Heat engines operate by converting heat into work. Examples of heat engines abound - abound! Says the Physics Kahuna - all around us. Gasoline engines, jet engines, diesel engines, steam engines, Stirling engines, &tc. Just about anything that burns fuel to generate heat is a heat engine. You could be considered to be a low-tech heat engine (but a really nice one, I’m sure we all agree).

Heat is added to the engine at a high temperature. Part of the heat is used to generate work and the rest of the heat is sent to some low temperature environment.

Fancy terminology used for heat engines involves things like: high temperature heat reservoir (the source of the input heat), low temperature heat reservoir (where you send the heat you don’t use) – sometimes this is called the low temperature heat sink.

For a great many heat engines, the heat sink will be the atmosphere – think of a gasoline car engine. Stationary electric power plants might use water from a river. A steam ship would use the sea, and so on.

Most engines have a cyclic operation – steps that are repeated over and over in order to do the work. A car engine has pistons that repeatedly move up and down producing power from the motion of the piston.

**Efficiency:** The heat that is not converted to work is called the waste heat. All heat engines produce waste heat. The waste heat is sent to the heat sink. The first law tells us that the engine can never produce more work than the heat that went into it. After all, energy has to be conserved, correct?

The ratio of output work to input work/energy is called efficiency.

\[
e = \frac{W_{out}}{Q_{in}}
\]

This is written as \[
\frac{W}{Q_H}
\]

for the AP Physics Test.

This simply says that the efficiency is the absolute value of the ratio of output work to input heat.

The first law says that the efficiency cannot be greater than one. But we’ve already said that some of the input heat does not become work. Doesn’t this mean that the efficiency has to be less than one? Well, so it is.

This was discovered in 1824 by Sadi Carnot (actually Nicolas-Leonard-Sadi Carnot – he had a lot of names for a French physicist). If an engine couldn’t be one hundred percent efficient, what was the best you could do? Carnot set out to find the answer to that question. In his pursuit, he invented thermodynamics.
Carnot imagined the best possible engine, this would be an ideal engine – it wouldn’t have friction between its moving parts and it wouldn’t lose heat through the walls of its cylinders, etc. An ideal engine would have no change in its internal energy. For the engine:

\[ \Delta U = Q + W \quad \Delta U = 0 \quad \text{so} \quad Q = -W \]

The work done by this ideal engine is simply equal to the change in heat, \( \Delta Q \).

\[ W = Q_H - Q_C \]

\( Q_H \) is the high temperature and \( Q_C \) is the low temperature.

We plug this into the efficiency equation (for like one cycle):

\[ e = \frac{W \text{ done in one cycle}}{Q \text{ added in one cycle}} \]

\[ e = \frac{W}{Q_{in}} \]

Plug in value for work we figured out:

\[ e = \frac{W}{Q_{in}} = \frac{Q_H - Q_C}{Q_{in}} \]

But \( Q_{in} \) is simply \( Q_H \).

\[ e = \frac{Q_H - Q_C}{Q_H} \]

\( Q \) is proportional to \( T \) so:

\[ e = \frac{T_H - T_C}{T_H} \]

This is the efficiency of an ideal energy. It is the best efficiency that it is possible to obtain.

\[ e = \frac{T_H - T_C}{T_H} \]

**The temperatures must be in Kelvins!**

No real engine can achieve this efficiency, but it is possible to come close. The equation is important because it shows us that the efficiency depends on the two temperature extremes used by the engine. The higher the temperature the engine operates at and the lower the temperature of the heat reservoir, the higher the efficiency will be.
Diesel engines are more efficient than gasoline engines because they operate at a higher temperature.

It is expensive to build machines that can operate at high temperatures – you need costly metal alloys that can handle the temperature, the engine must be stronger because it is doing more work, so its components have to be beefed up, and so on. Engine designers have to balance tradeoffs like that between efficiency and manufacturing expense.

