to the nearest hundredth.

a. The cylinder pictured below.



b. A cone with a radius of 3 ft and a height of 10 ft.

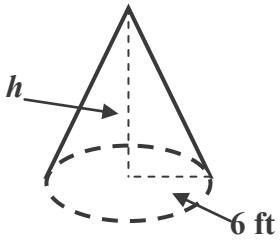
c. A glass tree ornament is a gold sphere. The diameter of the ornament is 4 inches.

Sample Problem #2

Find the missing measurement for each object desribed below. Draw and label a picture if needed.

a. The volume of a regular can of soda pop is approximatley 23.7 in^3. The height of the can is 4.83 inches. Find the diameter of the can.

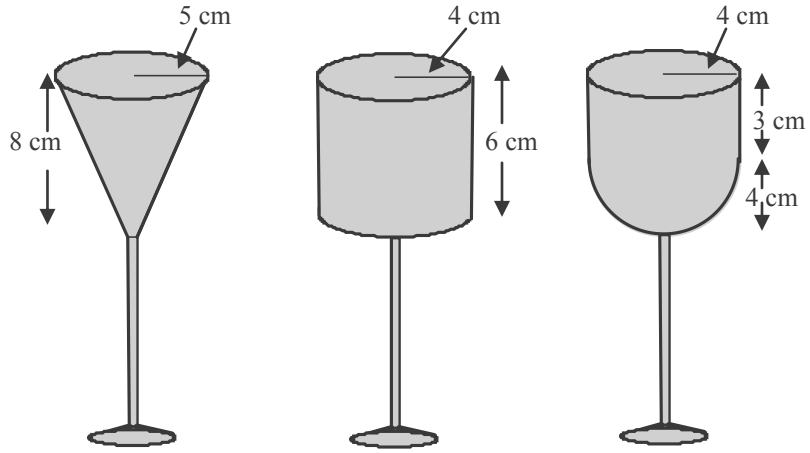
b. The volume of the cone below is approximatley 377 ft^3. Find the height of the cone.



c. A sphere has a volume of 113.1 mm^3. Find the radius of the sphere.

Sample Problem #3

Suzy is throwing a party and is choosing from the glasses below to serve her punch. Use the information below to answer the questions that follow.



Glass 1 Glass 2 Glass 3

- The shape of Glass 1 is a cone with a radius of 5 cm and a height of 8 cm.
- The shape of Glass 2 is a cylinder with a radius of 4 cm and a height of 6 cm.
- The shape of Glass 3 is a hemisphere with radius of 4 cm with a cylinder on top of it with a radius of 4 cm and a height of 3 cm.

a. Suzy wants to choose the glass that has the smallest volume so that she doesn't have to use as much punch. Find the volume of each glass to determine which glass she should choose.

b. Suzy really wants to use the cylinder shaped glass. What would the approximate height of the cylinder shaped glass need to be to hold the same amount of punch as the cone shaped glass. Would this be practical?