- A steam engine operates on a warm 28.0 °C day. The saturated steam operates at a temperature of 100.0 °C. What is the ideal efficiency for this engine?
An important thing to remember is that the temperatures in the equation must be in Kelvins.

\[ T_{\text{hot}} = 100.0^\circ \text{C} + 273 = 373 \text{ K} \]

\[ T_{\text{cool}} = 28.0^\circ \text{C} + 273 = 301 \text{ K} \]

\[ e = \frac{T_h - T_c}{T_h} \]

\[ e = \frac{373 \text{ K} - 301 \text{ K}}{373 \text{ K}} = 0.193 \]

The efficiency can be left as a decimal fraction or it can be expressed as a percent.

To increase the efficiency of a real engine, one can:

- Increase the temperature of the high temperature heat reservoir.
- Decrease the heat sink temperature.
- Do both.

The other thing that can be done is to try and make it behave as close to an ideal engine as possible. This can be done by eliminating friction, decreasing the mass of moving parts, &tc.

No real engine can have this efficiency. Eliminating friction and all the other good engineering things that can be done will help the engine approach the ideal efficiency. Unfortunately it will never reach this level of performance.

What a shame.

**PV Diagrams:** Pressure/volume graphs are of tremendous value in analyzing the performance of heat engines.

Let’s look at part of a thermodynamic cycle. We will look at the work for each step in the process.

Here is the first step - the process $a \to b$ represents an isobaric compression of a gas. From $a$ to $b$, the pressure remains constant – its value is $P_1$. The volume decrease from $V_1$ to $V_2$. This represents work done on the system. It takes work to compress the gas. The work is the area under the curve, which is conveniently shaded in so you can see it. The work is: \[ P_1 \Delta V. \]

\[ W = P \Delta V = P_1(V_1 - V_2) \]

Now let’s look at the next step in the cycle. You can see the PV diagram to the right.
Process $b \rightarrow c$ is an isochoric compression. Isochoric because the volume does not change but the pressure increases. The work for this step is zero. This is because $\Delta V$ is zero.

$$W = P\Delta V = P(0) = 0$$

The next step is process $c \rightarrow d$. This is an isobaric expansion. The gas expands from $V_1$ to $V_2$, doing work as it expands. The amount of work is equal to the area under the curve.

$$W = P\Delta V = P_2(V_2 - V_1)$$

The final step to complete the cycle is process $d \rightarrow a$. This is an isochoric expansion. The volume stays constant, so no work. The pressure decreases.

$$W = P\Delta V = P(0) = 0$$

The whole cycle looks like the graph to the right.

The net work done by the machine in one cycle is the area enclosed by the curve. It is the sum of the work, positive or negative, for each step in the cycle.

The work is just the difference in the two areas under the curve that we had.

$$W = P_2(V_2 - V_1) - P_1(V_1 - V_2)$$

The work done in a cycle depends on the path. Let’s look at the previous cycle and compare it with one that has a different path.
Cycle 1 is the one we just looked at. The gas is compressed from $V_2$ to $V_1$, compressed from $P_1$ to $P_2$. Then it is expanded from $V_1$ to $V_2$ and expanded from $P_2$ to $P_1$. Pretty much the same thing happens in cycle 2, except that the two expansions take place in only one step instead of two. This difference is significant however as the net work for cycle 1 is greater (twice the magnitude) than the net work for cycle 2.

- A heat engine’s cycle is shown in the PV diagram to the right. $P_1 = 345$ kPa, $P_2 = 245$ kPa, $P_3 = 125$ kPa, and $P_4 = 225$ kPa. $V_1 = 35.0$ L and $V_2 = 85.0$ L. What is the net work done during one cycle of the engine?

We can solve this by finding the area under the curve for the $a \rightarrow b$ step. Then we find the area under the curve for the $c \rightarrow d$ step. The net work is the difference in the two areas.

Find volume in m$^3$:

\[
V_1 = 35.0 \times \frac{10^{-3} m^3}{1 \text{ L}} = 35 \times 10^{-3} m^3 = 0.0350 m^3
\]

\[
V_2 = 85.0 \times \frac{10^{-3} m^3}{1 \text{ L}} = 0.0850 m^3
\]

Area for $a \rightarrow b$: \[W_{ab} = A_{ab} = A_{rec} + A_{tri}\]

\[
A_{rec} = P_2 (V_2 - V_1) = 245 000 Pa \left(0.085 m^3 - 0.035 m^3\right) = 12 250 J
\]

\[
A_{tri} = \frac{1}{2} (P_1 - P_2)(V_2 - V_1)
\]
\[ A_{Tri} = \frac{1}{2}(345\ 000\ Pa - 245\ 000\ Pa)(0.085\ m^3 - 0.035\ m^3) = 2500\ J \]

\[ W_{ab} = 12\ 250\ J + 2\ 500\ J = 14\ 750\ J \]

Area for \( c \rightarrow d \):
\[ W_{cd} = A_{cd} = A_{rec} + A_{Tri} \]

\[ A_{rec} = P_3(V_1 - V_2) = 125\ 000\ Pa\left(0.035\ m^3 - 0.085\ m^3\right) = -6\ 250\ J \]

\[ A_{Tri} = \frac{1}{2}(P_4 - P_3)(V_1 - V_2) \]

\[ A_{Tri} = \frac{1}{2}(225\ 000\ Pa - 125\ 000\ Pa)(0.035\ m^3 - 0.085\ m^3) = -2\ 500\ J \]

\[ W_{ab} = -6\ 250\ J - 2\ 500\ J = -8\ 750\ J \]

\[ W_{Net} = 12\ 250\ J - 6\ 250\ J = 6\ 000\ J = \boxed{6.00\ kJ} \]

- A substance undergoes a cyclic process shown in the graph. A work output occurs along path \( a \rightarrow b \), a work input is required along path \( b \rightarrow c \), and no work is involved in constant volume process \( c \rightarrow a \). Heat transfer occurs during each process in the cycle. (a) what is the work output during process \( a \rightarrow b \)? (b) how much work input is required during process \( b \rightarrow c \)? (c) What is the net work done during the cycle?

(a) \[ W_{ab} = A_{ab} = A_{rec} + A_{Tri} \]

\[ W = (1\ atm)(\frac{1.013 \times 10^5\ Pa}{1\ atm})(40\ \text{ml})(\frac{10^{-3}\ m^3}{1\ \text{ml}}) + \frac{1}{2}(4\ atm)(\frac{1.013 \times 10^5\ Pa}{1\ atm})(40\ \text{ml})(\frac{10^{-3}\ m^3}{1\ \text{ml}}) \]

\[ W = 122 \times 10^2\ J = \boxed{1.22 \times 10^4\ J} \] Expansion process, so negative work.

(b) This is positive work. \( W = \) area of rectangle

\[ W = -(1\ atm)(\frac{1.013 \times 10^5\ Pa}{1\ atm})(40\ \text{ml})(\frac{10^{-3}\ m^3}{1\ \text{ml}}) = -40.5 \times 10^2\ J = \boxed{-4.05 \times 10^3\ J} \]

(c) \[ W_{net} = W_{ab} + W_{bc} + W_{ca} \]
The Carnot Cycle: Sadi Carnot envisioned a perfect machine that would have the greatest possible efficiency that it could possibly have. We’ve already seen how the equation for this efficiency was developed. But what kind of machine could do that? Well, the machine that Carnot came up with is a simple piston/cylinder device. The operating sequence of the thing is called the Carnot cycle.

Here is the device (the drawing above). The sides and top of the cylinder are insulated so heat cannot flow in or out of the system. The bottom is made of an ideal conductor so that heat can flow in or out of the system through the bottom of the cylinder. Three stands are available for cylinder placement; a hot stand, a cold stand, and a perfect insulator stand. Let’s see how the cycle works.

Step 1: Isothermal expansion. The cylinder is placed on a high temperature heat sink that is at $T_H$. The heat is conducted through the bottom of the cylinder and the gas absorbs heat. We call this heat $Q_{in}$. As a result of the added heat, the gas expands, pushing the piston upward. This step does some work. We call this step isothermal expansion because the temperature stays constant (isothermal means constant temperature) and the volume increases. The work done is equal to $P\Delta V$. This step is represented as the curve $AB$ on the PV diagram below.

Step 2: Adiabatic expansion: The cylinder is immediately moved from the hot stand to the insulated stand. Once it is placed on the insulated stand, heat can no longer flow into the system (or out of it). The gas continues to expand, but since heat is no longer entering or leaving, this is an adiabatic expansion. The pressure in the cylinder drops to its lowest value. This is represented by the curve $BC$ on the PV diagram. Work continues to be done by the system during this step.

Step 3: Isothermal compression: The cylinder is immediately place on the low temperature heat sink. Heat, which we call $Q_{out}$, flows from the cylinder into the heat sink. The system loses
heat, the volume decreases, and the gas is compressed isothermally. This is represented by the curve $CD$ on the PV diagram.

**Step 4: Adiabatic compression:** The cylinder is placed on the insulated stand. Heat no longer can enter or leave, so the system undergoes adiabatic compression back to its original state along the curve $AD$. The heat engine is now ready to undergo another cycle.

Here is the PV diagram for the Carnot cycle.

![PV diagram for the Carnot cycle](image)

- $A$ to $B$ -- isothermal expansion.
  - $V$ increases and $P$ decreases
- $B$ to $C$ -- adiabatic expansion.
- $C$ to $D$ -- isothermal compression
- $D$ to $A$ -- adiabatic compression

Work is done along the curves $AB$ and $BC$.

The efficiency of the Carnot cycle, how well it operates, depends on the absorption of heat and the loss of heat in the respective steps of the cycle. One of the key factors that controls the flow of heat is the temperature difference. This would be the temperature of the hot stand and the cold stand. When placed on the hot stand, heat flows into the cylinder and it reaches (if left long enough) the same temperature as the hot stand. When placed on the cold stand, the temperature difference is equal to the hot temperature minus the cold temperature (of the cold stand).

If $\Delta T$ is increased, heat will flow faster and the machine will operate more efficiently.

So the higher the hot temperature reservoir (the hot stand), the greater the amount of heat absorbed by the system. Also, if we decrease the cold temperature reservoir, this too will increase the amount of heat that flows.

Increase the heat flow and you increase the efficiency of the system.
Dear Doctor Science,
You know those little birds filled with red stuff that bob up and down drinking from a glass of water? I can't make mine stop, even when I take away his water. He won't stop. Please help!
-- Joey Terry from Springfield, MO.

Dr. Science responds:
Those little birds are actually perpetual motion machines. Once you get them started, they'll never stop -- ever -- not until the sun winds down and our galaxy goes nova. Even then, somewhere in the imploded black holes that was once our solar system, little birds will be bob-bob-bobbing -- even in the noiseless vacuum of space. The "red stuff" inside these birds is actually a neutrino solution kept in place by a mysterious new force in the universe called hypercharge, which is a cross between a nuclear bond and superglue.
Fortunately, most of these birds are what science calls "broken" and will not bob at all. This is true, at least for most of us, who bought these little birds at airport gift shops and we can only thank the powers-that-be. Otherwise, they'd just give us the creeps. Bobbing, bobbing, bobbing through eternity, through entropy, bobbing, bobbing, bobbing, bobbing...

**Solitude** – Ella Wheeler Wilcox

Laugh, and the world laughs with you;
Weep, and you weep alone.
For the sad old earth must borrow its mirth,
But has trouble enough of its own.
Sing, and the hills will answer;
Sigh, it is lost on the air.
The echoes bound to a joyful sound,
But shrink from voicing care.

Rejoice, and men will seek you;
Grieve, and they turn and go.
They want full measure of all your pleasure,
But they do not need your woe.
Be glad, and your friends are many;
Be sad, and you lose them all.
There are none to decline your nectared wine,
But alone you must drink life’s gall.

Feast, and your halls are crowded;
Fast, and the world goes by.
Succeed and give, and it helps you live,
But no man can help you die.
There is room in the halls of pleasure
For a long and lordly train,
But one by one we must all file on
Through the narrow aisles of pain.